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Efficiency Prediction of Splash-Lubricated Bevel Gear Drives via Energy-Equivalent Spur Gear Transformation

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Abstract: Accurate efficiency prediction of splash-lubricated bevel gear drives is challenging due to the continuously varying pitch radius along the conical face width and the absence of closed-form analytical formulations tailored to conical geometries. This study presents a novel energy-equivalent transformation that converts a partially immersed bevel gear into an equivalent spur gear while preserving the hydrodynamic energy associated with oil churning. By defining the face width along the cone generatrix and deriving a radius-weighted integral representation of the immersed region, an equivalent spur diameter is obtained for both partial and full immersion conditions. The transformed geometry enables the direct application of established spur-gear loss models to evaluate meshing, bearing, seal, and splash lubrication losses within a unified framework. A rated-condition efficiency evaluation procedure is developed in which the load torque is determined from a consistent power balance equation, ensuring physical compatibility between transmitted power and total losses. The resulting formulation provides a geometrically rigorous and computationally efficient method for predicting total efficiency of bevel gear drives operating under oil-bath lubrication. The proposed approach bridges the gap between conical gear geometry and spur-based analytical loss modeling, offering a practical tool for efficiency estimation, preliminary design, and optimization of bevel gear transmissions in splash-lubricated environments.

Keywords: Bevel gear transmission, Splash lubrication, Churning losses, Energy-equivalent transformation, Gear efficiency prediction, Oil-bath lubrication, Rated efficiency analysis.

INTRODUCTION

The efficiency of geared transmissions is a critical factor in modern mechanical systems, directly influencing energy consumption, thermal behavior, reliability, and overall system performance. In automotive, aerospace, and industrial applications, even small improvements in gear efficiency can lead to significant reductions in power losses and operating costs. Consequently, accurate prediction of gear transmission efficiency has long been an important topic in mechanical engineering research.

Early investigations into gear efficiency primarily focused on load-dependent meshing losses. Anderson and Loewenthal [5] examined the influence of geometry and operating conditions on spur gear power loss, highlighting the role of sliding velocity and friction coefficient. Xu et al. [2] developed analytical models for predicting mechanical efficiency of parallel-axis gear pairs, incorporating contact mechanics and load distribution effects. Experimental studies on helical and planetary gear sets further quantified meshing efficiency under realistic operating conditions [1], [4]. For multistage gear trains, overall efficiency prediction methods were proposed by Kuria and Kihiu [3], emphasizing the cumulative influence of individual stage losses. Comprehensive treatments of gear loss mechanisms, including meshing, bearing, and seal losses, are summarized in classical references such as Jelaska [6].

While load-dependent losses have been extensively studied, load-independent losses-particularly those associated with splash lubrication-have gained increasing attention in recent years. Under oil-bath conditions, gears partially immersed in lubricant generate hydrodynamic drag due to oil churning, significantly affecting efficiency at moderate and high rotational speeds. Albers [9] investigated oil churning losses in gearboxes, emphasizing the importance of housing geometry and oil level. More recently, Laruelle et al. [7] conducted experimental investigations on churning losses of splash-lubricated spiral bevel gears in a rectangular sump, identifying strong dependence on immersion depth and Reynolds number. Their results demonstrated the existence of regime transitions and highlighted the influence of enclosure effects. Similarly, Hu et al. [8] employed computational fluid dynamics (CFD) to analyze churning power losses in a gearbox with spiral bevel gears, showing that speed scaling deviates from classical cubic assumptions and that housing configuration significantly affects oil flow patterns. Experimental and numerical analyses of oil distribution and churning behavior have also been reported by Mastrone et al. [10], further confirming the complexity of splash lubrication phenomena in practical gearboxes.

In recent years, Danh et al. [11] investigated the multi-objective optimization of a two-stage bevel helical gearbox with the dual goals of reducing gearbox bottom area and increasing gearbox efficiency. Their formulation explicitly decomposed total power loss into gear meshing, bearing, and seal components and employed analytical expressions for gear efficiency based on sliding mechanics and contact geometry (see Section 2.2, Eqs. (53)–(97) in [11]). Although the study successfully integrated Taguchi and grey relational analysis for parameter optimization, the efficiency model considered only mechanical losses and did not incorporate splash-induced churning losses under partial oil immersion. Consequently, while [11] provides a solid mechanical-efficiency framework for bevel helical transmissions, the influence of oil bath hydrodynamics on overall rated efficiency remains insufficiently addressed.

Despite these advances, efficiency prediction of splash-lubricated bevel gears remains non-trivial. Most analytical loss models are developed for spur or parallel-axis gears with constant pitch radius, whereas bevel gears exhibit a continuously varying radius along the cone generatrix, leading to non-uniform circumferential velocity and complex hydrodynamic behavior. As a result, analytical treatment of conical churning losses is limited, and existing approaches rely largely on empirical correlations or CFD simulations.

To address this issue, this study proposes an energy-equivalent spur transformation for partially immersed bevel gears. By preserving the hydrodynamic energy of the conical surface through a radius-weighted integral formulation, an equivalent spur diameter is derived, enabling direct application of established spur-based loss models within a unified framework. A rated-efficiency evaluation is then performed using a consistent power-balance formulation, ensuring compatibility between transmitted torque and total losses. The resulting method provides a practical analytical bridge between conical geometry and cylindrical loss modeling for bevel gear transmissions operating in oil-bath conditions.

TOTAL POWER LOSS AND EFFICIENCY FRAMEWORK

The efficiency of a gear transmission operating under splash lubrication is governed by the balance between transmitted power and total losses. For a steady-state operating condition, the power balance can be written as:

$$P_{\text{in}} = P_{\text{out}} + P_{\text{loss}} \tag{1}$$

where $P_{\text{in}}$is the input power, $P_{\text{out}}$is the output power, and $P_{\text{loss}}$is the total power loss.

The total loss is decomposed into four primary components:

$$P_{\text{loss}} = P_{\text{mesh}} + P_{\text{bearing}} + P_{\text{seal}} + P_{\text{ch}} \tag{2}$$

where P_{mesh} is the tooth meshing loss, P_{bearing} is the bearing friction loss, P_{seal} is the seal friction loss, and P_{ch} is the oil churning loss under splash lubrication.

The total efficiency of the transmission is therefore defined as:

$$\eta_{\text{total}} = \frac{P_{\text{out}}}{P_{\text{in}}} = 1 - \frac{P_{\text{loss}}}{P_{\text{in}}} \tag{3}$$

For rated-condition analysis, a specified output power P_{out} is imposed. The transmitted torque is then obtained by solving Equation (1) such that:

$$P_{\text{out,target}} = P_{\text{in}}(T_r) - P_{\text{loss}}(T_r) \tag{4}$$

where $T_r$denotes the rated load torque. This approach ensures physical consistency between transmitted power and all loss components.

MECHANICAL LOSS COMPONENTS

The first three components in Equation (2) are mechanical losses directly related to transmitted load and are evaluated using established formulations.

Meshing Loss (Buckingham-Based Formulation)

The meshing loss originates from sliding friction along the path of contact between mating involute teeth. Following Buckingham's analytical treatment [12], gear efficiency is derived from the work of friction during engagement. For spur gears, Buckingham gives the efficiency as:

$$\eta_{\text{mesh}} = 1 - \left(1+\frac{1}{u}\right) f \,\frac{\beta_a^2 + \beta_r^2}{2(\beta_a + \beta_r)} \tag{5}$$

where $u = \frac{N_2}{N_1}$ is the transmission ratio, f is the coefficient of friction, and $\beta_a$and $\beta_r$are the arcs of approach and recess, expressed in circular measure on the pitch circle. These factors can be calculated by [12]:

$$\beta_a = \frac{\left(R_{aev2}^2 - R_{0v2}^2\right)^{1/2} - R_{v2}\cdot\sin\alpha}{R_{01}} \tag{6}$$

$$\beta_r = \frac{\left(R_{aev1}^2 - R_{0v1}^2\right)^{1/2} - R_{v1}\cdot\sin\alpha}{R_{0v1}} \tag{7}$$

In which, R_{aev1} and R_{aev2} are outside radious of equivalent pinion and gear, respectively; R_{v1} and R_{v2} are pitch radius of equivalent pinion and gear, respectively; R_{0v1} and R_{0v2} are base radius of equivalent pinion and gear, respectively; α is pressure angle.

$$\mathrm{R_{v1}} = \mathrm{R_1}/\cos\delta_1 \tag{8}$$

$$\mathrm{R_{v2}} = \mathrm{R_2}/\cos\delta_2 \tag{9}$$

R1 and R2 are pitch radius of bevel pinion and gear at large end, respectively; δ_1 and δ_2 are pitch angle of bevel pinion and gears, respectively.

$$\mathrm{R_{aev1}} = \mathrm{R_{v1}} + \mathrm{a_p} \tag{10}$$

$$\mathrm{R_{aev2}} = \mathrm{R_{v2}} + \mathrm{a_g} \tag{11}$$

a_p and a_g is addendum of pinion and gear, respectively.

$$\mathrm{R_{0v1}} = \mathrm{R_{v1}} \cdot \cos\alpha \tag{12}$$

$$\mathrm{R_{0v2}} = \mathrm{R_{v2}} \cdot \cos\alpha \tag{13}$$

with α is pressure angle. The friction coefficient f can be determined as follows [13]:

- If v ≤ 0.424 (m/s),

$$\mathrm{f} = -0.0877 \cdot \mathrm{v} + 0.0525 \tag{14}$$

- If v > 0.424 (m/s),

$$\mathrm{f} = 0.0028 \cdot \mathrm{v} + 0.0104 \tag{15}$$

The meshing power loss is therefore:

$$P_{\text{mesh}} = P_{\text{in}}(1 - \eta_{\text{mesh}}) \tag{16}$$

Buckingham further demonstrates that bevel and spiral gears may be evaluated through equivalent cylindrical representations using the same meshing framework [12]. Thus, in the present work, the meshing efficiency of the bevel gear pair is evaluated using the spur-equivalent geometry derived later in Section 5.

Bearing and Seal Losses

Bearing losses are estimated using classical friction models based on bearing reaction forces [6]:

$$P_{\text{bearing}} = \sum_j f_b\, R_j \omega_j r_{s,j} \tag{17}$$

where, f_b is the bearing friction coefficient; R_j is the reaction force at bearing j; $r_{s,j}$ is the shaft radius; and ω_j is shaft angular velocity.

Seal losses are estimated through empirical correlations [6]:

$$P_{\mathrm{s}} = \textstyle\sum_{i=1}^{2} \mathrm{P_{si}} \tag{18}$$

Where i is the ordinal number of the seal (i = 1 ÷ 2) and P_{si} is calculated by

$$P_{si} = [145 - 1.6 \cdot t_{oil} + 350 \cdot log\, log(VG_{40} + 0.8)] \cdot d_s^2 \cdot n \cdot 10^{-7} \tag{19}$$

Where VG_{40} is the ISO VG number (or ISO Viscosity Grades number); d_s shaft diameter, and n is rotational speed.

CHURNING LOSS IN SPLASH-LUBRICATED BEVEL GEARS

Unlike meshing and bearing losses, splash-induced churning loss in bevel gears does not admit a simple closed-form analytical expression. For cylindrical spur gears, several models exist to estimate churning losses based on pitch-line velocity, immersion depth, and housing geometry. However, due to the continuously varying pitch radius along the conical face width of bevel gears, direct application of cylindrical churning models is not straightforward.

Experimental investigations of spiral bevel gears under splash lubrication have demonstrated strong dependence of churning losses on immersion depth and rotational speed [8]. CFD analyses of bevel gearboxes have further shown deviations from classical cubic speed scaling [9]. Nevertheless, a practical analytical formulation suitable for design-level efficiency prediction of partially immersed bevel gears remains limited.

To address this gap, the next section proposes a geometrically consistent energy-equivalent spur transformation that enables the use of established cylindrical splash-lubrication models while preserving the hydrodynamic energy characteristics of the bevel gear.

PROPOSED ENERGY-EQUIVALENT SPUR TRANSFORMATION FOR CHURNING LOSS

Motivation

As discussed in Section 4, direct analytical prediction of churning losses in splash-lubricated bevel gears is complicated by the continuously varying pitch radius along the conical face width. In contrast, cylindrical spur gears admit established splash-lubrication formulations that relate churning torque to pitch-line velocity, immersion depth, and housing geometry.

To bridge this gap, a geometrically rigorous transformation is proposed:

- The partially immersed bevel gear is replaced by an energy-equivalent spur gear such that the hydrodynamic energy associated with oil churning is preserved.
- This approach extends the equivalence principle used by Buckingham for meshing efficiency to the domain of splash-induced losses.

Geometry of Partial Immersion

Consider a bevel gear modeled as a truncated cone with the large-end pitch radius $r_1 = D_1/2$, the small-end pitch radius $r_2 = D_2/2$, and the face width $b$measured along the cone generatrix.

The coordinate system is defined along the cone generatrix as illustrated in Figure 1. The axial coordinate z is defined such that: $z = 0$ at the large end ($r = r_1$), and $z = b$ at the small end ($r = r_2$).

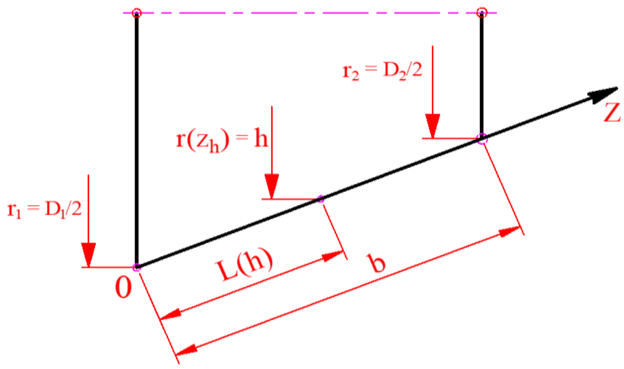

Fig.1: Geometric representation of a partially immersed bevel gear. The coordinate zis defined along the cone generatrix, with $\mathbf{z = 0}$ at the large end ($\mathbf{r_1 = D_1/2}$) and $\mathbf{z = b}$ at the small end ($\mathbf{r_2 = D_2/2}$). The oil level $\mathbf{h}$ determines the immersion boundary through $\mathbf{r(z_h) = h}$, and the immersed face width is denoted by $\mathbf{L(h)}$.

The pitch radius varies linearly along the cone generatrix:

$$r(z) = r_1 - \frac{r_1 - r_2}{b} z, 0 \le z \le b \tag{20}$$

where $r_1$is the large-end pitch radius, r_2 is the small-end pitch radius, and b is the face width measured along the generatrix. For a vertical oil level h, immersion begins at the large end. The immersed face width is:

$$L(h) = b\frac{r_1 - h}{r_1 - r_2} \tag{21}$$

with limits:

$$L(h) = \begin{cases} 0, & h \ge r_1 \\ b\dfrac{r_1 - h}{r_1 - r_2}, & r_2 \le h \le r_1 \\ b, & h \le r_2 \end{cases} \tag{22}$$

Hydrodynamic Energy Argument

Oil churning loss arises from viscous drag acting on the rotating gear surface. The local circumferential velocity is:

$$v_t(z) = \omega r(z) \tag{23}$$

Experimental and theoretical considerations indicate that splash-induced power dissipation scales approximately with the cube of local velocity. Therefore, the hydrodynamic energy contribution of the immersed region can be expressed as:

$$\Phi(h) = \int_0^{L(h)} r(z)^3 \, dz \tag{24}$$

Substituting Equation (9) and integrating analytically yields:

$$\Phi(h) = \frac{b}{4(r_1 - r_2)} (r_1^4 - h^4)(r_2 \le h \le r_1) \tag{25}$$

For full immersion ($h = r_2$):

$$\Phi_{\text{full}} = \frac{b}{4(r_1 - r_2)}(r_1^4 - r_2^4) \tag{26}$$

Equation (14) demonstrates that the large-end radius dominates churning behavior due to the fourth-power dependence.

Definition of Energy-Equivalent Spur Diameter

To enable the use of cylindrical splash-lubrication models, an equivalent spur gear is defined such that the energy-weighted integral of the bevel gear equals that of a spur gear of radius r_e and face width $L(h)$:

$$r_e(h)^3 L(h) = \Phi(h) \tag{27}$$

Solving for the equivalent radius:

$$r_e(h) = \left(\frac{\Phi(h)}{L(h)}\right)^{1/3} \tag{28}$$

and the equivalent diameter:

$$D_e(h) = 2r_e(h) \tag{29}$$

For full immersion:

$$D_e = 2\left(\frac{r_1^4 - r_2^4}{4(r_1 - r_2)}\right)^{1/3} \tag{30}$$

The equivalent spur gear thus preserves the hydrodynamic energy contribution of the partially immersed bevel gear at identical rotational speed.

Churning Loss Evaluation Using Equivalent Geometry

The bevel gear under partial immersion is transformed into an energy-equivalent spur gear characterized by:

$$D_e(h) = 2r_e(h) \tag{31}$$

where $r_e(h)$ is obtained from Equation (11). The pitch-line velocity becomes:

$$v_t = \omega r_e(h) \tag{32}$$

Churning torque under splash lubrication may be expressed in exponential form consistent with spur-gear splash models [6]:

$$T_H = C_{sp} C_1 \exp\left(\frac{v_t}{v_{t0}}\right) \tag{33}$$

where, C_{sp} is a splash coefficient depending on housing confinement; C_1 is an immersion-dependent geometric coefficient; v_{t0} is a reference velocity.

The corresponding churning power loss can be found by [6]:

$$P_{ch} = \sum_{i=1}^{k} T_{Hi} \cdot \frac{\pi \cdot n_i}{30} \tag{34}$$

Where k is the total number of gear pairs in the gearbox; n is the number of revolutions of the driven gear; T_{Hi} can be found by [6]:

$$T_{Hi} = C_{Spi} \cdot C_{1i} \cdot e^{\frac{C_{2i} \cdot v}{v_{t0}}} \tag{35}$$

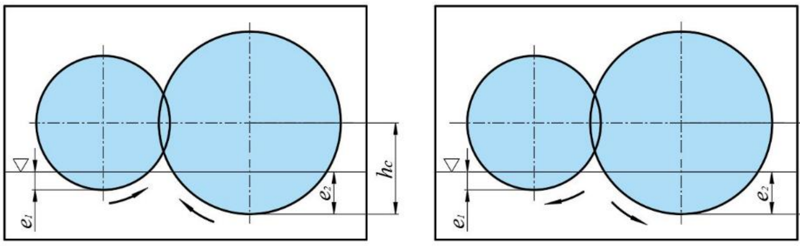

Fig.2: Schema for calculating lubrication factors.

In (35), $C_{Spi} = 1$ for stage 1 under the circumstance wherein the involved oil must pass the mesh, and for stage 2, C_{Spi} is computed using the following equation (Fig.2):

$$C_{Spi} = \left(\frac{4 \cdot e_{max}}{3 \cdot h_C}\right)^{1.5} \cdot \frac{2 \cdot h_{Ci}}{l_h} \tag{36}$$

where l_{hi} is determined by [6]:

$$\mathrm{l_{hi}} = (1.2 \div 2.0) \cdot \mathrm{d_{a2i}} \tag{37}$$

In (26), C_1 and C_2 are determined by [6]:

$$\mathrm{C_{1i}} = 0.063 \cdot \left(\frac{\mathrm{e_{1i}+e_{2i}}}{\mathrm{e_0}}\right) + 0.0128 \cdot \left(\frac{\mathrm{b_i}}{\mathrm{b_0}}\right) \tag{38}$$

$$\mathrm{C_{2i}} = \frac{\mathrm{e_{1i}+e_{2i}}}{80 \cdot \mathrm{e_0}} + 0.2 \tag{39}$$

In which $e_0 = b_0 = 10$ (mm).

Physical Interpretation and Theoretical Consistency

The proposed transformation:

1. Preserves the correct immersion sequence (large end immersed first).
2. Retains the dominant influence of large pitch radii.
3. Reduces conical churning prediction to a cylindrical problem.
4. Extends Buckingham's equivalence principle (originally used for meshing) to splash-induced hydrodynamic losses.

Thus, the transformation provides a unified mechanical and tribological framework for efficiency prediction of splash-lubricated bevel gear drives.

NUMERICAL EXAMPLE FOR TECHNICAL ILLUSTRATION

This subsection provides a numerical example to illustrate the application of the proposed efficiency framework, in which splash-induced churning losses are included in the total power-loss balance.

Operating conditions and material assumptions: The following rated inputs are adopted: Output torque: $T_{\text{out}} = 500000$Nmm; Input rotational speed: $n_{\text{in}} = 1450$rpm; Overall transmission ratio: $u_h = 15$; Gear material: steel, with allowable contact stress $[\sigma_H] = 420$MPa for both stages. Accordingly, the stage ratio is defined as $u_2 = u_h/u_1$, while the remaining geometric coefficients follow the design ranges used in the computational program.

Computational procedure: For each candidate value of u_1, the geometry of both stages is determined from the same set of relationships used throughout the manuscript (center distance, pitch diameters, face widths, and module standardization). Power loss components are then calculated by Equation (2) with the note that $P_{zo} = P_{zo1} + P_{zo2}$ represents the splash-induced churning loss of the two stages. The input power is computed from the transmitted torque and speed, and the loss ratio and total efficiency are reported as:

$$\text{Loss}(\%) = \frac{P_l}{P_{in}} \times 100, \eta(\%) = 100 - \text{Loss}(\%).$$

Table 1 summarizes representative results obtained using the proposed model, including the individual churning-loss contributions P_{zo1}, P_{zo2}, and total P_{zo}, together with P_{in}, P_{zp}, P_{lb}, P_s, P_l, Loss(%), and Efficiency(%).

Table 1: Total efficiency of the bevel gearbox including churning losses (proposed model).

u_1	P_{in} (kW)	P_{zp} (kW)	P_{lb} (kW)	P_s (kW)	P_{zo1} (kW)	P_{zo2} (kW)	P_{zo} (kW)	P_l (kW)	*Efficiency (%)*
1.76	*5.57*	*0.01*	*0.05*	*0.16*	*0.08*	*1.41*	*1.49*	*1.71*	*69.30*
2.07	*5.57*	*0.01*	*0.05*	*0.16*	*0.10*	*1.02*	*1.11*	*1.33*	*76.09*
2.38	*5.57*	*0.01*	*0.05*	*0.16*	*0.11*	*0.77*	*0.88*	*1.10*	*80.23*
2.69	*5.57*	*0.01*	*0.05*	*0.16*	*0.13*	*0.60*	*0.73*	*0.95*	*82.94*
3	*5.57*	*0.01*	*0.05*	*0.16*	*0.15*	*0.48*	*0.63*	*0.85*	*84.71*
4.49	*5.57*	*0.01*	*0.05*	*0.16*	*0.18*	*0.12*	*0.29*	*0.52*	*90.74*

Key Observation and Implication

The computed total efficiency varies substantially with the stage ratio allocation. In this example, the total efficiency changes from 69.3% to 90.74% as $u_1$increases from 1.76 to 4.49. This wide range indicates that the distribution of the overall ratio between stages significantly influences both mechanical losses and splash-induced churning losses. Therefore, it is necessary to formulate and solve an optimization problem to identify the most suitable design variables (notably $u_1$and related geometry coefficients) that maximize efficiency under the given constraints-this will be considered in further research.

CONCLUSIONS

This study proposed an analytical framework for predicting the efficiency of splash-lubricated bevel gear transmissions. Total power loss was evaluated through a consistent power-balance formulation, combining Buckingham-based meshing efficiency with bearing, seal, and splash-induced churning losses. A geometrically rigorous energy-equivalent spur transformation was introduced to model churning losses of partially immersed bevel gears, enabling the use of established cylindrical splash-loss formulations while preserving hydrodynamic consistency.

A numerical example ($T_{out} = 500000$ Nmm, $n_{in} = 1450$rpm, $u_h = 15$, $[\sigma_H] = 420$MPa) showed that total efficiency varies from 69.30% to 90.74% as $u_1$changes from 1.76 to 4.49. In the most unfavorable case, churning losses accounted for approximately 87% of total dissipation, highlighting the critical role of oil-bath hydrodynamics.

The proposed method provides a practical analytical tool for efficiency evaluation and supports future optimization of bevel gear transmissions operating under splash lubrication.

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