

Photon Not Massless: A Possible New Theorem

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ABSTRACT

As we all know, the photon(P), or *light particle*, is universally considered as a massless particle, in the sense that it is attributed a zero *restmass*, due to the fact that there is no P at rest, since its main characteristic is that it is always in motion, travelling at relativistic speeds. However, a fair portion of the scientific community admits that, being also a *quantum of energy*, when it is fired, the P can be considered as having a mass, which it would retain throughout its journey. It is like saying that the P has a *non-zero relativistic mass*, which can plausibly be considered as the *dynamic mass of P* which, in our opinion, would coincide with the *momentum(p)* of the P. It is the Bohr Complementarity Principle that would *hide* this mass when the P is in motion. For example, our calculations show that the *p-value* of an optic P is $p = 1.325 \cdot 10^{-22}$ [g· cm/s]. On the other hand, there are numerous mechanical effects induced by the possible mass carried by P. Indeed, the P produces a *recoil effect* in the atom that emits it, or deflects the path of the affected particle (*scattering effect*), as well as, by striking an atom, it can detach electrons from it (*photo-electric effect*), or it can even transform itself, alone or by colliding with another P, into a real and tangible mass. As is well known, this is the *materialisation of energy* whereby, from the collision between two photons (considered massless), an electron-positron pair is generated, as if from nothing, each having a *restmass* equal to 0.511 MeV!

Keywords: Photon (P); Electromagnetic Radiation (EMR); Photon Electric Effect (PEE); Planck constant (*h*); Mass-Energy Equivalence Principle (MEEP); *momentum (p)*

INTRODUCTION

As we all know, Planck and other eminent scientists had been racking their brains for years trying to discover the law of the spectral distribution of *black body radiation*, until the Meeting of the German Physical Society, in Berlin, on December 14, 1900.

On that occasion, the central assumption presented by Planck was the supposition (also known as the *Planck postulate*) that the electro-magnetic (EM) energy could only be emitted in *quantized* form; as to say that the energy could only be a multiple of an elementary energetic unit: the *Energieelement* [1].

It should also be borne in mind that it is equally well known that the photon (P) also has a corpuscular nature, which is amply documented by various practical or experimental evidences, of which we mention e.g. the photo-electric effect (PEE)[2],[3],[4], the Compton effect [5], or the Raman effect [6].

Just about the *corpuscular or particle nature* of P, Feynman told his Caltech students: "I want to emphasise that light comes in this form: particles. It is very important to know that light

behaves like particles, especially for those of you who have gone to school, where you were probably told something about light behaving like waves. I'm telling you the way it DOES behave: like particles. Light is made of particles. One could argue that it is the photomultiplier that reveals light in the form of particles. But no: every measurement made with any instrument (sensitive enough to detect very faint light) has always led to the same conclusion: light is made of particles. I have spoken of the photomultiplier in order to illustrate an essential aspect, which was perhaps not as well known to you, namely that light is made up of particles, but I hope that now there is no longer any doubt about this either"[7].

Thus, P travels like a wave, but also behaves like a particle.

In this regard, even before Feynman, Fermi had written: "In the present state of science it may be said that a theory that satisfactorily accounts for optical phenomena is lacking, for while the *wave theory* is perfectly suited to the explanation of interference, diffraction, polarisation and all that complex of phenomena that constitute the so-called classical optics, it appears insufficient to account for what happens every time the action of light on a single atom is examined. Thus, the *quantum theory of light* was constructed just to explain this last class of phenomena. Whenever an atom or molecule absorbs light energy, the amount of energy (E) that is absorbed cannot take just any value, but is related to the frequency (ν) of the light absorbed by the relation:

$$E = h \nu \quad (1)$$

where h is Planck's constant ($= 6.54 \times 10^{-27}$ erg sec). Thus, *light quanta* have a content of energy (E) directly proportional to frequency (ν)"[8].

As we now know, the exact value of h corresponds to $6.626 \cdot 10^{-27}$ [erg · sec].

Thus, in Planck's words, "Considering that - and this is the crucial point of the whole calculus - the energy, oscillator energy, is made of a defined number of finished and same parts; we can use to this purpose the natural constant $h = 6.55 \cdot 10^{-27}$ [erg·sec]. If this constant is multiplied for the normal oscillators' oscillating frequency, (ν), we get the *Energieelement*, ϵ , expressed in erg·sec" [1].

The following year, Planck further specified: "The essential point is to consider Energy, at each frequency, as made of a certain number of *energy elements* (*Energieelemente*), all equal to each other, indistinguishable and indivisible" [9].

As is well known, that represented by Eq. (1) is the famous *Planck-Einstein formula*, which shows us the possible energy value (E) carried by a *quantum of electromagnetic radiation* (EMR) [1],[9] or *light's quantum* [4].

With reference to the *Planck-Einstein formula*, it is interesting to quote from a Seminar by Rinaudo, who asks: 'What is there to understand in Planck's report $E = h \nu$? This *formula* links energy (E) to frequency (ν) with a proportionality coefficient h , or *quantum of action*, which is a natural constant.

Energy and frequency are concepts that belong to different phenomena, because *energy* is a *properties* that, in Classical Physics, we usually associate with a material body, well localised in space-time, to which, however, it is difficult to associate a *frequency*.

In turn, *frequency* is characteristic of a periodic phenomenon, with which, however, it is difficult to associate a well-localised *energy* in space-time"[10].

It must also be kept in mind that "in Classic Physics, *energy* often plays a marginal role (compared to other concepts), but in Quantum Mechanics *energy* is the crucial quantity: *energy* must be traced back to its role as a 'state variable' and, as such, a fundamental and unifying descriptor of phenomena.

Moreover, Planck's relationship *is revolutionary* because it expresses a link between energy and *frequency* of an electromagnetic field (EMF): ***energy is typical of material corpuscles***, while the *frequency* represents a characteristic property of a *wave*"[10].

Adds Rinaudo: 'According to Einstein's interpretation, the energy of an EMF is *quantised*. The value of the *elementary quantum of energy* (E), of frequency ν , will be: $E = h \nu$. The quantisation of the energy does not occur at the moment of interaction with matter, but it is EMF itself that behaves as a particle (the quantum of light, or photon), which simultaneously has corpuscular aspects (E) and undulators (ν)"[10].

Therefore, since "*energy is typical of material corpuscles*"[10] and is referred to as E to the 1st member of Eq. (1), a *material* must be represented in one of the two parameters placed at the 2nd member. Since the parameter ν indicates the oscillation frequency of the *quantum of action*"[1], we are left with the "*natural constant, h* " [1].

Therefore, for *Planck's formula* to be valid, it is the value of h that it must include, absolutely, a *mass* value: according to mathematical formalism, this is a *conditio sine qua non*.

And indeed, this is how it turns out, since the value of h is expressed by Planck himself in "erg·sec"[1]. As is well known, *erg* is measured in the basic units of the metric system *cgs* (centimetre per gram per second). Thus, *erg* is expressed in grams per square centimetre per square second, i.e.: $[g \cdot cm^2 / s^2]$.

And since Planck expressed the value of the "*natural constant, h* ," in *erg* [1], consequently h must be measured in grams x centimetres per second: exactly how the value of a massive particle is measured!

MATERIAL AND METHODS

Photo-Electric Effect (PEE)

As Fermi reminds us, the results of the study of the absorption of light energy by the matter can be summarised in the following Einstein's Law, which finds application in a very large number of phenomena; to mention only the main ones, we will mention the photo-electric effect (PEE), whereby a metal surface, when illuminated with light of sufficient frequency, emits electrons.

Well, it is found experimentally that the maximum kinetic energy (w) of emitted electrons is related to the frequency (ν) from the report:

$$w = h \nu - w_0 \quad (2)$$

where w_0 represents the low energy required to extract an electron from the metal. Therefore, in accordance with *Einstein's Law*, represented by Eq. (2), the total energy communicated to the electron by light ($h \nu$) is precisely equal to $w + w_0 = h \nu$ [8].

Of course, as Fermi points out, 'for photoelectric emission to occur, it is necessary that the energy communicated to the electron is sufficient to extract it from the metal, i.e. it must be $h \nu \geq w_0$. This is the interpretation of the fact that only light with a frequency (ν) above a certain limit is effective to produce PEE' [8].

As Asimov reminds us, 'At the beginning of last century, Lenard had discovered that when light hit certain metals it caused the emission of electrons from their surface, just as the light had the power to push out the electrons from the atoms' [11].

Well, we consider it appropriate and fundamental to dwell on this last sentence. Why?

For what this sentence of Asimov's first and foremost highlights is that a single *quantum of light*, considered to this day to be an object, a corpuscle, a *quantum object* (QO), but without mass, i.e. without any *intrinsic mass* of its own, not even infinitesimal *mass*, can have 'the power' [11] to move a particle such as the electron. It is truly astounding!

Indeed, according to Mechanics, the act of 'to push out' [11] an electron from its atom is first and foremost, and beyond doubt, a purely mechanical action which, moreover, as Einstein's Law dictates (see Eq.2), also requires a significant and sufficient impact force exerted by the incident particle in order for the phenomenon to take place.

Yet Planck himself said explicitly: "we can use to this purpose the natural constant $h = 6.55 \cdot 10^{-27}$, expressed in erg·sec" [1]. Thus, to make the accounts add up and brilliantly solve the enigma of *black body radiation*, while remaining in agreement with the experimental facts, Planck expresses the value of the constant h in erg per second, that is in $[g \cdot cm^2/s^2] \cdot s$

Therefore, considering the value of the constant h calculated by Planck, which is $6.55 \cdot 10^{-27}$ erg·sec,

we have:

$$h = 6.55 \cdot 10^{-27} [g \cdot \frac{cm^2}{s^2}] \cdot s \quad (3)$$

that is:

$$h = 6.55 \cdot 10^{-27} [g \cdot \frac{cm^2}{s}] \quad (4)$$

Thus, already with Planck we have that the value of h , i.e. *the quantum value of EMR* [1], or *Planck's grain*, or '*light's quantum*' [4], is given in grams!

In short, Planck's Communication of 1900 [1] is *doubly revolutionary* in that it presents the international scientific community for the first time with two absolute novelties: 1) EMR does not propagate as a continuous flow, as water normally does, but in many *discrete packets*, distinct and separate from each other; 2) the *quantum of EMR*, or *quantum of action* [1] is expressed in energy, but first and foremost in grams!

Yet, this second content of Planck's 1900 Report, which appeared perhaps too innovative in its most profound aspects, went unnoticed by the scientific community, which continued to follow and embrace, now as then, the widespread notion that *light particles*, the photons (P_s) are massless.

This is indeed a gross contradiction: a particle, such as the *quantum of light*, whose essence, its *soul*, is represented precisely by *Planck's constant*, which Planck himself expresses in *grams per centimetre per second*, is considered to be completely massless.

Well, we all know that the term *gram* is used to indicate the quantitative value of a mass, inherent in both an object of the macroscopic world and a particle of the microscopic world. And in fact, Eq. (4) indicates precisely the mass value that Planck attributes to the *natural constant* h .

Forget massless *light*!

Furthermore, a massless *quantum of light*, or photon (P), is also in open and paradoxical antithesis with the Special Theory of Relativity. Indeed, as is well known, the best-known expression churned out by Relativity Theory (as well as the most famous equation in the world) is the one indicating the *Mass-Energy Equivalence Principle* (MEEP) [12]:

$$E = m c^2 \quad (5)$$

In this respect, Einstein points out: 'From this formula it follows that, considering a particle at rest, mass and energy are essentially similar, i.e. they are just expressions of the same thing. The mass of a body is not a constant, but varies as its energy varies' [13]. Obviously, this also applies to the P : particle for all intents and purposes [7] so that, as the energy of a moving P varies, the value of the mass transported, if any, must also vary, as Einstein's MEEP shows [14].

On the Zero Mass Photon

Regarding the fact that when light travels through a medium other than *vacuum*, it slows down its speed significantly (in water, it loses about 1/3 of its speed), it is worth mentioning a very witty observation and physical explanation from a Mariotti seminar. She says: "We know that photons can behave like massive particles: this happens when they travel in a medium other than empty space. The physical reason for this is that the propagation of the electric and magnetic field (i.e. the photon) interacts with the medium. The resulting effect is the 'slowing

down' of the propagating wave, which is the equivalent of an effective mass for the propagating photon" [15].

So, as Mariotti explains, we have the physical effects of the actual behaviour of the photon (P) in front of our eyes every day: they look just like the typical effects of a not massless P!

The explanations, including mathematical explanations, that are commonly given in order to justify the belief that P can never possess mass, are not at all convincing from a physical, nor logical point of view! On the contrary, a completely massless P should not slow down at all in water, glass, or other media with refraction index, $n_o > 1$.

Thus, although considered as a massless object, *kinematically* P behaves like a real and *massive* particle, capable of inducing various well-known phenomena of a clearly mechanical nature (such as PEE, Compton and Raman effects, etc.).

Feynman points out: 'Newton thought that light was made of particles, which he called *corpuscles*, and he was right. Today we know that the light (meaning all the electromagnetic waves) is made of particles, because if we take a very sensitive tool, making a clicking when hit by the light, if we make the light dimmer, the intensity of every single click remains unchanged: they are just less frequent. Light is made of photons. We use the photomultiplier to detect a single photon. When the photon hits a small plate it causes the emission of an electron from one of the atoms of the plate' [7].

Hence, this shows the assumed *mechanical effect* determined even by a single photon (P), and able to hit a motionless electron and move it away [16]. Yet, the P is considered massless.

At this point, one wonders: how can a particle, without the least mass, have the same effect of a billiard small ball which hits and moves away the opposing ball? The latter is certainly bigger than the small ball, just as the electron compared to the P. It seems more likely that it is a body having a mass to move the electron from the metallic plate: it would be like saying that the P transports also a mass (a mass *equivalent to the energy* transported) [14].

To this purpose, we already saw these phenomena with the photoelectric effect (PEE). Other phenomena that highlight the corpuscular nature of light, and thus also that mechanical action exerted by light itself, are the well-known Compton effect [5,1923] and Raman effect [6], as well as the *recoil effect* (induced by P in the emitting atom).

In fact, it was the experimental evidence carried out by Compton [5] that supported definitively the even corpuscular nature of light, such as the particle nature of the photons (P_s).

Similarly, in other circumstances, and without our being able to explicitly discern the intimate mechanism underlying the phenomenon under consideration, a mechanical action, a *push effect*, induced by *electromagnetic radiation* (EMR) occurs again.

This occurs, for example, when we want to carry out a *measurement* in the subatomic world, or a microscopic observation: in these cases the P deviates the electron, in order to see it [17].

In other words, the observation of a phenomenon modifies the phenomenon itself, as it happens with the deviation of the observed electron. At this regard, Feynman writes: "To observe electrons, we need a light because the light rebounding on electrons make them visible. Nevertheless, the light affects the result because the result of *light on* is different from that of *light off*. We can say that the light affects electron behaviour. The electrons are very sensitive. When light is sent on an electron, it makes the electron vibrate so that the electron because of light, behaves in a different manner'[18].

In short, we can infer that the mechanical effects induced by photons (P_s), verified by now, is not the same for all, otherwise also a radio wave would move the electron, but it does not, since it does not have sufficient impact force to move it (direct consequence of the oscillation frequency of the P): it has a very weak mechanic effect on the electron, too weak to the purpose.

Hence, the higher the frequency of the EMR, and consequently the number of light's quanta travelling with the wave, the bigger the mechanical effect induced by the electromagnetic wave (EMW). A confirmation comes from the well known 'penetrating effect' of the γ and X P_s , which move electrons away from most of the objects they hit, unfortunately from organic molecules too [19].

To this purpose, Feynman adds: 'The freed electron is attracted by a second plate, with positive charge, hitting the plate the electron will free three or four more electrons' [7].

In this respect, it is important to emphasise that the P behaved just as the electron it freed, with the only difference that the P freed only one electron, whereas the electron frees three or four.

In brief, according to Feynman, there is only a quantitative difference between the mechanic power, or *push effect* of the electron and the P's. In other terms, there is not a qualitative difference, since they both behave in the same way: they are able to move electrons. That is, they have an analogous corpuscular behaviour, with analogous mechanical effects (or '*mass effects*'). There is only a difference in the intensity of the force exerted: much higher the electron's. "This will repeat as in a chain reaction, till the last plate is reached by billions of electrons, enough to make a measurable current. The latter sent to a loudspeaker will make an audible clicking. Every time a single P, of a certain colour, hits the photomultiplier, it is possible to hear a 'click' of uniform intensity. Besides, from experiments with photomultipliers acting at the same time around a luminous source, it was possible to ascertain that the light is not divided into 'half particles': that is the P is indivisible. Again: the light comes as particles. Light is made of particles. The light behaves as a flux of particles' [7].

Furthermore, we cannot overlook a very important concept, which is summarised in *Einstein's prediction*: The P *feels* the Gravity Interaction (GI).

To this end, Kane writes: 'Einstein's prediction, based on his General Relativity's Theory, says that the light *feels* the GI. Therefore, during a Sun eclipse it has been possible to observe that the light of far away stars, going pass the Sun, rather than spreading in a straight line, made a curved path towards the Sun itself' [20].

As is well known, in fact, Kane refers to the famous solar eclipse of 1919, the account of which we quote: 'The simplest interpretation of the deflection of the light beam is the one that considers it as an effect of the weight of light' [21], although the latter were already familiar with the concept of General Gravity, according to which a more or less significant gravitational source distorts the surrounding space-time closer together.

At the dinner of that Meeting, Lord Eddington read out some verses he had composed; we will quote the last quatrain: 'We will compare the measures taken, One thing at least is certain, Light has weight. One thing is certain and the rest debate. Light rays, when near the Sun, do not go straight' [22].

Thus, the "measures taken" by Dyson, Eddington and Davidson [21] are in full agreement with Einstein (the *P feels* the GI, confirming exactly his prediction of light deviation) and with our conviction that light carries with it also a mass (the *dynamic-mass* of *P*) [23]. In fact, Eddington pointed out: 'Light has weight' [22].

Someone might argue that it is the GI to bend the space around the star, so the light is bent too, appearing deviated. However, this phenomenon, as Newton said, happens only on bodies having a mass, thus the light, which is considered massless, should have gone straight on when close to the Sun, without undergoing the gravitational attraction [19].

In this respect, Hawking points out: 'Gravity is the force exerted between two particles of matter' [24]. Hence, the *P* may be energy, but in this case it is mass too.

Kane adds: "All particles, including the mediator particles of forces, *feel* the GI" [20]. Thus, the sensibility to GI of a massless particle, makes us think that the *P* might conceal an extremely small mass, impossible to catch directly, but *detectable* indirectly, as when it interacts with the matter, i.e. with another particle, or when it undergoes the gravitational attraction.

The same happens near a Black Hole: also in this case the *P* is attracted by the GI, till it is completely swallowed. Therefore, it is clear that the *P* is sensitive to the gravitational attraction [19]. Yet, a massless particle could not *feel* the gravitational field! Or it is not true that the *P* is completely massless.

At this regard, Chandrasekhar writes: 'It is the mass which (via the GI) gives weight to a particle and determines the way it moves when a force is applied to it' [25].

Hence, a force has to act on something material in order to move it.

Therefore, according to Newton [26], the GI should act on a mass. It may be useful to state that the mass determines the sensibility of a particle to the GI.

During the evolution period of the Universe, 'when finally, the radiation de-couples from the matter, the path of the photons is slightly diverted by the gravitational field' [27]: Sachs-Wolfe effect. Thus, if the path of photons (P_s) is diverted they should have a mass. In this case it means that there is an extremely small mass which goes with the energy of the *P*, making one body

[28]. This mass might be *concealed* during the motion of the P, in fact going along its path with relativistic speed it might add to its energy also a *kinetic energy*: this would allow it to *hide* more easily its probable mass.

The Photon's *Hidden Mass and Complementarity Principle*:

Since our P is in motion, we can say it travels as a wave, an EMW: in this case it shows its *undulation aspect* and *hides*, shall we say, its *corpuscular appearance and behaviour*. It could not do anything else! Why?

In order to respect the well known Bohr Complementarity Principle [29], according to which a particle can show itself only with one of two *aspects*: *undulation* or *corpuscular*. But always one at a time, never simultaneously!

In fact, these parameters are *complementary*, similarly to the *complementary parameters* of the Heisenberg's Uncertainty Principle (HUP) [30]: energy-time, or position and momentum of a particle. Thus, the more accuracy we have in knowing a parameter, the more uncertain the measure of the complementary corresponding parameter will be [31].

Hence, according to the Complementarity Principle, the more information we have about the *undulation aspect* of the P, the less, in the same instant, we have of its *corpuscular-particle aspect*.

Thus, in agreement with the Complementarity Principle, if the *Planck grain*, i.e. the P (or any other particle) is in motion we can catch its *kinetic energy*, adding it to its main base energy, but we will never be able to have news, simultaneously, about its *corpuscular* characteristics.

Hence, from the P in motion (wave-like aspect) we can have news about its energy, but we can never check its possible mass, since the mass is *hidden, masked* by the *wave behaviour* assumed by the moving quantum object, i.e. the P [32].

At this regard, in fact, what Feynman writes may be interesting: "The *momentum*, as a mechanical quantity, is difficult to *hide*. Nevertheless, the momentum (\mathbf{p}) can be *hidden* -in the electro-magnetic field, for example. This case is another effect of relativity" [18].

It is like saying that the *momentum* carries, albeit *hidden*, also a *dynamic-mass*, of which it manifests its *pushing effect*, i.e. a clear mechanical action only when it interacts with another particle [33]. Instead, as the Complementarity Principle dictates, the *quantum of action* of Planck, when in motion, can never show us its *corpuscular dress*, but always and only its *undulatory dress*!

It is a rule of nature, ingeniously intuited by Bohr [29].

On the contrary, according to Bohr, when the light's particle interacts, it slows almost completely its run, however without stopping completely: the HUP would not allow it to [30]. Thus, the P will cease to show its *wavelike appearance* and will show us its *corpuscular* one, allowing us to determine its mass (in case it has some!) [19].

To this purpose, Penrose points out: "The particle aspect of the wave-particle object shows itself only to the detector, when the *measurement* is finally performed. The *measurement* makes clear the holistic nature of the Wave Function of the *measured* particle, in the sense that the particle always appears and only at one point" [34].

Hence, only when the motion almost stops (and its *wave aspect* disappears) will the light quantum be able to show its *corpuscular aspect*. Only then, as a corpuscle, the P will show us, at last, its probable mass: maybe indirectly, showing us the probable mass-effects or mechanical effects [35].

In short, the P in motion corresponds to the *undulation aspect*, whereas the interacting P corresponds to its *corpuscular aspect*.

And yet, someone might say: it is wrong! The light quantum is always in motion, so it will never show us its *corpuscular appearance*, and thus its possible mass.

On the contrary, according to Penrose [34] and in agreement with the experimental evidence provided by particle detectors, it is very likely that the P *wears* its *corpuscular aspect*, only in the very brief instant in which it interacts.

In support of what we claim, there are several examples of *push-effect*, or mechanical effect induced by light quanta. To this purpose, it is possible to mention the photo-electric [3],[4], Compton [5] and Raman effects [6]: these are unequivocal experimental evidences of the corpuscular nature of P and its probable *mass-effect*.

The Photon's *Momentum*:

At this regard, Feynman writes: "That light carries energy we already know. We also know that the energy (E) of a light-particle is h (the Planck constant) times the frequency (ν): $E = h \nu$. We now understand that light also carries a *momentum* equal to the energy divided by c , so it is also true that these effective particles, these photons, carry a *momentum* (p):

$$p = \frac{E}{c} = \frac{h \nu}{c} \quad (6)$$

where c is the light speed in the *vacuum*. The direction of the *momentum* is, of course, the direction of propagation of the light"[18].

As it is known, the *momentum* was introduced in order to calculate how much a body in motion *weighs*. Newton was the first one to fully deal with this subject. In the first pages of 'Philosophiae Naturalis Principia Mathematica', Newton reported the following definition: '*Quantitas motus est mensura ejusdem orta ex Velocitate et quantitate Materiæ conjunctim*', that is, the *momentum* is a measure in itself, since it depends on both the speed and the quantity of matter [26].

The only mass or speed do not therefore describe what happens in real cases. Newton therefore defined this vector magnitude in the following way:

$$\vec{p} = m \cdot \vec{v} \quad (7)$$

The Eq. (7) describes the *quantity of motion* (p) of a body having a mass m and moving at a speed v .

Thus, the *momentum* of a particle is the product of 2 quantities, the particle's mass and its velocity.

In other words, the *linear momentum* (p) is a vector quantity: it has both magnitude and direction, and direction and line coincide with those of v . In fact, the vector p has the same direction and the same line of the speed v and its module is the mass times the speed module [32].

Furthermore, Rinaudo points out: 'As the energy, the role of *momentum* is also poorly used in Classic Physics. It is used mainly to solve problems dealing with the Momentum Conservation Law, and not as a descriptor of a characteristic property of the object in motion, such as it happens in Quantum Mechanics. Even the *momentum* has to be considered as a descriptor of a characteristic of the moving object, and not only of its interaction with other objects. In Classic Mechanics, the approach to motion has a Newtonian perspective, which ends up conditioning it.

Similarly, to *energy*, considered as a *state variable*, that is, something that the body (or the physical system) carries and characterises, the *momentum*, p , should also be seen as the true dynamic property describing the body in motion.

The *momentum* is much more important than *speed*, which instead describes the kinematic characteristic of body motion and which, in Quantum Mechanics, loses its meaning.

On the contrary, the *momentum* keeps its important role, also in Quantum Mechanics: p is the operator that describes the spatial variation of the Wave Function!"[10].

Moreover, as shown by Eq. (7), we find of particular value, as well as rich in meaning and potential, to point out that the *momentum* of a quantum object, i.e. a particle, is directly proportional to the mass of the object.

At this point, in truth, we feel compelled to make a reflection. Since 'the light also carries a *momentum*' [18], Eq. (7) obligatorily implies that to every photon (P), in some way, a mass must also be associated! This can no longer be denied: otherwise, as Eq. (7) shows, according to the most elementary rules of Arithmetic, the value of p would also cancel out!

In a nutshell, P, the *quantum of light*, with its *momentum* (p) is very likely to also carry a mass, and one of non-zero value: it should be a non-zero *relativistic mass*, that *dynamic mass*, that *hidden mass*, previously mentioned, probably *incorporated* and carried by Planck's constant (h). Yet this conclusion could have been reached as far back as 1900, when the *quantum of radiation* was expressed by Planck in erg ·second, i.e. in grams! [1].

As it is known, in Quantum Mechanics p is described by the *de Broglie formula* [36]:

$$p = \frac{h}{\lambda} \quad (8)$$

where λ is the wavelength of the considered particle and h indicates the Planck's constant.

In fact, taking inspiration from Einstein's intuitions, de Broglie proposed a similar process, in reverse, to be applied to particles. So, without experimental data, he suggested to give particles (including photons) the same property as waves. Hence, he gave each particle an its own wavelength, (λ), depending only on the *momentum* of the particle itself.

To this purpose, as Penrose reminds us, any particle with a *momentum* (p) seems to be something periodic, like a wave, with a universal relation between the wavelength of the particle, indicated by λ , and the modulus p of its *momentum* [34].

According to Weinberg, the mean wave length of a photon in the optical band corresponds to about $5 \cdot 10^{-5}$ [cm] [37] and in agreement with *de Broglie formula* its p is:

$$p = \frac{h}{\lambda} = \frac{6.626 \cdot 10^{-27} [\text{erg} \cdot \text{s}]}{5 \cdot 10^{-5} [\text{cm}]} \quad (9)$$

As the *erg* value is expressed in $[\text{g} \cdot \text{cm} / \text{s}^2 \cdot \text{cm}]$, that is in $[\text{g} \cdot \text{cm}^2 / \text{s}^2]$, we have:

$$P = \frac{6.626 \cdot 10^{-27} [\text{g} \cdot \frac{\text{cm}^2}{\text{s}}]}{5 \cdot 10^{-5} [\text{cm}]} \quad (10)$$

that is:

$$p = 1.325 \cdot 10^{-22} [\text{g} \cdot \frac{\text{cm}}{\text{s}}] \quad (11)$$

As shown by Eq. (11), it is clear that the *momentum* (p) of a visible photon (P), expressed in grams, should carry out a *hidden dynamic-mass*. Moreover, this *hidden dynamic-mass* carried by the *momentum* of an optic P is bigger than the *rest mass* of 100 protons [38].

No surprise! At this regard, indeed, Penrose says: 'The mass of photon is an impalpable type: it is pure energy' [39].

Mass-Energy Equivalence Principle (MEEP):

Another significant confirmation of the fact that energy is also associated with mass, in full agreement with Einstein's Mass-Energy Equivalence Principle (MEEP) [12], we read from Feynman: "*The mass of the object which is formed when two equal objects collide must be twice the mass of the objects which come together.* You might say, 'Yes, of course, that is the conservation of mass'. But not 'Yes, of course,' so easily, because *these masses have been enhanced* over the masses that they would be if they were standing still, yet they still contribute to the total mass (M), not the mass they have when standing still, but *more*. Astonishing as that

may seem, in order for the conservation of *momentum* to work when two objects come together, the mass that they form must be greater than the rest masses of the objects, even though the objects are at rest after the collision!"[18].

So, we have just read that the *momentum* of a particle in motion also increases its mass. As if to say that its *kinetic energy*, at the end of the run, remained stored in the particle in the form of mass! Since P, in addition to energy, 'also carries a *momentum*' [18], this must also apply to the particle of light: no more massless P!

Feynman goes on: "Suppose that our two equally massive objects that collide can still be 'seen' inside the total mass (M). Then, although we might at first expect the mass M to be $2m_o$, we have found that it is not $2m_o$, but $2m_w$. Since $2m_w$ is what is put in, but $2m_o$ are the rest masses of the things inside, the *excess* mass of the composite object is equal to the *Kinetic Energy* brought in. This means, of course, that *energy has inertia*"[18]: it would be a bit like saying, verisimilarly, that the energy of the particle, through its *momentum*, also incorporates, transports a mass, *inertial* or *dynamic* as it may be, and in full compliance with MEEP: $E=mc^2$ [12], as shown by Eq. (5).

Of course, this also applies to the particle of light, which is endowed with energy and *momentum*.

Feynman adds: 'It is still true that the mass is the total energy that has been put in. So, we see that the conservation of mass which we have deduced above is *equivalent* to the conservation of energy. Because of the *Kinetic Energy* involved in the collision, the resulting object will be *heavier*; therefore, it will be a different object. So, necessarily, the conservation of energy must go along with the conservation of *momentum* in the theory of relativity"[18].

The example given by Feynman represents a further and very authoritative confirmation of the *equivalence between mass and energy*. That is, we have seen that the addition of energy to a particle makes it '*heavier*', i.e. increases its mass!

This corresponds precisely to Planck's view that a heat transfer adds mass. In fact, Planck said: 'It seemed that a hot pot was heavier than a cold one, although exactly the same size'[40].

But the opposite can also happen, whereby a physical system, as it loses energy, in perfect accordance with MEEP, also loses mass, as might occur with Black Holes (BH_s). We discussed this at a Symposium held in Cambridge (Massachusetts) in 2010, where we presented a paper entitled "About the Specific Heat of Black Holes"[41].

In this regard, we cannot overlook the famous BH *evaporation* hypothesis proposed by Hawking. He writes: "It seems that any black hole will create and emit particles such as neutrinos or photons.... As a black hole emits this thermal radiation one would expect it to lose mass"[42].

It cannot be any clearer or simpler than this: the loss of 'thermal radiation', i.e. photons (P_s), over time causes the BH to lose mass in parallel!

It is a prestigious confirmation of the *mass-energy equivalence*, authoritatively, and without any hesitation, applied by Hawking to P as well, and of the no further sustainability, in our opinion, of the concept that P must be completely massless. A concept, moreover, that has never been demonstrated (except exclusively by means of arithmetical arbitrations). On the contrary, many experimental proofs, or practical applications in our daily lives, such as the Photo-Electric Effect (PEE) for example, clearly show *push effects*, i.e. real mechanical actions, albeit elicited by a particle considered massless.

But this would go against the primary rules of Mechanics, which only deals with phenomena that can be explained between massive physical systems.

Again, from the literature, you can read: 'By masses we also mean energies, as established Einstein through the famous equation $E=mc^2$ ' [43].

So, we ask ourselves once again: even from a purely logical point of view, how do you continue to maintain that a fully-fledged particle such as P, endowed with its energy and *momentum* values, is, however, devoid of even the slightest mass?!

It comforts us that Einstein himself felt the same way about this. In fact, Einstein wrote to his friend Conrad Habicht: "It has come to my mind a consequence of the study of Electrodynamics. The Principle of Relativity, in association with Maxwell fundamental equations, requires that the mass is a direct measure of the energy contained in a body; the *light carries a mass*" [40]. To this end, Galison specifies: 'Einstein was unsatisfied with the analyses of the light. Einstein stated that *to any kind of energy is associated a mass*' [40].

Thus, according to Einstein, there should be a mass associated with the photon (P), or *grain of Planck* [44]. In this respect, Galison adds: 'Planck stated that also the transfer of *heat* adds a mass' [40]. What is heat made of?

As we all know, *heat* is made of electromagnetic radiations (EMR_s), that is P_s. Thus, according to Planck, "a transfer of radiation, i.e. of P_s, from A to B will cause an increase in the mass of B. It was a new idea: in Newtonian physics there was nothing suggesting a variation in mass as a consequence of the energy" [40].

Thus wherever there is a body, or particle, having energy, there should be in a way (visible or *hidden*, concealed) a certain mass too, and *vice versa*: this is what comes from Eq. (5).

In fact, as Galison writes, "Einstein adds that based on the calculations of his article containing the equation $E=mc^2$, it emerges that a body that emits EMR_s necessarily loses mass" [40], just as would happen with the *evaporation* of Black Holes hypothesised by Hawking [42].

The Photon's Gravitational Mass:

Yet, despite the fact that the scientific community continues to consider the photon (P) to be completely massless, it is equally accepted by all that in the very first moments of the Universe's life, the P_s, i.e. the EMR, contributed significantly and consistently to making up the *gravitational mass* of the primordial Universe. This really seems like nonsense!

At this regard, Weinberg writes: "At the beginning of the history of universe, it was the total density of energy, of the various photons, electrons, positrons, etc. to provide the source of the gravitational fields of the universe" [37]. Therefore, if the P_s with their energy contribute to create the *gravitational field* of the primordial Universe, it may mean that P_s *hide, contain* a mass in their energy [19]. This would also be in perfect agreement with e.g. Penrose: "The mass of any body must receive a substantial contribution from the magnetic fields within it" [39].

Hence, not only are gravitational fields generated by the mass of the particles but by any form of energy too. At this regard, we read: 'Any particle having energy (mass) creates a gravitational field' [20].

In other words, if the energy rises the Gravity Interaction (GI) of a body (it doesn't matter if it is hot or cold), which moreover already emits a gravitational field, this should mean that the energy behaves like a mass: this might help explain why P_s contribute to enrich the source of a gravitational field. In this case the energy has behaved like a mass. This is another example of how the P may *hide* a mass under its energy [19].

Hence, an energetic particle, such as the P , should have a mass, otherwise it could not spread around a gravitational field: Newton's gravitational equation, indeed, includes only bodies having mass; on the contrary, it does not consider those having energy:

$$F = G \frac{m_1 m_2}{r^2} \quad (12)$$

where F is the force between the masses or Gravity Interaction (GI), G is the Newtonian gravitational constant ($=6.674 \times 10^{-11} \text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$), m_1 is the first mass, m_2 is the second mass, r is the distance between the centres of the masses [26].

Thus, no massless body or particle can undergo the effect of the GI, since if we multiply the mass of the first body times 0 (the mass of the second body, i.e. the massless P), the result will be 0: this means that the GI is null, it cannot be performed. In other words, the *Newtonian Universal Gravitation equation* itself imposes that each of the two bodies must have a mass, and a mass different from 0. This means that the GI could not act on massless P_s , and so it is much more likely that P_s carry a mass.

It cannot be excluded that, still talking about the Newtonian Law of Universal Gravitation, if a body or a particle feels of the GI, this involves that it carries a mass, or *subtends* a mass, even if in that instant it is in motion, showing only its undulation and energetic aspect.

In this way it is likely that, according to the Bohr's Complementarity Principle [29], the P in some cases shows the mass, in others the energy: it never shows both at the same time [19].

Furthermore, as Kane reminds us, "according to Einstein, in its gravitation theory (General Relativity), mass and energy are related, so any object having energy attracts other object gravitationally"[20]. Therefore, apparently the energy has a *gravitational effect*, that is an

action induced by a mass (namely the *equivalent mass*), a mass which should be intrinsic in that energy. It may be a very small mass, but it cannot be equal to 0!

Otherwise, as Einstein says, that energy could not have a gravitational effect. Why?

Because Newton's gravitational equation would not be satisfied: none of the two bodies in gravitational attraction will be able to have zero mass. A body with a zero mass would flee the GI, since the gravitational equation would be null. Consequently, we think that when a zero mass is applied to a charged particle, Einstein's Mass-Energy Equivalence Principle (MEEP) is broken!

Anti-Gravity Action Exerted by Photons:

From an authoritative source, we learn: 'In ordinary stars such as our Sun, the inward force of gravity is balanced by the outward hydrodynamic pressure of the hot gasses and, to a lesser extent, by the *radiation pressure of photons*' [45].

Thus, P_s contribute to counter-balance the huge gravitational pressure which pushes from the outward external layers of the star to the internal layers.

In short, the most frequent and widespread context in which this *Radiation Counter-Pressure* operates, is represented by a trial of strength that goes on uninterruptedly in the depths of the stellar *cores* between GI and P_s . That is, two opposing forces in an *arm wrestling*: the gravity (GI) on one side and the anti-gravity action exerted by P_s , or *photonic counter-pressure*, on the other [44].

Thus, these opposing forces can *fight* for a long time as it happens in the star's core.

At this regard, Margherita Hack points out: "The first force tends to compress the stellar gas until the *Photonic Pressure* reverses the motion, producing elastic oscillations" [46, Hack,164].

So, one may wonder: why, and according to what mechanisms, the '*Photonic Pressure* reverses the motion' [46] blocking the further gas compression?

In our opinion, in agreement with the *radiation compressibility limit* [18], it is the further *incompressibility* of the particles constituting this very hot photon gas, this plasma, to block the compression exerted by the GI.

To this purpose, let's analyse with Feynman the Compressibility (C) of the electromagnetic radiation (EMR): "We may give one example of the kinetic theory of a gas, one which is not used in Chemistry so much, but is used in Astronomy. We have a large number of photons in a box in which the temperature is very high. The box is, of course, the gas in a very hot star. The sun is not hot enough; there are still many atoms, but at still higher temperatures in certain very hot stars, we may neglect the atoms and suppose that the only objects that we have in the box are photons. Now then, a photon has a certain *momentum*, p , which is a vector. This p is the x-component of the vector p which generates the kick, and twice the x-component of the vector p ($2p_x$) is the momentum which is given in the kick. Thus, we find that the Pressure (P) is:

$$P = 2n \mathbf{p}_x v_x \quad (13)$$

where n is the number of atoms in the volume V , and v_x indicates the number of collisions, that is $n = N/V$ (N is the total number of atoms). Then, in the averaging, it becomes n times the average of $p_x v_x$ (the same factor of 2) and, finally, putting in the other two directions, we find:

$$P V = \frac{N \langle \mathbf{p} \cdot \mathbf{v} \rangle}{3} \quad (14)$$

That is the pressure times the volume is the total number of atoms (N) times $1/3$ ($\mathbf{p} \cdot \mathbf{v}$), averaged.

Now, for photons, what is $\mathbf{p} \cdot \mathbf{v}$? The *momentum* (p) and the velocity (v) are in the same directions, and v is the speed of light, so this is the *momentum* of each of the objects, times the speed of light. The *momentum* times the speed of light of every photon is its energy (E): $E = p c$, so these terms are the energies of each of the photons, and we should, of course, take an average energy, times the numbers of photons. So, we have $1/3$ of the energy inside the gas:

$$P V = \frac{U}{3} \text{ (photon gas)} \quad (15)$$

where U is the total energy of a monoatomic gas. U is equal to a number of atoms times the average kinetic energy of each.

For photons, then, we have discovered that the radiation in a box obeys to the law:

$$P V^{4/3} = C \quad (16)$$

where V is the volume and P is the Pressure of the photonic gas.

So we know the Compressibility (C) of radiation! That is what is used in an analysis of the contribution of *radiation pressure* in a star, that is how we calculate it, and how it changes when we compress it. What wonderful things are already within our power!"[18].

At this point, we must make a reflection: the latter Feynman's equation gives us a *limit*, beyond which the EMR cannot be further compressed. And why? Radiation is energy, let's say it is ethereal, it is made up entirely of photons (P_s), i.e. massless particles.

Furthermore, like all bosons, there can be a large number of P_s , even in a very limited space, since the Pauli's Exclusion Principle does not act on bosons [47].

So, Eq. (16) should have almost no limit at all. Instead it is not so: but then, what's underneath? For us, the possible photonic equivalent-mass places a limit on the *compressibility of radiation* [44].

Without considering the frequency of the involved P , indeed, it still remains its h which, as is known, is not zero, but $6.626 \cdot 10^{-27}$ [erg · s], as clearly shown by Eq. (10).

Likely, when in a stellar photonic gas the density becomes excessive, and the spaces between the particles are extremely reduced, the incompressibility of the radiation and the *consistency* of P_s come out. In fact, let's keep it in mind, P_s are also corpuscles, granules of Planck [44]. Hence, a repulsive action takes over, i.e. that repulsive, anti-gravitational force represented by the *Photonic Counter-Pressure* [28].

RESULTS

It should also be pointed out that the above is just the value of Planck's constant, not counting the number of oscillations per second. Yet, as can easily be seen, this value is by no means 0.

It is also important to note that the value of h was expressed by Planck in *erg* per second [1], i.e. in *grams*. And it is a positive value. So much for massless P !

In this regard, we read: 'It is the common experience of drivers that it is much easier to brake the car when travelling at low speed compared to when the car is travelling at high speed. At the same speed, however, it is much easier to brake a small car than a lorry. These simple observations allow us to state that we can define a certain physical quantity, called *momentum*, which in a certain way represents the *inertia of motion*' [48].

It is important to bear in mind that, also in accordance with Galilean Relativity, the term *inertia* implies a value of *mass*! That is, motion itself indicates a body or particle in motion, i.e. it must always be *something* moving, and not nothing.

One could object: in the case of P , it is its energy package that moves! Yes, but in that case, as Einstein's Special Relativity dictates, this energy must somehow correspond to a proportional *equivalent-mass*! So, in one way or another, this *inertia* must unequivocally be represented by *something* real, physical, i.e. a positive value, probably carried by P with its *momentum*. As Eq. (8) shows, $p=h/\lambda$, Einstein's MEEP ($E=mc^2$) also applies to P (since P has its own *momentum*) so that, if in motion it can only show us its wave-like appearance (as dictated by Bohr's Complementarity Principle), likewise the P in motion will also carry a positive value of mass, exactly *equivalent* to its energy value!

In fact, reading on: 'The car at high speed has a greater momentum than when travelling slower, while whatever speed a car and a lorry travel at, the latter has a much greater *momentum* than the former.

In mathematical terms, the *momentum* of a body, denoted by p , is defined as the product of the mass (m) of the body multiplied by its velocity (v): $p = m v$.

From the definition, it is understood that *momentum* is a vector, instant by instant parallel to the velocity vector and always having the same direction as it (m is always positive)"[48]: therefore is also referred to as *linear momentum*.

In summary, it is of great importance and scientific significance to always bear in mind that Planck's constant, h , represents Planck's *grain*, i.e. the *soul* of the P .

A common P in the optical band, as shown by Eq. (11), has a *momentum*, $p = 1.325 \cdot 10^{-22}$ [g·cm/s], i.e. it has such an impact force that it is capable of pushing an electron away from a metal surface, as Lenard clearly demonstrated with his experiments [3], later substantiated by Einstein [4].

About the extreme importance of the role played by h , and its actual physical value, Barroww writes: "The non-null value of Planck constant (h) is important for the *stability of matter*. In the impacts between the atoms and the electro-magnetic radiation, the value of h is large enough to take a rather strong 'stroke' to push the electrons to the immediately higher permissible level" [49]. And yet, a massless P is capable of inferring such a stroke, besides giving "stability to matter" [49]! Unless the P is not so massless.

In short, the Planck's constant is a real value, represented by an *intrinsic* value, a "natural constant", quoting the Planck's words [1], which therefore can never represent a null value (in full agreement with Barrow [49]).

Hence, h expresses the value of the density of energy- (equivalent mass) of the *Planck quantum of action*.

Thus, reducing the h value to zero, in full disagreement with Planck and Einstein's MEEP, but in order to be in accordance with *the gauge theories* [50], is not applicable to the Planck constant and would also cancel the existence of the P_s and, consequently, also the energy of light.

In this circumstance, we would have a world everywhere dark and totally devoid of power! No, it is not possible [51], nor further acceptable or conceivable.

Photon's Rest-Mass

In this regard, let us consider Eq. (5), $E=mc^2$, which as we know expresses the Mass-Energy Equivalence Principle (MEEP). As Galison reminds us, that's how Einstein commented upon his MEEP: "The value of the considered mass refers to the value of an *inertial mass*" [40].

Let's now apply the Eq. (5) to the photon (P), keeping in mind that one of the three parameters are well known, that is c , the speed of light in *vacuum*, corresponding to 299792.458 (± 0.4) Km/sec [52]. The 2nd parameter, E , is the *energy* of the P, which is described by Eq. (1): $E=h \nu$, where h is the Planck's constant, corresponding to $6.626 \cdot 10^{-27}$ [erg·sec] and ν indicates the frequency of oscillation (10^n) of the P considered, where n indicates the number of oscillations per second [c/s] [53].

If we want to consider the *energy* of the P in its *inert state*, indicated with E_o , we should have:

$$E_o = h \cdot \nu = h \cdot 10^n \text{ [c/s]} \quad (17)$$

$$E_o = 6.626 \cdot 10^{-27} \text{ [erg·s]} \cdot 10^n \text{ [c/s]} \quad (18)$$

$$E_o = 6.626 \cdot 10^{-27+n} \text{ [erg]} \quad (19)$$

Moreover, as the *erg* is expressed in $[g \cdot cm/s^2 \cdot cm]$, that is in $[g \cdot cm^2/s^2]$, we have:

$$E_o = 6.626 \cdot 10^{-27+n} [g \cdot cm^2/s^2] \quad (20)$$

This should be the Energy value of a P at an *inert state*.

Thus, in the case of a P at the *inertial state*, that is when it interacts with another particle, so it stops running, at least for that infinitesimal moment it will oscillate much less. However, we will never be able to know how much! That is, we will never be able to know with *accuracy* how much an interacting P can oscillate, i.e. what could be the number of oscillations $[c/s]$ in that instant [17].

In short, the P stops running when hitting another particle, so it will not oscillate as when it was running, though it never stops running completely: it is the *Heisemberg Uncertainty Principle* (HUP) to deny it, since in this case we would know simultaneously the *position* and the *momentum* of the P [30]. Therefore, it is the Quantum Mechanics to avoid a P with $v = 0$, since a P which does not oscillate is a motionless P, and in this case we would know simultaneously *two complementary parameters* of the same particle: its position and *momentum*.

Thus, also in the *inertial state* the oscillating frequency (ν) of the P can never be 0, but always $\geq 1/s$, that is \geq one oscillation per second (if not even $\frac{1}{2}$ oscillation per s., or a fraction of its).

Thus, from Eq. (20) we can have information, with a certain approximation, about the 2nd parameter of Eq. (5), referred to the P. Hence, we can easily have the 3rd parameter, the *equivalent rest-mass* or *equivalent inertial mass* (m_o) of the P:

$$m_o = \frac{E_o}{c^2} = \frac{6.626 \cdot 10^{-27+n} [g \cdot \frac{cm^2}{s^2}]}{(2.9979 \cdot 10^{10})^2 [cm/s]^2} \quad (21)$$

$$m_o = \frac{6.626 \cdot 10^{-27+n}}{(2.9979)^2} \cdot 10^{-20} \cdot \frac{[g \cdot \frac{cm^2}{s^2}]}{\frac{cm^2}{s^2}} \quad (22)$$

$$m_o = \frac{6.626}{(2.9979)^2} \cdot 10^{-27-20+n} \cdot \left[g \cdot \frac{cm^2}{s^2} \right] \cdot \frac{s^2}{cm^2} \quad (23)$$

$$m_o = \frac{6.626}{(2.9979)^2} \cdot 10^{-47+n} [g] \quad (24)$$

and we have:

$$m_o = 7.372 \cdot 10^{-48+n} [g] \quad (25)$$

What we get is that the *inertial mass*, or *rest mass* of the P corresponds to 10^{-48+n} grams.

Hence, if the value of n was 10^0 , that is one oscillation per second, m_o would be $10^{-48}[g]$.

Whereas if n was 10^3 oscillation per second, we would have $m_o = 10^{-45}$ [g]. Of course, in all cases it is an extremely small value, but it is $\neq 0$, in agreement with Quantum Mechanics, i.e. according to HUP [30, Heisenberg] and *Zero Point Motion* [25], [54.].

Photon's Rest-Mass Measured by Famous Physicists

Of course, we were not the only ones who tried to measure or calculate the possible values of the *photon's rest-mass*, or the values of the possible mass carried by a single P, since we have been preceded over the years by a multitude of prestigious Physicists! They indicated a limit to the photon rest-mass, m_γ , the value of which they expressed in grams (g).

In order not to go into too much detail, we will just report the results of their research, without going into the details of the specific methodologies used by each.

For the various authors, the photon (P) under study is that of the optical band.

To the best of our knowledge, the first research on this subject dates back to 1769, with Robison, who pointed out a limit to the *photon's rest-mass* (m_γ), corresponding to $\leq 4 \cdot 10^{-40}$ [g] [55]. Regarding this author, it is interesting to note that "In 1769, he announced that balls with like electrical charges repel each other with a force that varies as the inverse-square of the distance between them, anticipating Coulomb's law of 1785" [56].

A few years later, in 1773, an attempt to quantify the value of the P-mass was made by Cavendish, whose famous Physics laboratories at Cambridge University are named after him. Cavendish's research revealed a limit to the *photon rest mass*, m_γ , of 10^{-40} [g] [57]. Some ten years later, Coulomb tried this and came to the conclusion that the value of m_γ should correspond to $\sim 10^{-39}$ [g] [58], i.e. a result roughly superimposable on those of Cavendish and Robinson.

Just a century after Cavendish, it was the turn of Maxwell, whose research revealed that *photon rest-mass* would correspond to the value of 10^{-41} [g] [59].

Then we have the careful tests carried out in 1936 by Plimpton and Lawton, from which a *photon rest mass* of $3.4 \cdot 10^{-44}$ [g] [60].

About 30 years later, Cochran and Franken tried this, according to whom the possible *photon rest mass* would correspond to $3 \cdot 10^{-45}$ grams [61].

Bartlett's *team* came to rather similar results in 1970, whose research showed a *photon rest mass* (m_γ) of $3 \cdot 10^{-46}$ [g] [62].

The following year, Williams' *team* found values for m_γ corresponding to $1.6 \cdot 10^{-47}$ [g] [63].

Subsequently, research by Crandall (1983) showed a *photon rest mass* $\leq 8 \cdot 10^{-48}$ [g] [64], values perfectly coinciding with those calculated by us: see Eq. (25).

In turn, in 1985, Ryan's *team* showed values of m_γ roughly corresponding to 10^{-42} [g] [65]. In the same year, Fulcher found values that were slightly different from these, but completely the same as those found in the research carried out by Williams' *team*, i.e. $1.6 \cdot 10^{-47}$ [g] [66].

Finally, one cannot overlook what emerges in this regard from Penrose's masterful volume, 'The Road to Reality', in which he writes: 'The mass of the photon, if not 0, should be $<10^{-20}$ electronic masses for good observational motives'[34].

As it is known, the mass of the electron is $9.1 \cdot 10^{-28}$ grams, so if the photon (P) is $<10^{-20}$ electronic masses, we have: $9.1 \cdot 10^{-28-20}$ [g]. Thus, in agreement with Penrose a P which is not massless must have a mass very close to $< 9.1 \cdot 10^{-48}$ [g].

Penrose's calculations, among the greatest living mathematicians (and Nobel Prize in Physics, 2020), are completely superimposable on ours: $7.372 \cdot 10^{-48}$ [g], as shown in Eq. (25).

This is of great honour for us and greatly comforts us.

Possible New Theorem

As Stewart reminds us, regarding Euclid's '*Elements*', the best way to understand the *Elements* is perhaps to consider them as a study of the *Logic* of spatial relations: if a shape has certain properties, these may *logically* imply other properties. For example, if a triangle has all three sides equal (an equilateral triangle), then the three angles must all be equal.

This type of *statement*, which involves establishing certain hypotheses and then deducing their *logical consequences*, is called *Theorem*. This particular theorem relates a property of the sides of a triangle to a property of its angles (a less intuitive and more famous example is the Pythagorean theorem).

For modern mathematicians, the most interesting aspect of Euclid's *geometry* is not the content, but the *logical structure*. Unlike his predecessors, Euclid does not simply state that a certain theorem is true, but provides a *proof* of it [67].

Then Stewart clarifies Euclid's concepts in more detail: 'What is a *proof*? It is a kind of mathematical account, in which each step is a *logical consequence* of some of the preceding steps. Each *statement* must be motivated in relation to the preceding ones, and *demonstrated* as a *logical consequence* of them. Euclid understood that this process could not go back indefinitely: it had to have a *beginning*, with certain *starting statements*, which could not themselves be proved'[67].

In this respect, Euclid started with a list of *definitions*: clear and precise statements in about the meaning of certain technical terms, such as line or circle. A typical *definition* is: an Obtuse angle is an angle greater than a right angle.

Definitions provided Euclid with the terminology he needed to expound his undemonstrated *statements*, which he classified into two types: *common notions* and *postulates*.

A typical *common notion* is: things that are equal to the same thing, are equal to each other, so if A is equal to B, and B=C, then A will also be equal to C.

A typical *postulate* is: all right angles are equal to each other.

Today, we would put these two types of *statements* together to speak overall of *axioms*: the axioms of a mathematical system are the fundamental assumptions we make about it, they are the rules of the game, and we are careful that the game is played according to the rules [67].

Yet, as Stewart recounts, in Euclid's time, and for the next 2,000 years or so, mathematicians did not think that way at all, as they saw the *axioms* as self-evident truths, so obvious that no one could question them. On the contrary, Euclid did his best to make his *axioms* obvious, and he succeeded with almost everyone.

Thus, starting from these simple beginnings, Euclid's *Elements* proceed, step by step, to provide increasingly sophisticated *demonstrations* of geometric theorems. For example, the Proposition 12 of Book 1 of *Elements* proves that the angles of a triangle, when added together, are equal to 180°. Or, Proposition 47, from Book 1, is the Pythagorean Theorem.

In short, Euclid deduced each theorem from previous theorems and/or several axioms.

In this way he built a *logical tower* that rose higher and higher into the sky: the *axioms* were the foundation of the tower, while *logical deduction* was the mortar needed to hold the bricks of the tower together [67].

We, too, have actually tried to use *logical deduction* to derive some results, on the basis of which we could glimpse a possible new theorem.

Non-Zero Mass Photon (P)

Hence, what emerges, from what has been stated or reported above, consists essentially in the fact that many natural or experimental events, or those often habitually (and mostly unconsciously) used by all of us during the course of our daily lives, probably comparable to the *common notions* used by Euclid (or *axioms*), may represent a clear manifestation and *demonstration* of the performance of a purely mechanical action induced by light, by the 'so-called *Radiation Pressure*, or *Light Pressure*'[18].

To be exact, this phenomenon in particular is generated by the mechanical action exerted by the *momentum* (p) of the photon (P), right when the *quantum of light* strikes a particle.

Well, this is no small matter, i.e. it is not an insignificant impact, since a single P in the optical band hits a particle, the electron for example, with an impact force represented, as Feynman puts it, by its *push momentum* [18]. This *push momentum* (p) as shown by our calculations -see Eq.(11)- is equal to $p = 1.325 \cdot 10^{-22}$ [g· cm/s].

This is truly amazing, since the P hits the particle with a *mass-effect*, i.e. an impact force, transmitted by its *momentum* (p), corresponding to well over 100,000 times the rest mass of the electron itself!

So much for massless P!

Indeed, this can be compared to a Euclidean *starting statement*, which in turn could be classified as a *postulate*, or a *common notion*, say an *axiom*.

And it is precisely on this *foundation* that, as the '*logical tower of Euclid*' [67] for the formulation of his *theorems*, the construction of a new theorem, the *nonzero mass photon*, could also take hold.

In this regard, it is necessary to clarify what the material capable of holding this possible theorem might be, i.e. the material comparable to the *mortar* that holds the bricks of Euclid's *logical tower* together. In other words: what are the *logical deductions* capable of building and holding in feet this possible new theorem?

Well, as explained at length above, the *logical consequences* (or *deductions*) are, first and foremost, the unequivocal effects, purely mechanical in nature (*push-effect* or *mass-effect*), exerted by the light, which constitute, without a shadow of a doubt in our opinion, a clear and practical *demonstration* of the *non-zero mass* of the photon (P).

With regard to these mechanical effects or actions performed by P_s , we refer to e.g. to the photo-electric effect (PEE) [2],[3],[4], the Compton effect [5], the Raman effect [6], or the mechanical action performed within atomic nuclei by individual P_s , capable of moving an electron from a more internal orbit to an external one, just as we were reminded by Barrow [49] or Hawking [24].

Another piece of *evidence* for the *push-effect*, i.e. the *mass-effect* induced by P_s , comes to us provided by a very authoritative source such as The National Academy of Science, USA which, as mentioned above in § 2.2.5, emphasises the significant and powerful mechanical action exerted by P_s , often for very long periods of time, in order to counterbalance the powerful gravitational pressure which pushes from the outward external layers of the star to the *star's core* [45].

That is, there is a real conflict between 2 powerful opposing *forces*, between 2 opposing *pressures*: one side is represented by the relentless Gravity Interaction (GI) exerted by the entire stellar mass, which presses inexorably on the layers at its center, where on the other hand there is the opposing side, represented by hot gasses and P_s which, on the contrary, exert a completely opposite *action*, exerting a true *anti-gravitational action*, a *Photonic Counter-Pressure* [68].

In other words, in order to perform this anti-gravity action, P_s have to *base it on something*, as though they had an equivalent-mass [12, MEEP] [14, PuccEquiv.Mass]. That is, it could be the

equivalent mass of lots of billion..of P_s , which summed up may contribute, together with the hydrodynamic pressure of the hot gases, to prevent the collapse of the star [44].

This titanic clash goes on for a very long time, even for several hundred million years, just because it is also kept in equilibrium by light. And all this does not occur once in a while, or here and there throughout the Universe, but in all the stars *in equilibrium*, i.e. an approximate average of as many as 200 billion of the stars that populate, on average, every galaxy in the Universe.

It is indeed difficult to give a clearer, more obvious and more widespread example of a mechanical action, and one of considerable power, performed by light than this.

Yet, according to common belief, a massless P should be able to do all this and for so long, uninterruptedly! What is more, a multitude of P_s , but all massless, should be able to erect a *wall* of such magnitude, i.e. to create and hold up for so long an anti-gravity action, a *photonic counter-pressure* so powerful, that it would counteract for so long the inexorable collapse of a star [44].

And on what concrete, physical, material basis could a multitude of massless P_s erect this powerful counter-pressure, able to withstand and counterbalance so well and for so long the intense gravitational pressure exerted by the star towards its interior?

No, it really is not possible, it is not *logical* to accept this *imposition* from *gauge theories* in order to safeguard and make the relevant equations apparently congruous.

Furthermore, we consider it pertinent to bear in mind that the first astronomical observation regarding the *effect mass* induced by light, i.e. inherent to the mechanical action, i.e. the *force*, the *pressure* exerted by electromagnetic radiation (the so-called *Radiation Pressure* or "photon's *Pushing Momentum*" to use the words of Feynman [18], was reported over 400 years ago by Johannes Kepler.

In 1619, Kepler pointed out the concept of the *Radiation Pressure* exerted by the sun's rays to explain the observation that a tail of a comet always points away from the Sun [69].

"This *Force*" as Feynman reminds us "is called *Light Pressure*. Thus, light makes a pressure when it collides with an object. It is a very small pressure, but it can be measured with extremely sensitive instruments"[18].

In short, the obvious question arises: how can a P that is completely devoid of mass be able to perform all these obvious mechanical actions described (if anything, even at a considerable distance), to starting with the well-known photo-electric effect (PEE)?

In the case of PEE, in fact, see § 2.1, we have that a massless particle, as P is considered to be, succeeds in having "the power to push out the electrons from the atoms" [11], even managing to override the *binding energy* that keeps the electron bound to the nucleus of its atom.

In summary, with PEE, it is verified that a massless particle is able to expel from an atom a electronic particle, with a *rest-energy* of no less than 0.511MeV and an *equivalent rest-mass* of $9.1 \cdot 10^{-28}$ [g]. This is truly astounding! No, it cannot be.

On the contrary, precisely this significant mechanical action, this *mass-effect* performed even by a single P implies that, in some way, the P can also carry a *mass*, or something that functions as such.

Therefore, in our opinion, following the methodology of Euclidean *logical deductions*, Einstein's Mass-Energy Equivalence Principle (MEEP) can be a valid (albeit indirect) *demonstration* of *not massless P*.

On the other hand, regarding the possible transported mass of the *electromagnetic radiation quantum* [1], Planck himself, in his famous *Communication* of 14 December 1900, says: "We can use the natural constant, h , expressed in erg·sec" [1].

This is a truly *revolutionary* concept, even for that era. And why?

For the common belief, then as now, that electro-magnetic radiation (EMR) carries no mass: the beam of light is *massless*!

In contrast, Planck expresses the value of h in erg per second.

Since erg is measured in the basic units of the *cgs* metric system, it follows that erg is expressed in grams per square centimetre per square second, i.e.: $[g \cdot cm^2/s^2]$.

About the erg, we read from Chandrasekhar: 'Erg is the name of the unit of energy. An object with a mass of 2 grams and travelling one centimetre per second has a kinetic energy of one erg' [25]. It follows easily that the *erg* measures, of the same particle, both the value of its energy, than the value of its mass, expressed in grams. Which implies, consequently, that any energy particle should also be associated with a mass value, expressed in grams.

In fact, already with Planck we have that the value of h is given in grams [1].

Instead, it seems that this novelty revealed by Planck went unnoticed, not taken up by the community scientific, then as now, in its proper consideration, importance and relevance.

And yet, it is *logical* and elementary to conclude that if a particle, such as the *quantum of light*, is expressed in grams, implies that in reality it is not so ethereal, evanescent, insubstantial, as is believed, i.e. a particle with no mass at all, but that it also carries a mass value, *equivalent* to its energy value [12], [14].

On the other hand, as previously reported by Rinaudo (see § 2.2.2), "*energy* is a property that, in the Classical Physics, we usually associate with a material body'[10]. It is obvious, therefore, that also the *quantum of EMR*, i.e. the photon (P), the *particle of light*, and therefore *energy* carrier, can also be regarded as a '*material body*' [10], in the sense that it carries with it a *mass*

equivalent (to its energy), thus conferring on it, so to speak, a certain *consistency*, capable therefore of making it perform a mechanical action, which is realised just when P interacts with matter.

On the other hand, the mechanical effects induced by EMR are not pure fantasy, but applied in many ways in our daily lives, such as the aforementioned *photo-electric effect* (PEE).

We use PEE every time we automatically open the door of a supermarket, e.g., or when handling the TV remote control.

Admittedly, these are minimal, apparently null values, those attributed by Planck to the 'natural constant h' [1], but they are still different from zero [51].

Here, in our opinion, lies the fundamental importance of the value expressed by *Planck's constant*: this is the *core* of our work.

In other words, if it is taken into its proper consideration, i.e. in all its meanings and potential effects (including the deepest or least evident), this apparently insignificant and negligible value, inherent to the value of h , of the *Planck's grain*, then a multitude of doors open in so many fields, both Physics, Astrophysics, and Mathematics, such as the possible solution of the well-known *divergences* emerging in various equations of the Perturbative Calculus, or of the Quantum Fields Theory, or of the Yang-Mills theories [70],[71]: it is like saying that we are entering *a new Physics*.

On the contrary, it is all these mechanical actions that represent numerous and valuable *demonstrations* (following the *logic* used by Euclid) in favor of the proposed *theorem*.

Moreover, in our opinion, the contrary thesis seems to be weaker and no longer tenable, which, relying on *gauge theories* [50],[72],[73],[74] and maintaining the *integrity* (or *non-breaking*) of *symmetries*, not only denies mass to P, but also to all other particles, which would therefore also lack their own *intrinsic mass*! In this regard, from the height of his authority, Weinberg expresses himself as follows: "However, one of the consequences of *Electro-Weak Symmetry* is that, if no other ingredients are added to the theory, all elementary particles, including electrons and quarks, are massless, and this is patently false. Therefore, it is necessary to add something new to the theory, some new type of material or field" [75].

According to *gauge theories*, in fact, if the Brout-Englert-Higgs Mechanism [76] and the Higgs *Field* [77] had not been *created ad hoc*, a massless *gluon* should categorically have a range extended to infinity! [78]

We are truly in the midst of *illogic* and in open conflict with physical reality and the Mathematics. It is well known, in fact, that the bosons of a Nuclear Force can only and exclusively operate in the range of an atomic nucleus [79], in full agreement with the Yukawa Principle [80].

This is why Weinberg decisively declares that *it is patently false to consider all elementary particles to be massless* [75] and, of course, this also applies to P, since P is also considered to be a particle to all intents and purposes. In this respect, we have already quoted what was expressed, e.g., by Fermi [8] or Feynman [7], similarly we read from Chandrasekhar: 'We know that light also has corpuscular properties and we have even given a name, *photons*, to the particles that compose it' [25].

But let us return to the P, commonly considered massless for all intents and purposes, and therefore lacking a even infinitesimal mass, as the *mass-energy equivalence principle* (MEEP) would instead impose.

Instead, just the MEEP, in our opinion, represents one of the main *demonstrations* in favour of the proposed theorem, in accordance with the criteria followed by Euclid. Furthermore, the MEEP itself represents an insurmountable, truly insurmountable obstacle for the thesis, never demonstrated in all respects (that is to say: never concretely, nor *physically*), of the P massless! On the contrary, it is precisely the most elementary rules of Mathematics that peremptorily deny, and once and for all, the sustainability of the concept of a P completely devoid of its mass, since Arithmetic prohibits *a priori* the insertion of a relative null value for the mass (*m*) inherent to the particle (whether it is a P, or any other physical system) present in the MEEP: $E=mc^2$.

In this regard, we are reminded of Penrose's words: "The famous formula from Einstein's Special Relativity, $E=mc^2$, tells us that mass (*m*) and energy (*E*) are interchangeable.

For example, when a uranium atom decays, splitting into smaller pieces, the total mass there of each of these pieces, if they could be restored to rest, would be *less* than the original mass of the uranium atom; but if the *energy of motion* (*Kinetic Energy*) of each piece is taken into account, we actually find that the total is *unchanged*. In fact, the mass (*m*) is conserved"[39].

Therefore, we consider it necessary to emphasise that the phenomenon described by Penrose, which occurs continuously in reality, clearly shows that in the processes of transformation of mass into energy (and vice versa) there is full *conservation of mass* (and thus of *energy*), in total compliance with the Laws of Conservation of Mass and Energy, and in accordance with Einstein's MEEP *formula*. Obviously, this applies to physical systems, atoms and elementary particles, whatever particle is considered, including the photon (P).

Well, it is indeed the MEEP, the world's most famous equation, also known to the man from the road, to categorically annul, we might even say: *brutally*, the common belief in P massless. No! The *quantum of light*, *Planck's grain* cannot be massless at all: massless P is in total conflict with the basic criteria of Relativity Theory and the simplest Arithmetic, the one that is already taught in Elementary School.

As is well known, Einstein applied MEEP first of all to light, to his *quantum of light* [4], whereby if in the equation $E=m \cdot c^2$ the value of the mass, *m*, was 0, we would have:

$$E= 0 \cdot c^2 \tag{26}$$

In that case, the most basic Mathematics tells us that any physical or mathematical system, or any numerical value that is multiplied by 0 will cancel out, i.e. it will also become 0. It will therefore be:

$$E = 0 \quad (27)$$

Ultimately, we have that the energy value (E) of P also cancels out!

That is, Eq. (27) tells us that a massless P must also be *energyless*! So, a massless P , as the most elementary Arithmetic dictates, would also be *energyless*, so not only would it not be able to emit electromagnetic energy, to radiate light, but it would even be devoid of its own energy (E): it is as if it had never existed.

Furthermore, it follows from Planck-Einstein formula $E=h \nu$, shown by Eq. (1), that if the value of E were equal to 0, as shown by Eq. (27) concerning a massless P , even a natural value, such as the *Planck constant* (h), would cancel out and so would its *frequency* of oscillation per second, indicated by ν .

But no, that is not possible! We all know that h is a natural constant with its own value of well-known *intrinsic mass*, exactly equal to $6.626 \cdot 10^{-27}$ [erg·sec].

Ultimately, the key to supporting the proposed *theorem* in favor of P *not massless* lies in the actual value of the mass of the *Planck constant* (h).

In this respect, in fact, we can proceed backwards, by steps, analogous to the way Euclid constructed his *theorems*, in which each *step* is a *logical consequence* of the preceding *steps* [67], but probably with the further difference that in our case each *step* is not only a *logical consequence*, but also a *mathematical consequence* resulting from a succession of equations, which in turn also act as *demonstrations*.

Thus, we start with the *Planck constant* (h), a real value of mass and, proceeding backwards, the 1st *mathematical consequence* will be that, for a real, massive value of h , the value of the energy E , given in the Planck-Einstein formula $E=h \nu$, will also have to be a real, concrete value, rather than 0, as would be the case if P were massless, as shown in equations (26) and (27)!

This is not only a *logical* and *mathematical deduction* in favour of the construction of the theorem proposed, but at the same time undermines the conceptual foundations of the presumed belief that, like other elementary particles, P must also be exclusively massless in order to satisfy *gauge theories* [74] and not nullify the relevant equations, since a massive particle would break its *symmetry*, inducing the so-called *spontaneous symmetry breaking* (SSB) [71].

In this regard, Penrose writes: 'I question the reality of SSB! There are various difficulties in this idea of SSB' [34]. Witten adds: 'This proposal of the spontaneous breaking (SB) of electro-weak symmetry (EWS), or SSB, though simple and comfortable with known facts, probably does not tell us the whole story' [81].

Thus, proceeding backwards again, we arrive at the MEEP equation, $E=m \cdot c^2$, whose E (indicating the value of the energy of the particle in question, P in our case) corresponds exactly to the value of the energy (E) found in the formula $E=h \nu$, devised by Planck just to indicate the value of the energy of a *quantum of radiation* [1].

Following Euclid's *logical reasoning*, we can consider that a *mathematical consequence* (as well as a *logical* one) emerging from Einstein's MEEP consists in the fact that if the value of the energy (E) of P (placed at the 1st member of the equation) is a positive value, *it follows* that the value of the mass of P, denoted by m and placed at the 2nd member of the equation, must likewise show a real, positive, i.e. massive, non-zero value.

Instead, according to the alleged belief of P massless, it follows *mathematically*, as stated above in Eq.(27), that this value of the energy (E) inherent to P should also vanish, i.e. become equal to 0, thus making us to find ourselves in a world without light, totally in the dark, for the entire Universe! Nor could we ever see a starry sky: we fall into *science fiction*.

DISCUSSION

Thus, the MEEP represents an authoritative support and *logical deduction*, following the Euclidean model, as well as a valid *mathematical demonstration*, of the proposed *theorem*. From the MEEP, in fact, it is deduced that the value of the mass (m) of P cannot be zero at all, i.e. equal to 0, but *equivalent* to the value of the energy of P taken into consideration.

Another argument in favour of the proposed *theorem* is provided by Mariotti, already mentioned in §2.2, who writes: "Concerning Electromagnetism, we know that photons can behave like massive particles: this happens when they travel in a medium other than empty space.

The physical reason for this is that the propagation of the electric and magnetic field (i.e. the photon) interacts with the medium. The resulting effect is the *slowing down* of the propagating wave"[15].

Therefore, as Mariotti explains, we have before our eyes every day the physical effects of the real behavior of the photon (P): these are the typical effects of a non-massless P!

Indeed, it is precisely the slowing down of light (when it does not travel in the *vacuum*) that reveals that the *light's quantum* cannot be considered merely ethereal and insubstantial energy [82]. On the contrary, it is precisely the facts, the events that demonstrate that light also carries with it something real, massive, concrete which, by colliding with the medium it passes through, causes it to slow down [83].

This is why Mariotti states that 'P is a massive particle' [15].

We all know, in fact, that light slows down when it passes through glass, or when it travels through water, where it proceeds at a speed of about 224,000 km/s. In contrast, a massless P does not would have no reason to slow down its speed when crossing them.

Furthermore, in favour of the possible *theorem* we have presented, i.e. in support of the *not massless P*, an interesting *physical and mathematical demonstration*, as well as a *natural and experimental one* (since it also occurs spontaneously and continuously in nature), is provided to us by Penrose: "The well-known equation $E = mc^2$, indicates that mass and energy are interchangeable" [39].

In truth, just this initial sentence by Penrose would be enough to be able to peremptorily state that even *P*, being a particle endowed with its own energy, must equally also carry a mass which, in accordance with the Bohr Complementarity Principle [29], will be able to manifest its *mechanical effects* only when it interacts with matter.

Penrose continues: "To give a striking example, in which the effect of Einstein's mass-energy relation is present in an extreme form, let us consider the decay of a subatomic particle, the meson π^0 . It is a *material particle* with a well-defined (positive) mass. After about 10^{-16} seconds, it almost always disintegrates into just *two photons*. For an observer at rest with respect to the meson π^0 , each photon carries with it half the energy and, in fact, half the mass of the meson π^0 . Yet this photon *mass* is of the most impalpable kind: *pure energy*"[39], which is why this mass seems *hidden* and why one is inclined to assign no mass to *P*!

Well, this very phenomenon mentioned by Penrose, i.e. the creation of highly massive pairs of P_s (a natural event, which is widely and continuously repeated in the Universe), traces two completely opposite paths: 1) The 1st represents a further and indisputable *demonstration* of the proposed *theorem*: the *non-zero mass photon (P)*. 2) The 2nd conspicuously emphasises the no further sustainability of the opposite theory, according to which all the light, all the P_s , the entire range of electromagnetic radiations (EMR_s) would not carry the slightest mass, nor something like that!

In short, it is really surprising to have considered *P* massless to date, when it occurs continuously, as reported by Penrose [39] that a particle such as the meson, weighing about the equivalent of 140 MeV, splits into 2 P_s , each carrying a *mass-energy density* of 70 MeV. In this respect, as we read from Feynman, "One MeV is worth about $1.782 \cdot 10^{-27}$ grams"[7], so that each *P*, into which a neutral meson (π^0) spontaneously splits, will carry a mass of about $1.247 \cdot 10^{-25}$ grams, i.e. well over 2 *orders of magnitude* greater than the electron's rest mass: nothing but a massless *P*!

And such events occur all the time in nature, giving rise in such cases to massive P_s , i.e. carrying masses of this magnitude.

Another very obvious action of a purely mechanical nature, carried out by P_s , takes place in the well known Compton Effect [5], of which we quote Heisenberg's interesting description: "When X-rays arrive in a *Wilson Chamber*, it sometimes happens that they release an electron (*recoil electron*) in a molecule of the gas of the chamber, which is made visible by its fog wire. This phenomenon can be interpreted by assuming that the EMR (in our case X-rays) consists of individual corpuscles, which collide against the gas molecules (Einstein's *light quantum hypothesis*). Each *light's quantum* must therefore be assigned a certain energy *E* and a certain

momentum p , which are related to the frequency ν of the radiation by the relations $E = h \cdot \nu$ and $\lambda = h / p$ [84].

In fact, it was precisely this typical mechanical effect induced by X-rays, this *collision* of the P with the electron (a collision *sufficient* to detach the electron from the atom) that made the scientific community definitively accept the corpuscular nature of P_s. In fact, Heisenberg points out: "The application of the mechanical collision laws to the interaction between *quantum of light* and electron now provides in an elementary way a relation between the direction of recoil of the collided electron and the direction in which the scattered *quantum* recedes. Compton and Simon's experiment ([85]) allows direct verification of the consequences of this purely corpuscular theory of *X-quanta* scattering. In fact, Compton and Simon's results were thus able to demonstrate with certainty that the laws of *elastic shock* are fulfilled and thereby directly highlight the corpuscular nature of electro-magnetic radiation" [84].

In short, it must immediately be made clear that the *quantum of light* is represented by the basic value of h , which is a value expressed by Planck in *erg*: this is of fundamental importance, but unfortunately never taken into due consideration. In fact, as is well known, *erg* is expressed in grams per cm per second; this implies that, from the very beginning, Planck associated his quantum of electro-magnetic radiation (EMR) with a *mass*. It is certainly a very small value, but a positive one nonetheless, not at all zero!

But this should not surprise us! Precisely with regard to the meaning of the *mass* represented in Einstein's MEEP, Zeilinger asked: "What is the deep meaning of a relationship like $E=mc^2$? What is *hidden* behind these symbols? For many physicists the equation $E=mc^2$ is to say that energy and mass are the same thing, two faces of the same medal. There is, therefore, *equivalence* between mass and energy: energy is just another form of mass, and vice versa, mass is another form of energy"[86].

Well, as we all know, P is first and foremost an energy particle, so in agreement with Zeilinger, *energy is just another form of mass* [86]. Other than P massless!

It is also well known that one of the fundamental characteristics of P consists in its continuous oscillations: P always oscillates. In fact, the very value of the energy (E) of P is directly proportional to its oscillation frequency (ν), as described in Eq. (1), $E=h\nu$, also known as '*Planck's Law of Radiation*' [39].

In this regard, we read from Penrose: "Louis de Broglie in his doctoral thesis ([36]) proposed that *matter* particles themselves should sometimes behave like waves! The wave frequency ν , for any particle of mass m , also satisfies *Planck's Law* which, combined with Einstein's equivalence $E=mc^2$, tells us that the frequency of oscillation (ν) is connected to the mass (m) by:

$$h \nu = E = m c^2 \quad (28)$$

so:

$$m = \frac{h \nu}{c^2} \quad (29)$$

Thus, according to de Broglie's proposal, the dichotomy between particles and fields, which had been an important element of classical physics, is *not* respected by nature!

In fact, anything that *oscillates*, with some *frequency*, ν , can *only* occur in discrete units, of mass $h\nu/c^2$.

Somehow, nature manages to construct a consistent world in which *particles and oscillations of field are the same thing*" [39].

Well, P is also an *oscillating particle*, therefore, according to Penrose, it is permissible to be able to *deduce logically* (in the manner of Euclid) that P "can *only* occur in discrete units, of mass $h\nu/c^2$ " [39], as illustrated by Eq. (29).

In short, what Penrose writes, may congruently represent, in line with Euclid, another *logical deduction*, as well as a *mathematical demonstration*, in favour of the construction of the proposed *theorem*, since the mass of photon should correspond exactly to $h\nu/c^2$!

On the other hand, how can one deny that the photon (P) is related to a mass, a *mass-energy* density, after reading what Weinberg writes: "At the origins of the Universe, the situation was very different from what it is today: at that time, it was light that constituted the dominant ingredient of the Universe, while common matter was nothing more than a negligible contamination"[37]. This sentence, incidentally, is in full agreement with the Bible: 'In the beginning *fiat lux*' [The New Testament, Genesis 1].

Well, this sentence by Weinberg alone would be sufficient to fully reject the belief that P is massless. In fact, as the Laws of Conservation of Mass and Energy dictate, both *mass* and *energy* cannot disappear at all, but rather transform into each other (or one type of energy can transform into another form of energy).

Therefore, a good portion of that photonic energy of the primordial Universe, "then largely dominant with respect to matter" [37], certainly cannot have been lost, so all that remains for us to do is to incur a *logical deduction* (in the Euclidean manner), i.e. a *logical consequence*: that primordial light, certainly of a very high frequency [87], and therefore invisible in truth, must have been transformed both into frequencies of progressively lower intensity, as confirmed e.g. by the detection of the *Cosmic Microwave Background* [88], and into mass (*materialisation of electromagnetic energy*), as is amply testified by the enormous multitude of galaxies and planets scattered throughout the Universe, as well as the birth of the plant and animal world, including us of course!

This statement does not differ from Weinberg, who writes: 'Long before the contents of the Universe became transparent, it can be considered to have been composed mainly of *radiation*, with only a slight contamination of matter. The enormous energy density of radiation, present at the beginning of the Universe, was lost through the shift in the wavelengths of the photons

towards the red, coinciding with the expansion of the Universe. At the same time, what could be considered a contamination of nuclear particles and electrons was being organised to form the stars, rocks and living beings of the present Universe"[37].

Moreover, the *materialisation* of some of that primordial light energy began immediately, as the very high temperatures allowed for it. In fact, in describing the expansion of the Universe, Weinberg starts from the time one hundredth of a second after the Big Bang: "This is the earliest moment that we can speak of with any certainty. At that time, the temperature was 10^{11} degrees Kelvin ($^{\circ}\text{K}$) and the mixture of particles in the Universe had a density four billion times that of water. This cosmic mixture consisted first and foremost of P_s , as well as electrons and neutrinos, with their antiparticles, plus very small traces of protons and neutrons"[37].

Obviously, the materialising particles referred to by Weinberg are photons (P_s)! At this regard, Weinberg writes: "The process by virtue of which matter is produced from radiation can best be understood on the basis of the quantum conception of light. 2 quanta of radiation, or photons, can collide and disappear: all their energy and *momentum* give rise to 2 or more material particles. This process is observed indirectly in today's high-energy nuclear physics laboratories"[37].

Thus, it is continuously experienced that from the collision of 2 P_s (particles commonly considered massless) two or more particles of matter are created, i.e. with their own mass, often a substantial one: but then is it *magic*, or is there another explanation that we are still unaware of?!

Never, one will object: it is nothing more than a simple transformation of energy into mass!

And yet, in our opinion, there is something that does not add up, as if there were a more subtle explanation, as if something seemingly *hidden* from our eyes: as if inherent in P there was an imperceptible, *intrinsic* mass (most likely associated with *Planck's constant*) of which we cannot have any information because of the Complementarity Principle [29].

This concept, moreover, is in perfect agreement with what Penrose writes: "this 'mass' of the photon is of the most intangible kind: *pure energy*"[39].

In sum, the question arises: if two particles are completely massless, it is natural and *logical* configure them as ethereal, evanescent, insubstantial, *immaterial* corpuscles, so that, if their paths intersect, instead of giving rise to a *collision* (with the creation of matter with its own physical, real consistency, concrete substance) they should behave in the same way as neutrinos (also considered massless, until recently), i.e. act as '*ghost particles*', as Weinberg also calls them [37], and thus pass through each other and continue unchallenged, each along its own path.

As is well known, after the evidence of the neutrino *oscillation* [89], even the most *ardent* supporters of the massless neutrino (ν) have had to reconsider. Yet despite the fact that ν is provided with mass, it does not interact with any particles, not even the other neutrinos (ν_s).

In this regard, Weinberg writes: "The example that is usually cited to demonstrate how weak the interaction of ν_s is that, if we wanted to have an appreciable chance of stopping or spreading a given ν produced in some radioactive process, we would have to place several light-years of lead in its path" [37]. In this regard, after extensive studies and analyses, Bethe and Peierls came to the conclusion that it would be impossible to detect a ν , since it would pass, without interacting, through a lead wall of over 3500 light years [90].

So, we have two diametrically opposed and contradictory behaviours: on the one hand there is ν , a massive particle, which passes through any physical system, really like a *ghost particle*. On the other hand, there is the photon (P), a massless particle which, on the contrary, immediately interacts with the material bodies and particles it encounters on its way; indeed, if it encounters another P, there is a real *collision*, of a *material* type in our opinion, so much so that from such a collision, as if from nowhere, 2 or more massive particles emerge!

How to interpret these different and contrasting, even paradoxical behaviours!

And why *paradoxical*? Because a massless P should behave in the same way as ν (which is even massive), i.e. without interacting with the surrounding matter at all.

Instead, just the opposite happens: a massless particle, as P is considered, always interacts and, if conditions permit, even creates massive particles!

And it is true, so it happens in reality.

In short, the behaviour of P seems to be that of a massive particle, which, however, can only manifest its corpuscular aspect, and therefore its possible *mass-effects*, when it interacts with other particles, just as Bohr's Complementarity Principle dictates [29]. In this respect, in fact, it is Weinberg himself who gives us the *atout*, as he says that both the energy and the *momentum* (p) of each P contributed to the mass of the electron-positron pair (created by the collision of 2 P_s).

Well, Newton *docet*: $p = m \cdot v$ [26], as shown by Eq. (7). Therefore, if a particle is provided with a *momentum* (p), and all particles are, including P, this means to all intents and purposes (with respect to Newton's formula) that the photonic particle must also be provided with a mass (m), which can be either well evident, i.e. explicitly detectable, or, as Feynman puts it: "*hidden*" [18]. And why should the possible mass of P be *hidden*? This is suggested by the Complementarity Principle, according to which a particle in motion (motion expressed by the value of its *momentum* and acquired *kinetic energy*) can only manifest its wave-like behaviour, and not its corpuscular behaviour, which will only come out when the particle, or P, interacts.

That is why, being always in motion, the P is massless in the eyes of everyone, since its value *mass*, closely related to *momentum*, will not be detectable, but remains hidden by its *wave-like nature*!

Instead, only when P interacts with matter will it be able to show, for those few moments, its corpuscular behaviour and its *mechanical effects*, such as the *Compton Effect*, or the *photo-electric effect* (PEE), or transform itself into material particles, even of a certain consistency: *materialisation of Radiation*, as described above by Weinberg, Penrose, or Feynman.

Well, what has just been discussed could be considered as another point in favour of the construction of the possible new theorem: 'the not massless P '.

On the subject of the *materialisation of radiation*, Weinberg points out: 'For 2 P_s to be able to produce an electron-positron pair in a head-on *collision*, the energy of each P must be greater than the *quiet energy* (equal to mc^2) of the mass of an electron or positron. This energy is 0.511003 million electron volts. To find the *threshold temperature* at which P_s is likely to possess such a high energy, we divide the energy by the Boltzmann constant (k), equal to 0.00008617 eV, for each Kelvin degree ($^{\circ}\text{K}$) and find a *threshold temperature* of about 6 billion degrees Kelvin ($\approx 6 \cdot 10^9$ $^{\circ}\text{K}$). At each higher temperature, electrons and positrons are freely created as a result of *collisions* between P_s .

This threshold temperature is much higher than any temperature we encounter in the Universe today under normal conditions. Incidentally, in the centre of the Sun, the temperature is about 15 million degrees. This is why we are not used to seeing electrons and positrons coming out of space empty when the light is very bright" [37].

Thus, the *threshold temperature* necessary for the *materialisation* of a particle, i.e. for the transformation of energy into matter, must unequivocally be \geq to the value obtained by dividing the *inertial energy* of the considered particle for the Boltzmann constant (k). To this purpose, Chandrasekhar reminds, the *inertial energy* is identified with the *zero point energy* (ZPE) [25],[54].

In fact, in agreement with Weinberg, it is thus obtained that while for the electrons (with ZPE = 0.511 MeV) the *threshold temperature* corresponds to 5.93 billion $^{\circ}\text{K}$ [37], for the *nucleonic synthesis* (*baryogenesis*) really amazing temperatures are needed, which are obtained under very limited circumstances, sometimes only for short periods of time, equal to fractions of a second, as soon after the Big Bang: *primordial nucleosynthesis* [91].

Therefore, according to Weinberg, to obtain the formation of a proton (ZPE = 938.26 MeV) "the *threshold temperature* corresponds to 10888 billion $^{\circ}\text{K}$ " [37]. Similarly, the creation of a neutron (ZPE = 939.55 MeV) requires "a *threshold temperature* of 10903 billion $^{\circ}\text{K}$ " [37]. These are very high temperatures that, we could say, in nature are reached only in those situations of *singularities*, such as *Big Bang* and *Black Holes* [24],[92],[93], or stellar cores and *neutron stars*: the latter are direct consequences of the collapse of a Supernova [94].

Thus, with the *primordial nucleosynthesis* the lightest chemical elements were formed, namely only the first 3: hydrogen (H_1^1), helium (He_2^4) and lithium (Li_3^7), in addition to some isotopes related to these elements, among which deuteron (H_1^2) and helium-3 (He_2^3) [94].

Observing the Mendeleev Table, indeed, it is noted that there are no stable nuclei with *atomic mass* = 8, so the *primordial nucleosynthesis* stops at Li^7 [95].

In fact, it was enough for the temperature of the Universe to fall below the *threshold temperature* required for *nucleonic synthesis* that the creation of elements with *atomic number* >3 was no longer possible. After that, as is known, it will have to go through several hundred million years, until the conditions of gravity, pressure, density and temperature are sufficient to see again a natural *proton* and *neutron synthesis*, that is, a new *baryogenesis* [96].

This occurs in the star core: *stellar nucleosynthesis*.

In fact, through the stellar nucleosynthesis, including supernovae explosions, all-natural elements of the Periodic Table were formed: from the 4th to the 92nd.

Furthermore, and to close, it is of considerable significance what Weinberg writes: "At temperatures above the threshold temperature, a material particle behaves largely like a photon"[37]! Therefore, in accordance with the Euclidean methodological procedure, i.e. proceeding step by step, where "each step is a logical consequence of some of the previous steps" [67], it seems more than legitimate to be able to *logically deduce* that the photon (P), in turn, can also behave like a material particle!

And the facts amply confirm this: just bear in mind the numerous examples of mechanical actions performed by electromagnetic radiation (EMR) [97].

Finally, that the P can also transmit a *mass* value is indirectly told to us by Feynman, from whom we read: 'If electrons were *ideal*, and went from one point to another in space-time following *only* direct paths, there would be no problem: *n* would only be the electron's *mass* (determinable during observation) and *j* would be its *charge*, i.e. the probability amplitude of interaction between the electron and one or more P_s (also determinable experimentally).

But ideal electrons do not exist, and real electrons emit or absorb their P_s from time to time. Therefore, the mass of the electron (measured in the laboratory) depends on *j*, i.e. the amplitude of interaction with the P_s "[7]. This is like saying that the mass of an electron depends on the number of P_s in charge, i.e. the amount of P_s emitted or absorbed. In other words, Feynman, one of the most expert in the secret of light, is telling us that just one more P, absorbed by the electron, is able to increase the value of the electron's mass, while one P emitted, i.e. one P less (in charge of the electron) decreases its mass!

Just what Feynman has now described, in relation to electron-photon interactions, which are then the essential basis of Quantum Electro-Dynamics (QED) and "affect most usual phenomena, for example all chemical phenomena and all biological phenomena"[7], would be enough to fully authorize us to be able to propose the *new theorem* in question.

CONCLUSIONS

We have listed several natural events in which really massive P_s *emerge*, which *materialise* spontaneously following the decay of a particle. Or, the reverse phenomenon can occur. In this

respect, as Weinberg reminds us, 'as long as there is sufficient energy, all sorts of particle-antiparticle pairs will always be *possible* in the *collisions* between P_s ' [37].

What is more, the same *collisions* between P_s , like other particles, imply the presence of something real, physically *consistent*, possibly *material* (carried by the particles themselves), for the collision to occur, and if anything, the creation of new particles (of matter, moreover)!

On the contrary, *logic* leads us to believe that 2 P_s massless would have no reason *to collide* and, consequently, as ether particles, like *ghost particles*, they should not interact with each other at all, so that, without *colliding*, they should ignore each other and continue indifferently on their way, or even *pass through* each other, without producing any physical effect!

Moreover, we have reported well-known experimental proofs [5], [85], admirably described by Heisenberg [84], where the P_s collide with particles, deflecting or ejecting them from the atom, and thus behaving like real material particles.

In fact, it was precisely this explicit mechanical action characterising *the Compton Effect* that sanctioned definitively the corpuscular nature of P_s .

Nevertheless, it is believed that these purely mechanical actions would be performed by a massless particle. In this respect, it is natural to seriously question the veracity of the assumption that P must be absolutely massless.

On the contrary, as demonstrated by the decay of the π^0 meson reported by Penrose [39], upholding the concept of massless P will always result in a total and untenable conflict with natural events, experimental evidence and, moreover, with a Fundamental Law of Physics: the *Mass Conservation Law*! In this regard, we read from Feynman: "Ordinarily the energy changes represent extremely slight changes in *mass*, because most of the time we cannot generate much energy from a given amount of material; but in an atomic bomb of explosive energy equivalent to 20 kilotons of TNT, for example, it can be shown that the dirt after the explosion is lighter by 1 gram than the initial mass of the reacting material, because of the energy that was released, i.e., the released energy had a mass of 1 gram, according to the relationship $\Delta E = \Delta(mc^2)$ " [18].

Thus, as described by Feynman, we have that a single gram of bomb material turned into a frightening amount of energy, just equivalent to 20 kilotons of TNT (trinitroglycerin): it is a very clear example of the *transformation* of mass into energy, fully confirming both the *Mass Conservation Law* and Einstein's *Mass-Energy Equivalence Principle* (MEEP).

To this purpose, indeed, Feynman adds: "This theory of *equivalence of mass and energy* has been beautifully verified by experiments in which matter is annihilated-converted totally to energy: An electron and a positron come together at rest, each with a *rest mass* m_0 . When they come together they disintegrate and two gamma rays emerge, each with measured energy of $m_0 \cdot c^2$. This experiment provides a direct determination of the *energy* associated with the existence of the *rest mass* of a particle" [18].

Therefore, it is very important to emphasise that in this experiment two massive particles were transformed into 2 gamma rays, let us say 2 gamma photons (P_s), each carrying the *dynamic mass* (" $m_0 c^2$ " [18]) equivalent to the energy of an electron in motion.

In essence, and it is of considerable significance to emphasise this again, Feynman has described, without a shadow of a doubt, that massive P_s are habitually and routinely created in nature, each carrying a mass of no less than 0.511 MeV: and we are still talking about massless P_s !

In short, just what Feynman says completely coincides with the concepts behind the possible theorem proposed with this work.

In our comfort, we read from Penrose: "Maxwell electromagnetic field delivers energy. For $E=mc^2$, it must also have a mass. Maxwell's electromagnetic field is therefore also matter! Now we must certainly accept this notion, since the Maxwell's field has an intimate involvement in the forces that bind particles together. The mass of anybody must receive a substantial contribution from the magnetic fields within it" [39], probably analogous to a form of *intrinsic mass* inherent in the particle itself. It is pleonastic to specify that Maxwell's electromagnetic field is constituted and operated by *Planck's grains*, by P_s !

In this regard, it may be interesting to read: "To the question: 'Does the photon fit into the equation $E=mc^2$?' The answer is yes. Whereas to the question: 'Does the photon have mass?' The answer is not.

To understand better, a brief clarification must be made regarding the definition of mass. In this case, the confusion arises from a problem of definition. One often knows the expression for the energy of a photon ($E=h\nu$) and Einstein's famous formula $E=mc^2$. And so, from the equality of the two one is led to think that the photon has mass $m \neq 0$. The problem lies in the fact that the quantity m that appears in Einstein's formula is the so-called *relativistic mass*: this is a different conception of *mass* from the one we are familiar with; the relativistic mass is a measure of the energy of the particle, and varies with velocity. By convention, what we daily call 'mass' is not the 'relativistic mass' but, in modern terminology, is the so-called *invariant mass*" [98]. In truth, the author contradicts Einstein himself who, as quoted above, commenting on his equation $E=mc^2$, expressed himself thus: "The value of the considered mass, m , refers to the value of an *inertial mass*" [40], i.e. precisely the *invariant mass*, i.e. a value in itself fixed, constant that, on the basis of all that has been stated, following the Euclidean *logical procedure*, it can be *deduced* that the value of this mass (*invariant, inertial, or intrinsic*) corresponds first of all to the value of *Planck's constant* (h)!

In fact, "*Invariant mass is an intrinsic property of the particle and, as the name suggests, is independent of the reference system. Its 'invariance' in this sense makes it a useful and conceptually well-defined quantity. If one wishes, one can state that the photon has a non-zero 'relativistic mass', but this statement has little following from the point of view of practicality and physical sense*" [98]. On the contrary, in our opinion, this *non-zero relativistic mass*, attributed to P in motion, represents a pivotal point, both in confirmation of the possible veracity of the *theorem* proposed with this manuscript, and in confirmation of the *hidden mass*

of P, *hidden*, i.e., not retrievable (as dictated by the Complementarity Principle) when the particle is in motion (*photon's dynamic mass*), and probably identifiable with the values expressed by p , i.e., by P's *momentum*.

Furthermore, we are not aware that the *relativistic mass* of P "does not have much following from the point of view of practicality and physical sense" [98]. Nothing at all!

As for that *relativistic mass* 'having no practical consequence', we must instead point out that it represents precisely the *key* to understanding and explaining *practically, technically*, and in all their essence and explanatory modalities, the various mechanical actions (with their *effects*) induced and carried out through the EMR, and explainable in their intimate physical mechanisms only by a more or less massive P.

Furthermore, we do not agree at all that such *relativistic mass* 'does not have much of a physical sense'; far from it, as a massive P opens the door to a *new Physics*.

Indeed, a non-zero value of the mass of P can play a decisive role in many physical phenomena: in Optics, as in the atomic and sub-atomic world, as in the most refined Mathematics or in Astrophysics [51].

Thus, just to give a few examples, the *mass-equivalent* of P can help explain the intimate physical mechanism of well-known phenomena, already mentioned above, all of a purely mechanical nature [14]; we refer to the photoelectric effect, the Compton and Raman *effects*, the *recoil effect* generated by emitted P, or peculiar less known and less investigated phenomena such as the decrease in temperature and mass of Black Holes [42], or the negative value of their *specific heat* [41].

Furthermore, only a P not massless, in our opinion, can make us better understand the well-known *Arm Wrestling* between the Gravity Interaction (GI) and the *Radiation Pressure*. In fact, as reported above in §2.5.5, this *Arm Wrestling* goes on uninterruptedly in the depths of the stars. In fact, in ordinary stars the inward force of gravity is balanced by the outward hydrodynamic pressure of the hot gasses and by the *radiation pressure* of photons [45]. To this purpose, it is interesting to highlight that only a huge multitude of photons with mass (and forming a highly compressed '*photon gas*' [18]) can contribute to counterbalance the huge gravitational pressure which pushes from the outward external layers of a star to the internal layers. In other words, it could be the equivalent mass of lots of billions of billion. of *light quanta*, which summed up may exert for very long times a mechanical action, probably a mass effect acting as a *Radiation Counter-Pressure*.

Even in the mathematical field, a not-massless P can make a new and remarkable contribution. In fact, if we consider all the equations of Perturbative Calculus, Quantum Electro-Dynamics (QED), Quantum Fields Theory, and Yang-Mills theories, and replace the zero value, attributed to the mass of P, with the value of the *momentum* (p) relative to the P considered (as represented by our equations), then, as if by magic, all *divergences*, i.e. all zeroes and infinities, will suddenly disappear and, consequently, the limits imposed by the *Spontaneous Symmetry*

Breaking (SSB) will also disappear, so that there is no longer any need to deny the mass to the Nuclear Forces bosons [71].

Moreover, it is of particular importance what Feynman states: "From now on we can say that the total energy of an object is mc^2 . Therefore, we have a *new idea*: we do not have to know what things are made of inside; we cannot and need not identify, inside a particle, which of the energy is *rest energy* of the parts into which it is going to disintegrate. It is not convenient and often not possible to separate the total mc^2 energy of an object into *rest energy* of the inside pieces, *kinetic energy* of the pieces, and *potential energy* of the pieces; instead, we simply speak of the *total energy* of the particle. We 'shift the origin' of energy by adding a constant $m c_0^2$ to everything, and say that the total energy of a particle is the mass in motion times c^2 , and when the object is standing still, the energy is the mass at rest times c^2 " [18].

Furthermore, it is crucial keeping in mind that the value of the *density of mass energy* carried out by h , by the Planck's *grain*, although infinitesimal (and without considering its number of oscillations per second) will always be $\neq 0$! In this respect, as already shown with Eq. (4), it was Planck himself who first sketched the possible value, certainly *not without mass*.

In closing, it really is nonsense to continue to uphold the common notion that P is massless. This denies the reality of the facts and the manifold evidence, including experimental evidence, of the numerous mechanical effects induced by light!

And, as students of Physics are well aware, Mechanics implies mass [97].

The main subject, the 1st actor in all mechanical phenomena is the mass!

Well, it is truly astounding that in the face of this evidence and all the other mechanical phenomena listed above, and all of them activated or induced by light, by P_s , one can still maintain that electro-magnetic radiation (EMR) does not carry the slightest mass.

Of course, one could argue: it is the very energy of P_s that does all this. It may be, but still having to admit that with its energy, in order to explain the various mechanical actions, P also vehicles a mass value, e.g. an *energy-mass density*. In this respect, in order to better understand, and in part justify the possible objections, it is interesting to bear in mind what Penrose writes: "In fact, the mass (m) is conserved, but being composed, in part, of energy, today seems to be less clearly the measure of real substance" [39].

In other words, mass itself is also composed of energy!

In fact, what Penrose has just read is masterfully confirmed by the *primordial nucleosynthesis* described by Weinberg [37], from which it is clear that the first forms of *baryonic matter* (protons and neutrons) were assembled from P_s very hot, i.e. created by EMR brought to very high temperatures, i.e. " ≥ 10903 billion $^{\circ}\text{K}$ " [37]: as if to say that the matter, the mass, is nothing but highly concentrated energy!

In this regard, we also take up the endorsement given to us by Feynman: "Finally, associated with the relativity theory, there is a modification of the laws of *kinetic energy*, or whatever you wish to call it, so that *kinetic energy* is combined with another thing called *mass energy*. An *object* has energy from its sheer *existence*"[18]. Likewise, it is deduced that the *object* in question (a particle like P, e.g.) also possesses a mass, to be exact a proportional *equivalent mass energy* [99].

In short, just what Feynman reported completely coincides with the concepts underlying the possible *theorem* proposed in this paper.

Likewise, the considerable mass carried by the 2 P_s generated by the decay of the π^0 meson, as described by Penrose [39], cannot be denied at all. On the contrary, such massive P_s , spontaneously generated in nature, represent one of the most significant and valid *physical* and *mathematical demonstrations* of the real existence of *non-zero mass P_s* , i.e. of the possible truthfulness of the proposed *theorem*.

Lastly, again in relation to the close and intrinsic correlation between mass and energy in any particle, P included, we read from Feynman: "In the Einstein Relativity Theory, anything which has energy has mass, mass in the sense that it is gravitationally attracted. *Even light*, which has an energy, *has a mass*" [18].

And what is this *mass*?

According to Eq. (29), reported by Penrose [39], the mass (m_γ), carried by any photon (P), should correspond to:

$$m_\gamma = \frac{h \nu}{c^2} \quad (30)$$

where the parameter ν , of course, indicates the frequency of oscillation of the P under consideration.

More specifically, according to our calculations, a common P in the optical band, caught in full movement, as Eq. (11) shows, should carry a mass (m_γ) related to its *momentum* (p), i.e. a *hidden dynamic-mass* of $1.325 \cdot 10^{-22}$ [g·cm/s].

If, on the other hand, the same optic P is studied in its *inertial state*, in which it can only show its *rest energy* (E_o), as can be seen from Eq. (20) its *equivalent rest mass* (m_o) should correspond to $6.626 \cdot 10^{-27+n}$ [g·cm²/s²], where the parameter n , i.e. 10^n , indicates the frequency of oscillation of the considered photon.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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