TDOA Wireless Localization Comparison Influence of Network Topology

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ABSTRACT

The interest to wireless positioning techniques has been increasing in recent decades due to widespread of location-based services as well as constraints imposed by regulator on cellular operator to achieve an accepted level of cellular accuracy regardless of availability of GPS signals. Nevertheless, failure of some base stations cannot be fully avoided, yielding various cellular topologies, which, in turn would likely influence the accuracy of the positioning. This paper explores four types of cellular topologies: balanced, circular, U-shape and linear, which can be inferred from balanced topology structure. Assuming time difference of arrival technology and, up to some extent, time of arrival technology were employed, least square like methods are contrasted with maximum likelihood, Taylor, Chan and hybrid approaches in a simulation platform.

Keywords: wireless positioning, topology, network, TDOA

1 Introduction

With the substantial increase of location based services, which include E911 [1] emergency services where user is tracked with high accuracy using only operator’s cellular infrastructure, mapping and path finding, targeted advertising, location based social networking such as MySpace, Friendster or Facebook, the interest to wireless localization techniques has grown drastically in the last two decades. In addition, many ubiquitous applications, including systems like EasyLiving [2] and the Rhino Project [3], among others [4], would benefit from a practical location sensing system. RADAR [5] was one of the first systems to use radio frequency (RF) signal intensity for location-sensing. Small et al. [6] and Smailagic et al. [7] looked at how signal intensity varies over time and developed a location-sensing system based on these observations. Strictly speaking, several localization techniques have been reported in the literature in order to deal with wireless localization, depending on the available technology, which include time-of-arrival (ToA), angle-of-arrival (AOA), time-difference of arrival (TDOA), and received-signal strength (RSS) [8]. Likely the RSS method, where the signal strength from the base station as received in the mobile station is employed as key, which is the less demanding and cheap technology as it does not require any infrastructure change or additional hardware component, which motivates its use in some of above projects like radar [2, 5]. TDOA is recognized for its efficiency and high precision, but requires synchronization among base stations. Indeed, this requires a very accurate timing reference at the mobile which would need to be synchronized with the clock at the base stations. In commonly
employed CDMA system [9], TDOA can be implemented using the pilot tones from different base stations, where the pilot tone transmitted by each cell is used as a coherent carrier reference for synchronization by every mobile in that cell coverage area, which enables the mobile to differentiate each cell site’s pilot tone. Therefore the mobile measures the arrival time differences of at least three pilot tones transmitted by three different cells.

Most of the literature survey, including the survey of Guvenc and Chong [8], investigated the performance of the localization algorithms regardless the sensor infrastructure disposition. Although in GSM and UMTS network, it is acknowledgeable that the antenna positioning problem (APP) is one of the major design issues for any mobile operators. It is universally agreed that several factors influence such design. This includes, the (expected) traffic, type of antennas, allocated frequencies, interference, coverage, infrastructure nearby, among others. Since earlier work of Anderson and McGeehan [9] in antenna positioning problem, several other works have been published as well as several national and transnational research projects have been initiated. The idea of integrating several aspects of the network design problem is carried out by Reininger and Caminada [10], as part of the ARNO Project. In the latter, the authors partially relate APP and frequency allocation problem by “optimizing location and parametrization of the base stations on one shot”.

The integration of locating and configuring base stations is carried further to UMTS networks by Amaldi et al. [11], where the problem of selecting the location and configuring the base stations so as to minimize installation costs as well as to meet the traffic demand is considered. In [12] a trade-off is sought between minimum overlap and desirable cell shapes while the quality of radio coverage is controlled in the constraints. Zimmermann et al. [13] as part of EU ARNO project developed a multi-criteria model that involves a minimum cost, minimum interference and optimum cell shapes. This reveals that most of work in this area has rather been performed from operational research perspective where a multi-criteria decision making like approach has been pursued. Unfortunately less work has been achieved from wireless positioning accuracy perspective has been achieved, although this would significantly contribute towards the E911, for instance. This motivates the current work where some commonly employed techniques involving TDOA and ToA technology are contrasted and investigated with respect to the geometrical disposition of the antennas. More specifically, approximated least square solutions, Maximum likelihood estimation [8], Chan [14], Taylor [15] and a newly introduced combination of Chan-Taylor [16] are compared when considering several antenna topologies. The latter includes linear, circular, U-shape and balanced shapes. Such topology can straightforwardly be inferred from regular (optimal) cellular disposition when some blocking occurs making some BS disabled. The first section of this paper reviews the (eight) main localization techniques employed in this study. Section 3 highlights the simulation platform and comments the obtained results. Finally some conclusive remarks are reported in Section 4.

2 Review of Main TDOA Localization Techniques

Let us consider a general model for the two dimensional (2-D) estimation of a source, consisting of mobile station with Cartesian coordinates \((x, y)\) using \(M\) base stations of known locations \((X_i, Y_i)\), \(i = 1 \text{ to } M\). Then the measured distance between the mobile station and the \(i^{th}\) base station can be given as:

\[
d^i = \sqrt{(X_i - x)^2 + (Y_i - y)^2} + \epsilon_i = d^i + \epsilon_i = ct_i
\]
With \( \epsilon \sim \mathcal{N}(\sigma^2, 0) \) is the additive white Gaussian noise with variance \( \sigma^2 \). \( \hat{d}_i \) (i=1, M) stands for estimated distance from MS to \( i^{th} \) BS, and \( t_i \) is the TOA of the signal at the \( i^{th} \) BS and \( c \) is the speed of light. Consequently, for M measurements, the problem comes down to estimating \((x, y)\) from the following set of equations:

\[
\begin{bmatrix}
(X_i - x)^2 + (Y_i - y)^2 \\
\vdots \\
(X_M - x)^2 + (Y_M - y)^2
\end{bmatrix} =
\begin{bmatrix}
\hat{d}_i^2 \\
\vdots \\
\hat{d}_M^2
\end{bmatrix}
\] (2)

### 2.1 Least Square and Maximum Likelihood Solutions

Assuming that one base station, say \( r^{th} \) BS, acts as a reference, subtracting \( r^{th} \) row in (2) from other rows, yields, after some manipulations and defining \( K_r = x_r^2 + y_r^2 \) (i=1, M), to matrix equation:

\[
AX = \frac{1}{2}B,
\]

where

\[
A_{(M-1)\times1} = \begin{bmatrix}
X_1 - X_r & Y_1 - Y_r \\
X_2 - X_r & Y_2 - Y_r \\
\vdots & \vdots \\
X_M - X_r & Y_M - Y_r
\end{bmatrix};
B_{(M-1)\times1} = \begin{bmatrix}
d_i^2 - d_r^2 - (K_r - K_i) \\
d_i^2 - d_r^2 - (K_r - K_2) \\
\vdots \\
d_i^2 - d_r^2 - (K_r - K_M)
\end{bmatrix}
\] (4)

A linear least square solution to (4) yields the following \( LLT1 \) solution:

\[
X = \frac{1}{2}(A^T A)^{-1} A^T B
\] (5)

Another solution proposed in [17] assumes that each BS acts as a servicing BS, and therefore, concatenates the result yielding M (M-1) equations as described by the new \( A, B \) matrices as:

\[
A = \begin{bmatrix}
X_1 - X_2 & Y_1 - Y_2 \\
X_1 - X_3 & Y_1 - Y_3 \\
\vdots & \vdots \\
X_1 - X_M & Y_1 - Y_M \\
X_2 - X_1 & Y_2 - Y_1 \\
\vdots & \vdots \\
X_M - X_1 & Y_M - Y_1
\end{bmatrix}
B = \begin{bmatrix}
d_1^2 - d_2^2 - K_1 + K_2 \\
d_1^2 - d_3^2 - K_1 + K_3 \\
\vdots \\
d_1^2 - d_M^2 - K_1 + K_M \\
\vdots \\
\vdots \\
d_2^2 - d_3^2 - K_2 + K_3 \\
\vdots \\
\vdots \\
d_M^2 - d_1^2 - K_M + K_1 \\
\vdots \\
\vdots \\
d_M^2 - d_M^2 - K_{M-1} + K_M
\end{bmatrix}
\] (6)

Where the application of (5) yields what we will refer here as \( LLT2 \) solution

A third approach to least square solution was proposed in [18] where the average of all measurements is subtracted from each measurement equation in (2), yielding new matrices:
Again the application of (5) yields a solution referred to as \( LLT3 \).

A fourth least square solution is obtained when choosing the \( r^{th} \) reference BS as the one that induces the smallest distance among all other distances but yields same generic solution as (3). Such solution was suggested in [19] and is referred to here as \( LLT4 \).

The previous least square based solutions discard the knowledge about the uncertainty pervading the measurements (e.g., \( \varepsilon_i \)) as modelled by the associated variance-covariance matrix, in order to account for such effect, the maximum likelihood solution MLS yields as a counterpart of (5) [20]:

\[
X = \frac{1}{2} (A^T C^{-1} A)^{-1} A^T C^{-1} B
\]  

(8)

Where \( A, B \) are defined as in (4), while the variance-covariance matrix is given by, assuming without loss of generality \( \sigma_1 = \sigma_2 = \ldots = \sigma_M \):

\[
C = 4d_i^2 \sigma^2 + 2\sigma^4 + \text{diag}\{4\sigma^2 d_i^2 + 2\sigma^4 \ldots 4\sigma^2 d_i^2 + 2\sigma^4 \ldots 4\sigma^2 d_i^2 + 2\sigma^4 \}
\]  

(9)

### 2.2 Chan and Taylor methods

In Chan’s method [14], one assumes the knowledge of the TDOA with respect to a reference BS, say \( r \), so that the measurements are:

\[
d_{ir} = d_i - d_r = cT_{ir}
\]  

(10)

Where the \( T_{ir} \) is the difference of time arrival between \( i^{th} \) and \( r^{th} \) base stations, and \( d_i \) are as in (1). Similarly, one denotes \( X_{ir} = X_i - X_r \), \( Y_{ir} = Y_i - Y_r \). Squaring (10) and substituting in (1) yields after some manipulations to [14]:

\[
d_{ir}^2 + 2d_{ir}d_r = K_r - K_i - 2X_i - 2Y_i \quad (i=1,M, i \neq r)
\]  

(11)

(11) can be put on the form (3) where

\[
A_{ij[M-1]x1} = \begin{bmatrix} X_{ir} & Y_{ir} & d_{ir} \\ \vdots \ & \vdots \ & \vdots \\ X_{ij} & Y_{ij} & d_{ijr} \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ d_r \end{bmatrix} \quad B_{ij[M-1]x1} = \begin{bmatrix} K_i - K_r - d_{ir}^2 \\ \vdots \\ K_M - K_r - d_{M,r}^2 \end{bmatrix}
\]  

(12)

Where the unknown vector \( X \) contains redundant component \( d_r \), and the solution is approached when first assuming low impact of such dependency to the solution, which is then computed in a two-step strategy. Namely, a linear weighted least square is applied first yielding:

\[
X = (A^T Q^{-1} A)^{-1} A^T Q^{-1} B , \quad \text{with } Q = \text{diag}(\sigma_1, \ldots, \sigma_M).
\]  

(13)
In the second step, the estimate is refined as

$$X = \left(A^T \Psi^{-1} A\right)^{-1} A^T \Psi^{-1} B$$

(14)

With

$$\Psi = c^2 BQB$$, with $$B = \text{diag}(d_0^0, d_0^0, ..., d_0^0)$$

(15)

And $$d_i^0$$ stands for noise-free estimate of $$d_i$$, which is approximated assuming $$\text{cov}([x \ y \ d_i]) \approx (A \Psi^{-1} A)^{-1}$$, see [14] for detail.

On the other Taylor’s approach [15] to solve (11) in [x, y] starts with an initial guess $$(x_0, y_0)$$ of the unknown mobile position (x, y), and computes the deviations of the position location estimation:

$$\Delta = \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \left(G_i^T Q^{-1} G_i\right)^{-1} G_i^T Q^{-1} h_i$$

(16)

With

$$h_i = \begin{bmatrix} d_{2,1} - (d_1 - d_2) \\ d_{3,1} - (d_1 - d_3) \\ d_{M,1} - (d_1 - d_M) \end{bmatrix}, \quad G_i = \begin{bmatrix} X_1 - x & X_2 - x & Y_1 - y & Y_2 - y \\ d_1 & d_2 & d_1 & d_2 \\ X_3 - x & X_3 - x & Y_3 - y & Y_3 - y \\ d_3 & d_1 & d_1 & d_3 \\ X_M - x & X_M - x & Y_M - y & Y_M - y \\ d_M & d_1 & d_1 & d_M \end{bmatrix}$$

(17)

In the next iteration, $$x_0$$ and $$y_0$$ are set to $$x_0 + \Delta x$$ and $$y_0 + \Delta y$$. The whole process is repeated until $$\Delta x$$ and $$\Delta y$$ are sufficiently small, resulting in the estimated PL of the source (x; y). The Taylor-series method can provide accurate results; however, it requires a close initial guess $$(x_0, y_0)$$ to guarantee convergence and can be computationally intensive.

In [15], a combination of Chan-Taylor method has been put forward. The proposal assumed a linear combination of the two methods such that the global variance-covariance is minimized. This yield

$$X = \lambda \begin{bmatrix} x_{\text{Chan}} \\ y_{\text{Chan}} \end{bmatrix} + (1 - \lambda) \begin{bmatrix} x_{\text{Taylor}} \\ y_{\text{Taylor}} \end{bmatrix}$$

(18)

With

$$\lambda = \frac{P_{\text{Taylor}}(1,1)^2 + P_{\text{Taylor}}(2,2)^2}{P_{\text{Taylor}}(1,1)^2 + P_{\text{Taylor}}(2,2)^2 + P_{\text{Chan}}(1,1)^2 + P_{\text{Chan}}(2,2)^2}$$

(19)

Where $$P_{\text{Taylor}}$$ and $$P_{\text{Chan}}$$ stand for variance-covariance matrices associated to Taylor and Chan methods, respectively.

3 Simulation

Similarly to most studies investigating wireless localization techniques, the performances are often evaluated through a set of Monte Carlo simulations. A generic simulation platform is shown in Figure 1. The simulation assumes a set of base station at fixed locations (7 BS in Figure 1). As in practical implementations, the cells have hexagonal shapes in order to restrict the interference between cells as
no overlapping region exists. By abuse, we shall refer to such situation a balanced topology. Nevertheless in case where a blocking occurs in some cells, this yields different topology. For instance if the middle BS in Fig 1 is failed, this yields a circular topology. Similarly if the two first cells in the second row of cells in Fig 1 failed, the cells form a U-like shape, so this is referred to U-shape topology. In total, we shall consider here four different topologies: Circular, U-shape, linear and the balanced one as in Figure 1.

![Figure 1: Generic simulation platform (Balanced topology).](image)

Besides we shall consider a vehicle moving at a constant speed in one direction. We therefore, compute for each of the aforementioned localization technique, the localization accuracy with respect to a set of Monte Carlo simulations. The parameters of the simulations for each topology are described in Table 1. The three other topology structures are represented in Figure 2.

![Figure 2: Circular, U- and Linear shape topologies](image)

Typically, to the initial true mobile position is added a random perturbation generated by a zero-mean Gaussian noise with a given standard deviation. A pseudo code highlighting the functioning of the simulation is described in Figure 3.
Table 1: Parameters of the simulation setup

<table>
<thead>
<tr>
<th>BS Topology</th>
<th>Cell Radius</th>
<th>Noise Standard Deviation</th>
<th>MS Starting Position</th>
<th>Moving Distance</th>
<th>Time</th>
<th>Constant Velocity</th>
<th>Freq. of</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balanced</td>
<td>3000 m</td>
<td>0.1 us</td>
<td>[-5000, 0]</td>
<td>10000 m</td>
<td>50 s</td>
<td>200 m/s</td>
<td>Once / second</td>
</tr>
<tr>
<td>Circle</td>
<td>3000 m</td>
<td>0.1 us</td>
<td>[-5000, 0]</td>
<td>10000 m</td>
<td>50 s</td>
<td>200 m/s</td>
<td>Once / second</td>
</tr>
<tr>
<td>U-Shape</td>
<td>3000 m</td>
<td>0.1 us</td>
<td>[0, 0]</td>
<td>15000 m</td>
<td>50 s</td>
<td>30 m/s</td>
<td>Once / second</td>
</tr>
<tr>
<td>Line</td>
<td>3000 m</td>
<td>0.1 us</td>
<td>[0, 450]</td>
<td>3000 m</td>
<td>50 s</td>
<td>60 m/s</td>
<td>Once / second</td>
</tr>
</tbody>
</table>

\[ [MS, \text{RMSE}] = \text{LOCATION ESTIMATION (TOPOLOGY)} \]

RETRIEVE BS\(_i\), Vehicle Movement direction, Std \(\sigma\), Initial MS\(_0\)

FOR EACH sampling interval \(k\)

FOR EACH Monte Carlo iteration

MS = ComputePosition (MS\(_0\), \(k\))

Generate a realization of Noise = (\(\theta, \sigma\))

FOR EACH BS

Calculate distance \(d_i = \sqrt{(BS_x - MS_x)^2 + (BS_y - MS_y)^2 + \text{Noise}}\)

END FOR

Estimate Position MS = LocationAlgorithm (\(d, BS, \text{Noise}\))

END FOR

Calculate RMSE of current MS

END

Figure 3: Pseudo-code of simulation

In order to quantify the performance of the eight localization techniques, at each sampling interval along the trajectory of the vehicle, the RMSE of the averaged MS estimation over the 1000 Monte Carlo simulations is calculated for each location technique; namely,

\[
RMSE(t) = \sqrt{\frac{\sum_{i=1}^{n}((x_{true}(t) - x_i(t))^2 + (y_{true}(t) - y_i(t))^2)}{n}}, \text{ where } (x_i(t), y_i(t)) \text{ stands for MS (x, y) estimation at } i^{th} \text{ Monte Carlo simulation and } t \text{ sampling interval, and } n=1000.
\]

Figures 4, 5, 6 and 7 summarize the localization errors in terms of RMSE of the eight localization techniques when using balanced, circular, U-shape and linear topology.
**Figure 4:** RMSE value in case of balanced topology

**Figure 5:** RMSE value in case of Circular topology

**Figure 6:** RMSE value in case of U-shape topology
The discrepancy of the various positioning techniques when a change of a topology occurs demonstrates the influence of the topology on the accuracy of the underlying positioning method.

In the above simulation, at a given sampling interval, the measurements from all base stations are assumed available and aggregated in the localization technique. Although such data cannot be straightforwardly be available in cellular network in practice, where the mobile station is only connected to the base station providing the strongest signal, it is still available from network provider perspective. Besides, such approach is commonly employed in previous work that investigated the performance of cellular/wireless network positioning techniques as testified in the extensive review paper [8].

Looking at the range of the RMSE values with respect to various topologies reveals that the balanced topology produces the best performance with respect to all positioning techniques, while the linear shape topology yields the worst performance as its associated values RMSE go beyond 340 m as compared to less than 30 m in case of balanced topology. This shows that whenever possible the use of balanced topology should be persuaded. This is mainly due to quality of the obtained measurements, where, at least from geometrical perspective, yields comprehensive intersection of the underlying circles.

The combination method of Chan and Taylor shows on average that it marginally outperforms the remaining seven topologies regardless the topology employed.

The investigation of the low values of RMSEs in the above figures reveals that (almost) the least square like methods approach the minimum RMSE value at a sampling time corresponding to the time the vehicle comes close to underlying base station. While such phenomenon is less apparent in case of Chan, Taylor and Combined Chan-Taylor methods where less sensitivity is observed. This is mainly due to the global nature of the above positioning methods.

The above results have been obtained assuming low noise perturbation as testified by the low standard deviation shown in Table 1. Nevertheless, the influence of the noise intensity cannot
be excluded. On the other hand, few extra simulations with various noise intensities have shown that the generic trends issued from this analysis are not void when the level noise increases. To see it, a 3D graph is depicted in Figure 8 and Figure 9 for balanced and linear like topologies.

- So far, the metric employed for comparison is only related to the accuracy of the positioning technique. Nevertheless, one should bear in mind that some techniques are computationally significantly more expensive than others. From this perspective, LLS1 is computationally the most effective one, and also provides good balance between accuracy and computational cost. While Taylor and combined Chan-Taylor are the most expensive ones because of the iterative approach they do involve. Strictly speaking, even for the LLS1, the computational cost increases with the number of measurements available (value of parameter M). This is mainly due to the cost involved by the matrix inversion operation.

![Figure 8: Noise influence in case of balanced topology structure](image1)

![Figure 9: Noise influence in case of Linear shape topology](image2)

4 Conclusion

This paper highlights the importance of the antenna positioning when looking at the accuracy of the wireless positioning techniques. Four type of topologies, which can straightforwardly be generated by a regular balanced cellular topology when some blocking occurs, have been investigated. Wireless positioning techniques related to TDOA technology have been examined. This corresponds to four distinct least square based approaches, maximum likelihood, Chan, Taylor and a combined Chan-Taylor method. Simulation results have been obtained assuming a vehicle moving at a constant speed along the given topology. The results demonstrate the credibility of the topology influence on the positioning.
accuracy. Besides, the combined Chan-Taylor shows a marginally increased performance in terms of RMSE and sensitivity to base station positioning.

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