Setting an Optimization Problem of Distributing Traffic Across Multiple Independent Paths

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**ABSTRACT**

Communication network development is considered as an urgent need for the world. The proposed work in this paper is to solve the problem of the traffic engineering, using the multi-path routing mechanisms by applying a developed algorithm includes a multi-criteria optimization procedure to define the optimal number of paths. The values of the maximum flow and multi-path delay are used as partial criteria; however, a set of particular criteria can be extended when needed to ensure the required quality of service.

**Keywords:** Routing Protocols, QoS, TN, hybrid network, traffic engineering, traffic distribution, path delay, the Dijkstra’s algorithm

1 Introduction

The paper presents the optimization procedure providing traffic distribution in a critical mode of network operation based on [1-15]. In [2-4], network coding technique was applied to obtain the optimum transmission algorithm based on a minimum number of transmitted packets and hence obtaining the shortest path to be shorter than the traditional optimization algorithm.

The benefit of the proposed procedure is implementing the recursive space-time operations of information flows redistribution, resulting to ensure the network functioning stability, accordingly, the network reaches a state where the traffic is approximately evenly distributed over the corresponding routes, after which the normal dynamic routing procedure is started. The analysis of the developed method shows the existence of a limited number of the most significant routes, which depends on the connectivity of the network, and the amount of traffic that allows maximizing the compound index of the quality of service (QoS) such as in [5-8].

2 Optimization of Traffic Distribution

The single-product flow distribution between network node s to node t across m independent shortest paths is considered in this work. This problem is a special case of the multi-path routing problem considered in the literature [7-17]. The algorithm for solving such a generalized problem has a large computational complexity and the time of solution of such a problem cannot meet the requirements for the means of multipath routing. These requirements are applied to the routing algorithms traditional requirements, and its low computational complexity, rapid convergence and minimum volumes of created...
service traffic were proposed [7-9]. In addition, the conversion factor $q$, which determines the degree of filling of the arc, is determined only for a particular case. There are no general assumptions on the choice of this factor, which narrows the possibilities of practical use of this generalized model.

The notation system adopted in streaming models is used, where the network structure $G(N,E)$ is determined by the set of nodes $N=\{1,2 \ldots n\}$ and set of arcs $E \subset (N \times N)$. Let the arc flow $i_k, (k = 1, cardE)$ be $f_k$. For each arc $k$, a value of $c_k$ is assigned to define the upper boundary of the flow along the arc $k$. Let the set of independent shortest paths from noted $s$ to node $t, (s, t \in N)$ to be $P_n$, where $P \in G$.

The flow $Q$ specified value between a minimal cut (correct section) $C^*$ of the fragment $P$ of the network $G$ satisfies the following condition in (1):

$$Q < C^*, \text{ Where } C^* = \sum_{j=1}^{m} \min \{c_j\}, \; j = 1, r_j,$$

$j = 1, m, m=card$, and $r_j$ - rank of $j$- path of the flow $Q$ across $m$ shortest paths where more than one solution (not trivial solution) can be obtained, so, the setting optimization problem becomes possible. The requirement (1) is satisfied at the stage of choosing the number of shortest paths $m$ and constructing the set of these paths $P$.

One of the requirements for the flow distribution over $m$ independent paths is the minimization of the transmission time of the flow $Q$ from $s$ to $t$, which will be determined by the maximum transmission time of the corresponding parts of the flow $Q$ across the $m$ paths. Minimization can be achieved by equalizing the transmission time along all paths. Let the value of the flow routed along the $i$- path to be $x_i, i = 1, m$, $c_i = \min \{c_j\}, \; j = 1, r_i, h_i = \frac{c_i}{\sum_{j=1}^{m} c_j}$. Then the problem of optimal distribution of the flow of the value $Q$ across the $m$ paths with given minimum capacities $c_j$, can be formulated as a mini to max (minimax) problem:

$$\max \left| \frac{x_i}{c_i} - \sum_{j=1}^{m} h_j x_j \right| \rightarrow \min \text{ subject to } x \in \Omega$$

Taking into account that those values of the given flow $x_i/c_i$, that are less than the weighted average value, are not necessary to minimize, and hence formula (2) is transformed to the (3):

$$\max \left( \frac{x_i}{c_i} - \sum_{j=1}^{m} h_j x_j \right) \rightarrow \min \text{ subject to } x \in \Omega$$

Then the minimax problem (3) can be reduced to the linear programming problem:
\[ z(x) = \sum_{C=1}^{m} \frac{c_i}{C} \cdot x_i \rightarrow \max \]  

(4)

under the following constraints in (5), (6), (7), and (8):

\[ \sum_{i=1}^{m} x_i = Q \]  

(5)

\[ x_i \sum_{j=1}^{m} c_j - c_i \sum_{j=1, j \neq i}^{m} x_j \geq 0 \]  

(6)

\[ x_i \leq a_i c_j \]  

(7)

\[ x_i \geq 0 \]  

(8)

The constraint (5) is the requirement to distribute the entire flow, the constraint (6) is the requirement that the distribution to be proportional, the constraint (7) is the requirement to prevent the overloads in the "bottlenecks" of the \( i \)-path, and the coefficient \( a_i \) determines the maximum load of the \( i \)-path.

### 3 Modeling Results Analysis

The modeling algorithm was carried out for a network section in which there are 5 independent shortest paths from the source to the destination using Matlab simulation. As a load model in the node \( s \), a traffic source with a normal distribution and M.O. equal to 10 was used. To analyze the proposed algorithm, in the node \( s \), the flow was distributed without optimization procedures and with the use of optimization (fig. 1 a and b) respectively.

![Figure 1: Distribution of load across the paths without optimization (a), and with optimization (b).](image_url)

The (fig.1.a and b) shows the distribution in time of the incoming flow along the multiple paths. When distributed without optimization (fig. 1-(a) the maximum capacity path has the highest priority and is "filled" completely. The "remainder" of the flow is routed along the path that has the highest priority among the remaining paths. Path priorities are determined on the basis of the throughput of the path. As can be seen from the (fig.1-a), path having the maximum bandwidth 6 is loaded completely, then the next
path having the maximum value of 5, etc. Thus, when using such a load distribution scheme, some paths will be overloaded, while others will be idle. The fig. 1-(b) shows that when using optimization; all paths are loaded more compactly and values $x_i$ are proportional to the path capacity.

The fig. 2-(a) shows the average value of the flow $X_i$ along the $i$- path, so, can be seen, in the case of optimal load distribution, the average path loading is 52% (fig. 2-(a)), and the maximum value of loading is 58% (fig. 3). In contrast to this, when using load distribution without optimization several paths are loaded completely, while the others are idle (fig. 2 (a) and (b)).

![Figure 2: The path number n with and without optimization for the flow size Xi (a), and Ci /Xi (b).](image)

The Figure 3 shows the dependence $M \left[ \max_i \left( \frac{x_i}{c_i} \right) \right]$ on the value $n$ when using optimization. Expectation value in this case is 0.58, i.e. 58% of the value $\max_i \left( \frac{x_i}{c_i} \right)$. The value of this parameter when using the load distribution without optimization is 1.

![Figure 3: Expectation value of the parameter](image)
4 The Proposed Algorithm Computational Complexity Estimation

When solving the optimization problem; it is necessary to analyze the feasibility of the proposed routing algorithm in real time.

The real time is determined by the properties of the processes taking place in a certain separately taken system, therefore it is necessary to determine the time interval in which the routing task at the operation stage must be solved. As such a time interval, it is advisable to choose the time during which alternative paths for traffic transmission must be selected.

The multi-path routing algorithm proposed in the paper uses the methods of finding the shortest paths on a graph, solving the problem of multi-criteria optimization and mathematical programming for finding the paths and load distribution. The amount of calculations for finding the shortest paths using the Dijkstra’s algorithm is known to be \(O(N^2)\), where \(N\) is the number of vertices of the graph [13-14]. Then, assuming that the optimal number of independent shortest paths is equal to the connectivity of the graph of the network \(S\) (the worst case), the computation volume of this part of the problem can be estimated by the following formula:

\[ E_1 \approx N^2 \cdot S \]  

(9)

The amount of computation for solving the multi-criteria optimization problem is determined by the following procedures:

1) Calculation of utility functions according to formula

\[ X_m^0 = \arg \max_{x \in X} K_m(x), m = 1, n \]  

(10)

2) Finding the optimal solution for the given importance of particular criteria.

The assumption that \(\text{card} M = S\), where \(M\) is a set of shortest paths, \(S\) is the network graph connectivity, the calculation of the utility function for 2 criteria for \(S\) variants will be \(2 \cdot 3 \cdot S\) operations. The construction of an additive criterion and finding the maximum will constitute \(4 \cdot S\) operations. Then the amount of computation in solving the multi-criteria optimization problem can be estimated using the following formula:

\[ E_2 \approx 10 \cdot S \]  

(11)

To distribute the flow, in a given network, the linear programming problem was solved. As indicated by experts in the field of mathematical programming, the simplex method is considered to be the best procedure for solving linear programming problems [11]. There are many modifications of this method, however, according to experts; none of the versions of the simplex method gives a tangible gain of the time of calculations. It is shown in [12] that in the general case the linear programming problem is \(NP\)-complete. However, in [11-17] it is shown that when solving applied problems, such as finding the maximum flow, etc., the simplex method has a polynomial complexity. The computational efficiency of the simplex method according to [11] can be estimated using the following two parameters: number of iterations (admissible basic solutions needed to achieve the optimal solution of the problem) and the total machine time required to solve the problem. The experience of solving a large number of practical
problems shows that the upper bound of the iterations in the solution of the linear programming problems in standard form with m constraints and n variables it is possible to consider the value 2(m+n). 

\[
E_3 \approx 2(n + m)^2 \cdot (5m + 3) \tag{12}
\]

For the problem under consideration, the number of constraints is three times greater than the number of unknowns, then

\[
E_3 \approx 180n^3 + 54n^2 \tag{13}
\]

The multi-path routing problem time can be estimated using the following expression:

\[
T = (E_1 + E_2 + E_3) \cdot t_{on} \approx \left[180 \cdot S^3 + 54S^2 + S(10 + n^2)\right] \cdot t_{on}, \tag{14}
\]

Table 1 shows the calculation of time for solving the multi-path routing problem for networks of various sizes and connectivity for a device with \(t_{on} = 10^{-9}c\).

<table>
<thead>
<tr>
<th>Number of network vertices, (N)</th>
<th>Network connectivity, (S)</th>
<th>Problem time, (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>5</td>
<td>2,85 \times 10^{-5} c</td>
</tr>
<tr>
<td>30</td>
<td>20</td>
<td>1,5 \times 10^{-3} c</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td>2,1 \times 10^{-4} c</td>
</tr>
<tr>
<td>50</td>
<td>30</td>
<td>5,0 \times 10^{-3} c</td>
</tr>
<tr>
<td>100</td>
<td>20</td>
<td>1,02 \times 10^{-3} c</td>
</tr>
<tr>
<td>100</td>
<td>50</td>
<td>2,3 \times 10^{-2} c</td>
</tr>
</tbody>
</table>

From Table1; it is clear that the problem time does not exceed the allowable value of 50 ms, which makes it possible to use the proposed algorithm in MPLS-TE networks.

5 Conclusion:

The proposed algorithm in this paper allows to significantly reduce the load on individual sections of the network and to perform load optimization, distributing traffic along the optimal number of paths. As shown by the simulation modeling, in the particular proposed case, the use of the proposed multipath routing scheme allows achieving a reduction of up to 40% of the total load on single paths by redistributing the traffic.

REFERENCES


URL: http://dx.doi.org/10.14738/tnc.54.3346