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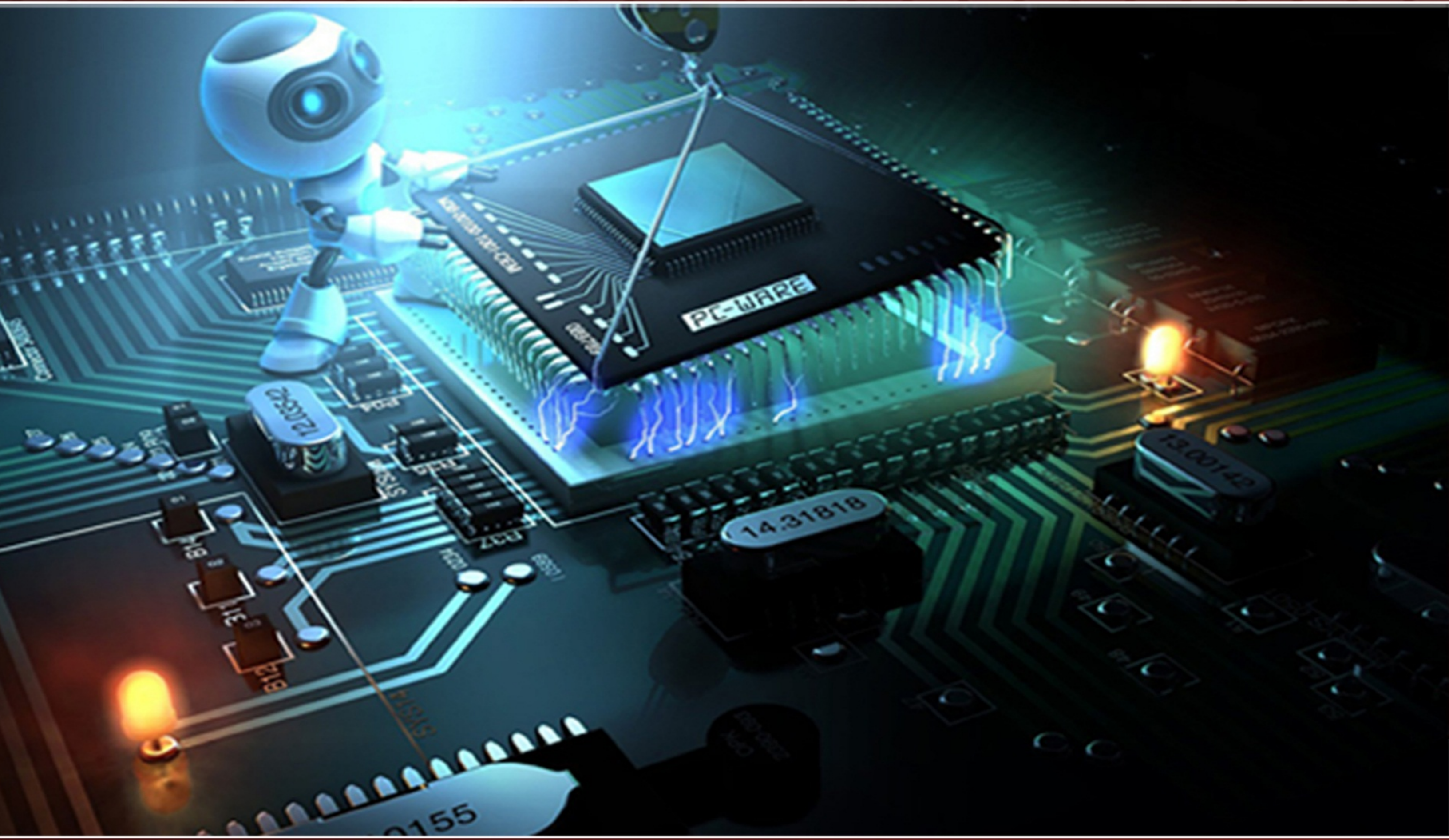

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Understanding the Cognitive Processes involved in Technological Entrepreneurial Opportunity Recognition

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ABSTRACT:

In the state-of-the art An Agenda for Future Research, Corbett and McMullen (2011) summarize the current state of the research in the following words - "Despite its many achievements, learning at the connection of free enterprise and cognition has focused primarily on the consequences of what happens when an industrialist profit from various cognitive characteristics, resources, or other dispositions. As such, cognitive study in free enterprise continues to experience from narrow theoretical articulations and weak theoretical practicalities that lower its role to the managerial sciences."According to Santos and Eisenhardt (2004)" entrepreneurs categorize innovative opportunities for the conception of worth, and construct a market around those opportunities" triumphant chance appreciation leads to successful start-ups and leads to the vibrancy of economy. However, very little work has been done in this field (Mitchell et al., 2014).

Key words: Industrial, Authority, Approach, Perceptive, Gratitude, Cognition, Experimental, Inspiration, Proficiency

Technical Details

The objective is to understand the Cognitive Processes involved in Technological Entrepreneurial Opportunity Recognition. This step involves creating a model with various skill sets that could influence opportunity recognition. Also, the modeling involves identifying the interrelationships between these skills. The second objective establishes the baseline data for the Indian technology entrepreneurs and compares it with their counterparts in USA. This benchmarking can help in establishing aspirational goals and also transforming our education system.

1 Introduction

In the state-of-the artarticle "The Cognitive Perspective in Entrepreneurship: An Agenda for Future Research," Grégoire, Corbett and McMullen (2011) summarize the current state of the research in the following words - "Despite its many achievements, scholarship at the intersection of entrepreneurship and cognition has focused primarily on the consequences of what happens when an entrepreneur benefits from various cognitive characteristics, resources, or other dispositions. As such, cognitive research in entrepreneurship continues to suffer from narrow theoretical articulations and weak conceptual foundations that lessen its contribution to the managerial sciences."According to Santos and Eisenhardt (2004)"entrepreneurs perceive new opportunities for the creation of value, and construct a market

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around those opportunities" Successful opportunity recognition leads to successful start-ups and leads to the vibrancy of economy. However, very little work has been done in this field (Mitchell et al., 2014).

2 Origin of the Proposal

Opportunity recognition is considered as a key cognitive skill that distinguishes an entrepreneur from the rest and is the beginning stage of the entrepreneurial journey (Christensen et al. 1994; Gaglio 1997; Knowlton, 1997; Shane and Venkataraman, 2000).Gaglio and Katz (2001:95) even describe its importance as "understanding the opportunity identification process represents one of the core intellectual questions for the domain of entrepreneurship" Baron (2006) identified three critical elements for the opportunity identification as actively seeking or searching for opportunities, attentiveness or alertness to potential opportunities and previous knowledge, which includes customers, market, technology and industry.

3 Definition of the Problem

To date, we have very little or no information about the cognitive processes involved in the technical opportunity recognition. As a result, we may be ineffectively training our future engineers and hampering their entrepreneurial mindset. Our goal is to understand technical opportunity recognition process and benchmark the abilities of Indian entrepreneurs. This study will be the first study to comprehensively examine the topic of opportunity recognition and establish the baseline information for the Indian entrepreneurs.

4 Objective

The objective is to understand the Cognitive Processes involved in Technological Entrepreneurial Opportunity Recognition. This step involves creating a model with various skill sets that could influence opportunity recognition. Also, the modeling involves identifying the interrelationships between these skills. The second objective establishes the baseline data for the Indian technology entrepreneurs and compares it with their counterparts in USA. This benchmarking can help in establishing aspirational goals and also transforming our education system.

5 Review of Status of Research and Development in the Subject

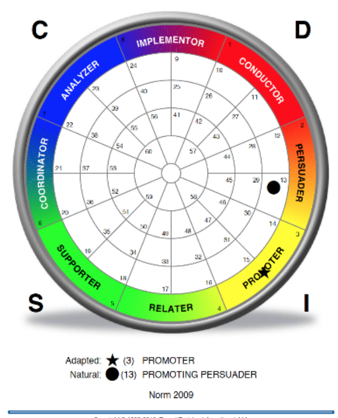
As Albert Einstein famously noted "Innovation is not a product of logical thought, although the result is tied to logical structure." As academicians, we often try to put a systematic procedure around this nonlinear process. In fact, we teach our engineering and business students systematic search models for opportunity recognition (Fiet, 2002). However, it is very well noted that entrepreneurs don't follow systematic search or heuristic methods (Shaver & Scott, 1991 and Busenitz& Barney, 1997). In a similar open-ended problem solving domain, Brooks (2003) notes that "…the rational models of the design process …is dead wrong and seriously misleading." In fact the models donot capture the process adopted by experts and therefore, results in "bizarre" results.Kirkner (1973 & 1979) introduced the concept of entrepreneurial alertness. It refers to the cognitive ability to identify opportunities without consciously searching for them. According to Kirzner, alter individuals experience an "aha" moment, which provides a different point of view and helps him/her to identify the opportunity in a very nonlinear fashion.

Groves (2011) empirically studied entrepreneurial cognition of entrepreneurs and concluded that the successful entrepreneurs have a versatile balance of the nonlinear thinking with more rational linear thinking. This landmark study finds differences in 219 professionals (not students), which includes 39 entrepreneurs, and uses the Linear and Nonlinear Thinking Stile Profile (LNTSP) instrument developed by Vance et al. (2007). This study is relevant to this proposal as it uses the same instrument to see the balance in Indian entrepreneurs.

DeTienne and Chandler (2004) empirically study the influence of entrepreneurial classroom using a Solomon Four-Group Designed experiment and propensity to innovate using Kirton Adaptor Innovator inventory (KIA) instrument. This particular study is very relevant to this proposal as it provides similar benchmarking data for young entrepreneurs in USA.

Finally, Pistrui (2013) uses a well-recognized survey instrument by Target Training Institute (TTI) to understand entrepreneurial mindset in engineering student in USA. The instrument creates a 62-page report that provides detailed qualitative and quantitative insight into behavior, motivation and skill. The survey results includes:

a) Insight into natural and adapted behavior in terms of DISC (Dominance, Influence, Steadiness, Compliance). According to TTI, DISC is "is the language of how we act – i.e. our behavior. Research has consistently shown that behavioral characteristics can be grouped together into four quadrants, or styles. People with similar styles tend to exhibit specific types of behaviors common to that style – this is not acting. A person's behavior is a necessary and integral part of who they are. In other words, much of our behavior comes from "nature" (inherent), and much comes from "nurture" (our upbringing). The DISC model merely analyzes behavioral styles; that is, a person's manner of doing things." The instrument provide both qualitative and quantitative analysis as shown in the figure below

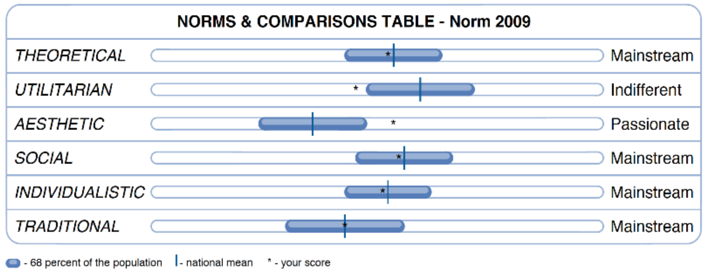


Sample DISC Wheel

b) Motivators are factors that drive our behavior or in other words, the motivators tell why we do things. The six values or motivators are

1. Theoretical is drive for acquiring new knowledge
2. Utilitarian is the drive for creating a value proposition that solves peoples problems.
3. Aesthetic is the drive for harmony in form.
4. Social/altruistic is the drive for helping the society
5. Individualistic is the drive for control and power.
6. Traditional is the drive for maintaining the order.

sample results are shown in the figure below.



Sample motivator

c) The instrument also ranks the core skills of the individual. Crawford et al. (2011) associated the seven professional skill clusters with the right brain thinking -

1. Experiences
2. Team Skills
3. Communication Skills
4. Leadership Skills
5. Decision Making/Problem Solving Skills
6. Self-Management Skills
7. Professionalism Skills

TTI survey expands them into lower level skills as shown in the sample data below.

Score	Mean	Description
9.6	7.6	Using Common Sense
9.0	8.3	Theoretical Problem Solving
8.9	8.0	Following Directions
8.9	8.0	Respect for Policies
8.9	7.6	Integrative Ability
8.9	7.7	Sense of Belonging
8.8	8.1	Self Improvement
8.8	8.0	Practical Thinking
8.6	7.6	Concrete Organization
8.6	8.0	Attention to Detail
8.6	7.9	Conveying Role Value
8.6	7.8	Monitoring Others
8.5	8.1	Understanding Motivational Needs
8.5	7.9	Emotional Control
8.4	7.9	Correcting Others
8.4	7.5	Sense of Timing
8.4	8.1	Empathetic Outlook
8.4	7.8	Systems Judgment
8.3	7.7	Evaluating Others
8.3	8.2	Realistic Goal Setting for Others
8.2	8.0	Material Possessions
8.1	7.7	Realistic Expectations
8.1	7.3	Project Scheduling
8.1	7.5	Quality Orientation
8.1	7.8	Relating to Others
8.0	7.3	Conceptual Thinking
8.0	7.0	Intuitive Decision Making
7.9	7.4	Project and Goal Focus
7.9	7.3	Results Orientation
7.9	7.9	Attitude Toward Others
7.9	7.8	Freedom from Prejudices
7.9	7.9	Leading Others
7.9	7.5	Problem Solving
7.9	7.9	Proactive Thinking
7.9	7.9	Sensitivity to Others
7.9	7.7	Evaluating What is Said
7.9	7.6	Status and Recognition
7.9	8.1	Personal Relationships
7.8	7.8	Persuading Others
7.8	7.3	Sense of Self
7.7	8.2	Respect for Property
7.6	7.4	Self Confidence
7.6	7.4	Handling Rejection
7.5	7.6	Long Range Planning
7.5	7.1	Role Confidence
7.5	7.4	Enjoyment of the Job
7.4	7.3	Surrendering Control

Score	Mean	Description
7.4	7.6	Realistic Personal Goal Setting
7.4	7.1	Internal Self Control
7.4	7.2	Persistence
7.4	7.4	Self Management
7.3	7.2	Personal Accountability
7.3	7.5	Accountability for Others
7.3	7.0	Balanced Decision Making
7.3	7.2	Taking Responsibility
7.2	7.3	Consistency and Reliability
7.2	6.7	Self Assessment
7.2	7.4	Developing Others
7.2	7.3	Job Ethic
7.1	7.0	Handling Stress
7.1	7.3	Sense of Mission
7.0	7.1	Role Awareness
6.8	7.1	Gaining Commitment
6.8	6.9	Meeting Standards
6.4	7.1	Personal Drive
6.4	6.9	Self Direction
6.2	6.9	Initiative

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Sample Skills

Pistrui et al. (2012) investigated the relationship between TTI assessment data and Entrepreneurial mindset based on a major survey of approximately 5000 undergraduate students and 300 practicing entrepreneurs. This is the most comprehensive empirical study performed in the area of engineering entrepreneurship. Once again, we plan to use this instrument to independently validate the model and also benchmark our engineering entrepreneurs.

As opposed to previous three empirical studies, Corbett (2007) based on an experimental task studied 380 technology professionals to understand how opportunities are identified based on the learning (acquiring and transforming information and experience). This study is important as it provides benchmark data and also provides another valid instrument to understand the opportunity recognition process.

6 International status

The primary studycomes from the Kern Entrepreneurship Education Network – a group of private higher education institutions in USA who are reforming engineering education to incorporate entrepreneurial mindset (Pistrui, 2013). The other groups studying include Rensselaer Polytechnic Institute and now Babson College which is ranked no 1 in entrepreneurship education (Corbett, 2005; Corbett et al. 2014) and Colorado State University (DeTienne and Chandler, 2004).

7 National status

Garud and Prasad (2013) from IIM Bangalore describe cognition in the R&D management in Indian hi-tech firms. Kundu and Rani (2008) conducted entrepreneurial attitude and orientation depending on gender and background in the Indian airforce trainees. A Conceptual Framework.Gangaiah and Viswanath (2014) studied the impact of management education in developing entrepreneurial aspiration and attitudes in the Indian context with business students. This research is centered on the problems in the management education. However, very little is done in the area of cognitive science as applied to technology opportunity recognition.

8 Importance of the Proposed Project in the Context of Current Status

Most studies of the opportunity recognition are in the area of business and not based on cognitive understanding. This study will provide an insight into how technology entrepreneurs think in India and also contrasts them with the engineering entrepreneurs. This study also benchmarks the skills of our entrepreneurs with the counterparts in USA.

The study helps to better understand the technology opportunity recognition process based on cognitive science. This in turn can help us can profoundly change the way we teach opportunity recognition as a subject and also in different courses in the engineering education. It also helps to identify the strengths and weaknesses of our engineering entrepreneurs so that we can systematically address them.

9 Methodology

The first part of the proposal requires creating an opportunity recognition model from the existing literature. During this phase, extensive literature review will be carried out to understand the opportunity recognition process particularly in engineering domain. Most models are very similar framework with four or five basic steps (shown in the figure below) with different levels of details. However, the focus of this particular study will be in understanding cognitive traits that play a crucial role in the process. The model will capture the current thinking in the domain of cognitive science and opportunity recognition.

Basic steps in the opportunity recognition process

Once the model is complete, the task of collecting empirical data to fine-tune the model begins. During this task, we adapt the three empirical studies to include an emphasis on technology opportunity recognition. For these studies, the principal investigator will recruit 250 entrepreneurs from non-technical areas and 250 entrepreneurs with technical background. These participants will be given the Solomon Four-Group Designed experiment, Kirton Adaptor Innovator inventory (KIA) instrument, andNonlinear Thinking Stile Profile (LNTSP) instrument. These instruments are used by Groves (2011) and DeTienne and Chandler (2004). Thus, these experiments will provide a bench mark data as applied to Indian ecosystem as well as provide additional insights as we are looking into the technology domain. Further, a select number of total sample (a total of 50 participants) will be given both TTI survey. Consistent with Pistrui study (2013), the Structural Equation Modeling or SEM techniquewhich is useful understanding and evaluating the relationshipbetween human behavior and human motivation (Wallgren and Hanse, 2007; Williams et al., 2003) and Cronbach's Alpha, an accepted measure of internal consistency or reliability will be used to perform factor analysis of a combined model (Cronbach, 1951).

These select group will also be asked to repeat Corbett's experiment with the additional verbal protocol analysis method. Verbal protocol analysis is a think aloud method where participants or subjects verbalize their thought processes while performing the design task. After the task is completed, these audio/video sessions are transcribed and analyzed to gain insights into the thought process thereby answering the research question.This type of analysis has been accepted as a valid research method (Ericsson, 1980). The protocol analysis method will be to replicate Corbett's experiment.

In the experiment, Corbett asks participants to identify new opportunities for the Bluetooth technology. The actual instructions are shown in the figure below. Note that in additional to Corbett's experiment instructions will be modified to asking participants to verbalize the thoughts. These verbal thoughts are both audio and video taped. It is typically acceptable to perform the protocol analysis on a much smaller set of participants. Also, they will be asked to perform a second task "design a innovative planter for people living in apartments for gardening." The task will require the participants to sketch as engineers are visual thinkers and Cobett's experiment doesn't capture the visual thinking.

Finding New Opportunities

In this section we would like to examine your ability to find new potential business opportunities. After reading the passage below on a new emerging technical protocol, take a few minutes to list any potential business opportunities based on this protocol that come to mind. The ideas you list may or may not be related to your current business.

Please note: It is extremely important for the validity of this survey that you take a few minutes to think creatively and try to answer this question as fully as possible. **The remaining questions will only ask you to tick boxes or provide one-word answers.** After taking a few minutes on this one question you will be done with this survey shortly. Thank you again for your consideration.

A wireless technology is currently being developed that operates over radio waves. This technology will allow all electronic devices to "talk" with one another without cable connections. One example of a business opportunity that has come from this technology is now your can connect to the internet on a portable computer wherever you are without being connected to a cable.

17. Use the following space to list any ideas for new products, services, or business opportunities based on the above technology. List all ideas that come to mind and list each separately as #1,#2, and so forth.

Corbett's experiment

These large quantitative empirical data coupled with detailed qualitative data obtained through the protocol analysis will help to fine tune the model, identify interrelationship between the various parameters, and spots trends in people who are good in opportunity recognition compared to others. It also provides insight into the development or a lack development of opportunity recognition skills during the course of engineering study. This information will be used to fine-tune the model with quantitative relationship between various cognitive skills.

Once, the model is completely developed, a large-scale study of 5000 participants will be undertaken. The verbal protocol study will be on a smaller sample size as it is time consuming to analyze the data and TTI survey will also be on a smaller size as it is expensive. The large-scale study will be used to validate the opportunity recognition model.

10 Organization of Work Elements

In the state-of-the art article "The Cognitive Perspective in Entrepreneurship: An Agenda for Future Research," Grégoire,Corbett and McMullen (2011)summarize the current state of the research in the following words - "Despite its many achievements, scholarship at the intersection of entrepreneurship and cognition has focused primarily on the consequences of what happens when an entrepreneur benefits from various cognitive characteristics, resources, or other dispositions. Assuch, cognitive research in entrepreneurship continues to suffer from narrow theoretical articulations and weak conceptual foundations that lessen its contribution to the managerial sciences."According to Santos and Eisenhardt (2004)"entrepreneurs perceive new opportunities for the creation of value, and construct a market around those opportunities" Successful opportunity recognition leads to successful start-ups and leads to the vibrancy of economy. However, very little work has been done in this field (Mitchell et al., 2014).

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Temporal Association Rule Mining: With Application to US Stock Market

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ABSTRACT

A modified framework, that applies temporal association rule mining to financial time series, is proposed in this paper. The top four components stocks (stock price time series, in USD) of Dow Jones Industrial Average (DJIA) in terms of highest daily volume and DJIA (index time series, expressed in points) are used to form the time-series database (TSDB) from 1994 to 2007. The main goal is to generate profitable trades by uncovering hidden knowledge from the TSDB. This hidden knowledge refers to temporal association rules, which represent the repeated relationships between events of the financial time series with time-parameter constraints: sliding time windows. Following an approach similar to Knowledge Discovery in Databases (KDD), the basic idea is to use frequent events to discover significant rules. Then, we propose the Multi-level Intensive Subset Learning (MIST) algorithm and use it to unveil the finer rules within the subset of the corresponding significant rules. Hypothesis testing is later applied to remove rules that are deemed to occur by chance. After which, multi-period portfolio optimization is done to demonstrate the practicality of using the rules in the real world.

Keywords: Temporal data mining, financial time series, Knowledge discovery, events, DJIA, hypothesis testing, multi-period portfolio optimization.

1 Introduction

With regards to financial market predictions or pattern recognitions, many economists believe in the Efficient Market Hypothesis. The Efficient Market Hypothesis states that the security markets are extremely efficient in reflecting information about the individual stocks or about the stock market as a whole [1]. The Efficient Market Hypothesis even implies that neither Fundamental Analysis nor Technical Analysis will give an investor a return higher than that obtained from holding a randomly selected portfolio of individual stocks. The Random Walk Hypothesis is a very famous hypothesis which is closely related to the Efficient Market Hypothesis. The Random Walk Hypothesis indicates that the stock prices are always unpredictable and random. It states that the price changes in the future are independent of the prices and the news today or in the past. For example, the price changes tomorrow will reflect only the news tomorrow and will be independent of the price changes and news today [2]. Since the news in the future is unpredictable, the price changes in the future will be unpredictable and random as well. By putting these two famous hypotheses together, the stock market can easily be illustrated as a perfect market, where the news are reflected in the market instantly and none of the analysis or rules will help investors

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to gain greater return without taking greater risk. Hence, all the news and the prices are completely random and every security in the market will always be presented by its true value.

However, by studying historical events, the markets are not always perfect. Famous and recent examples can easily be named, such as the 1997–2000 Internet Bubble and the 2001 - 2006 United States Housing Bubble. These were caused by over-valuation of many companies as well as assets in the marketplaces. If the markets are always perfect, the share prices of the companies should not be grossly over-valued and should be reflecting their true values all the time. Thus, history shows that the financial markets are often imperfect, especially in the short term. Some financial economists believe that the financial market maybe a voting mechanism in the short run; in the long run, it is a weighing mechanism [1]. The detection of the short term trends or patterns may be done by using either traditional financial analysis or more complex techniques, such as Knowledge Discovery in Databases (KDD).

KDD is a non-trivial process for identifying valid, previously unknown, potentially useful and ultimately understandable information from the data [3, 4]. It consists of five steps and they are data selection, data preprocessing, data transformation, data mining and interpretation. Data Mining is the fourth step of KDD, where the interested knowledge, rules, or patterns are recognized. A number of famous Data Mining techniques are actively used in analyzing the financial market. They include neutral networks, decision tree, factor analysis, nearest neighbor techniques, clustering, association rules, and inductive logic programming [5]. In particular, association rules are used in our proposed framework.

Associating rule mining [8] is a popular tool used in many applications such as bioinformatics, web usage mining, continuous production, intrusion detection. However, association rules that discover concurrent relationships between items within the same period, are insufficient for financial market prediction. Nevertheless, the following type of association rules might interest analysts: If the prices of Intel and General Electric shares showed an increasing trend over the past 1 month, there is a 60% likelihood that the price of Microsoft shares will show a positive trend in the next 1 month. These association rules with an added time dimension are referred as temporal association rules or time-series association rules, and will be the focus of this study.

The issues of discovering and acquiring the hidden knowledge from the data have been frequently reviewed in the literatures of KDD and Data Mining. For example, various techniques and applications were reviewed and considered in [9, 10]. [11] illustrated a Data Mining approach for time series databases, which emphasized on the preprocessing stage of KDD. The temporal relationships between items were then extracted based on the transformed database. Based on the study done in [11], the time series databases (TSDBs) could be transformed into transactional databases with an extra time dimension. Classical association rule learning algorithms, such as APRIORI and ECLAT, could then be applied to extract the frequent patterns or relationship between the items from the transformed TSDBs. On that basis, [12] demonstrated Data Mining algorithm, called MINEPI, which focused explicitly on the time parameter. Based on the previous study, [13] presented the Representative Episodal Association Rules (REAR) algorithm, which further would convert the TSDBs into discrete representations for the generation of association rules. This algorithm used the concept of episodes as defined in [12] and further reduced the dimension by converting episodes into discrete symbols with the aid of sliding window. Temporal association rules were then generated from the frequent episodes detected with the sweep of sliding window in the database. After introducing the REAR algorithm, [14] presented Minimum Occurrence With Constraints And Time Lags (MOWCATL) algorithm. This is an enhanced algorithm based on REAR. The

MOWCATL further classifies episodes into antecedents and consequents. The temporal association rules are then learnt with a time lag between the antecedents and consequents. The REAR and MOWCATL are the results of improvements from the Gen-FCE, Gen-REAR in [13], and MINEPI in [12].

Furthermore, [15] modified the MINEPI to MINEPI+ and EMMA for mining the frequent episodes in TSDBs. ARMADA, an algorithm capable of mining for temporal rules in databases based on interval data, was introduced in [16]. Recently, [6] introduced a heuristic methodology for learning association rules from the financial market TSDBs. This heuristic methodology is mainly based on the MOWCATL algorithm but ECLAT algorithm is used for the Data Mining process. It is mainly supported by the toolkit which they developed, namely CONOTOOL. Some of the studies were done for financial markets, such as Madrid Stock Exchange (IGBM), Spain, by [6], The Stock Exchange of Thailand (SET), Thailand, by [17], and Singapore Exchange (SGX) by [18].

The state of art temporal association rule mining techniques are able to unveil the underlying association rules from the financial TSDBs. However, curse of dimensionality could arise when dealing with high-dimensional data. To overcome this, we propose the Multi-level Intensive Subset-based Training (MIST) algorithm and use it to extract the subset data of the significant rules in a recursive fashion. Hence, this breadth-first association rules searching mechanism enables efficient discovery of finer rules as the complexity and the processing time are greatly reduced by removing unrelated elements in the subset. Another shortcoming of the current rule mining techniques is that some rules might be useful but are ignored if they do not satisfy the support threshold as they occur rarely. To resolve this, we suggest using hypothesis testing to retain rules if the occurrences are sufficiently large compared to chance occurrences. These two major additions will be further explained in the modified framework after we have described the general framework.

The rest of the paper is organized in the following manner. The general framework, based largely on the heuristic methodology for temporal association rule mining by Dante et al. [6], is introduced in Section 2. The limitations are discussed and will provide motivation for improvements. By making some modifications to the general framework, our proposed modified framework is elaborated in Section 3 and 4. Here, Multi-level Intensive Subset Learning (MIST) algorithm and hypothesis testing are introduced to overcome the limitations. In Section 5, the top four component stocks in terms of the highest daily volume from Dow Jones Industrial Average (DJIA) and DJIA itself are used as a case study. This section also analyses the rules and simulation results. Section 6 presents the usefulness of the rules in the real world using multi-period portfolio optimization. Section 7 concludes the paper.

2 Temporal Association Rule Mining

Let p_i where i = 1, 2, …, n be the time-series data which is used to form a time-series database where sliding time windows are introduced as the timing constraints. The time window for antecedents will be referred to as Input Window and window width is given by IW, while the time window for consequents will be referred to as Output Window and window width is given by OW. The definitions for antecedents and consequents will be elaborated later in this section. Sliding time windows are shown in Figure 1.

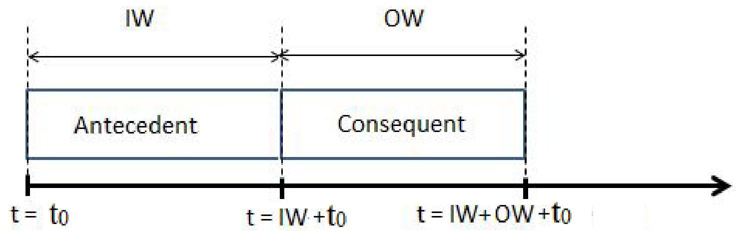

Figure 1 Illustration of sliding time windows

where unit of t is in days and t0 refers to a particular day.

p_i can be classified according to the respective range for each item. The Symbolic Aggregate Approximation (SAX) algorithm [17, 19] is chosen and used for this purpose because one of its key advantages is the ability to classify time series data based on its percentile position in the overall distribution.

To elaborate on SAX further, it is the first symbolic representation for time series that allows data points reduction and indexing with a lower-bounding distance measure. SAX is as well-known as representations such as Discrete Wavelet Transform (DWT) and Discrete Fourier Transform (DFT), except that it requires less storage space. One of the advantages of the SAX algorithm is its ability to convert time-series data of N size into time-series data of n size, where $n \le N$ and $\frac{N}{n} \in Z^+$, by applying the Piecewise Aggregate Approximation (PAA) technique. Z^+ refers to the set of positive integers. The idea of PAA technique is to divide time-series data of N size into n segments with equal length and the average value of each segment is used. The equation of the PAA technique to convert a time-series data, C, of the length of N into length n, in which the ith element of C, is calculated by the following equation [20].

$$\tilde{p}_i = \frac{k}{N} \sum_{j=\frac{N}{k}(i-1)+1}^{\frac{N}{k}i} C_j \qquad (1)$$

Figure 2 shows a time-series C that is represented by PAA (by the mean values of equal segments). In the example above, data points are reduced from N=60 to k=6.

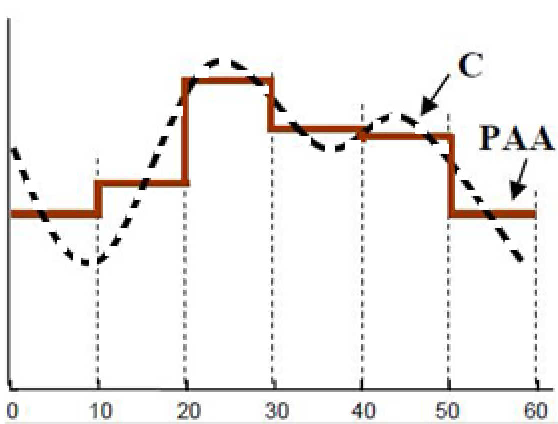

Figure 2 Illustration of PAA

In order to obtain the string representation after a time-series data is transformed into the PAA representation, symbolization region should be determined. According to [19], by empirically testing more than 50 datasets, it is defined that normalized subsequences have distribution that highly resembles the Gaussian distribution. Therefore, by calculating the mean and the variance of the subsequences PAA data and symbolizing each of the PAA data based on its percentile value, the SAX algorithm is then able to convert the whole time series data into symbolized string representations. This quantization process transforms $\tilde{p}_i$ to $\bar{p}_i$. The implementation of the SAX algorithm is contributed by the researchers [19]. Their implementation is chosen because it is widely used in the related research field and is also recommended by the SAX main research website to interested researchers; hence, the correctness of the implementation is ensured. Figure 3 gives the illustration of SAX.

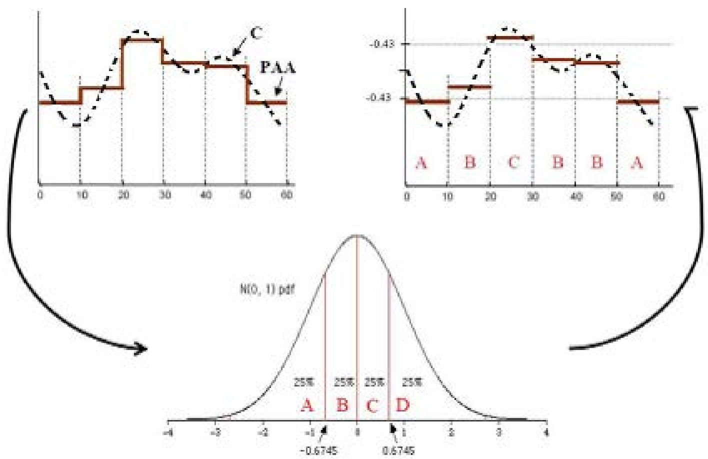

Figure 3 Illustration of SAX: The N-size time series data was first converted into k-size data using PAA technique. The k-size data are then normalized and symbolized by mapping it in the Gaussian distribution.

The SAX program requires four parameters as shown in Table 1.

Table 1 Explanation of the parameters of the SAX program

Parameter	Description
Data_Raw	The data to be processed by SAX algorithm.
N	The number of entries in the provided Data_Raw.
n	The number of output segments
alphabet_size	Number of discrete symbols used to represent the string.

Hence, the corresponding inputs and outputs can be given by $X_j = \left[\bar{p}_i, \bar{p}_{i+1}, \bar{p}_{i+2}, \ldots \right]$ and $Y_j = \left[\bar{p}_i \right]$ where i = 1, 2, …, n and j = 1,2, …, m. The inputs are known as antecedents and the outputs are known as consequents. A typical association rule can be written as

$$X \Rightarrow Y \tag{2}$$

The "support" $S(X \Rightarrow Y)$ of the rule is defined as the fraction of observations where the rule holds and the formula is as follows

$$S(X \Rightarrow Y) = \frac{N(X,Y)}{N(T)} \tag{3}$$

where N(X,Y) is the number of times when both the antecedent and consequent are observed and N(T) is the number of transactions in the database. The "confidence" $C(X \Rightarrow Y)$ of the rule is its support divided by the support of the antecedent.

$$C(X \Rightarrow Y) = \frac{S(X \Rightarrow Y)}{S(X)} \tag{4}$$

This can also be viewed as the conditional probability that Y occurs, given that X occurs [7]. We can then generate all possible rules based on the below constraints

$$S(X \Rightarrow Y) \geq s \quad and \quad C(X \Rightarrow Y) \geq c \tag{5}$$

where "s" is the support threshold and "c" is the confidence threshold.

For the temporal association rules extraction algorithm, we have chosen to use MOWCALT algorithm by Harms et al [14] as they provide a clear explanation with pseudo code for their algorithms and the implementation is relatively simple. The rules extracted in this manner are called significant rules. See Algorithm 1.

Algorithm 1

1) Generate Antecedent Target Episodes of length 1 which we denote as ATE_1;
2) Generate Consequent Target Episodes of length 1 which we denote as CTE_1;
3) Record occurrences of ATE_1 and CTE_1 episodes;
4) Prune unsupported episodes from ATE_1 and CTE_1 based on minimum support threshold;
5) k=1;
6) while ($ATE_k \neq \emptyset$) do
7) Generate Antecedent Target Episodes ATE_{k+1} from ATE_k;
8) Record each occurrence of the episodes;
9) Prune the unsupported episodes from ATE_{k+1};
10) k++;
11) Repeat Steps 5 – 11 for consequent episodes using CTE_{k+1};
12) Generate combination episodes from antecedent episodes and consequent episodes;
13) Record each occurrence of the combination episodes;
14) Return the supported combination episodes that satisfy the minimum confidence threshold and these are the relevant rules;

To evaluate the usefulness of the rules, we need to run simulation for test data and measure the performance against some benchmarks.

Refer to below for a recap on the general framework as described in this section.

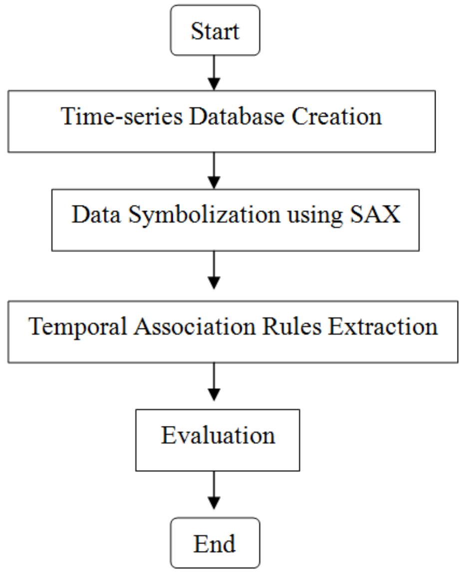

Figure 4 Flowchart of general framework

3 MIST Algorithm

The number of possible temporal association rules, H, can be easily formulated as the following.

$$H = \sum_{j=2}^{Q} \left(M^{j} \times \binom{Q}{j} \right) \tag{6}$$

Where M is the number of symbols, Q is the number of items, and j is the total number of both Antecedents' and Consequent's episodes in the corresponding rules. It can be observed that the number of possible temporal association rules is growing exponentially with the number of symbols and stocks. If there are only fifteen items to be symbolized with just five symbols, H will be 4.7018×10^{11}. Therefore, huge memory resources and processing time are required even for small values of M and Q. If items are required to be quantized with more symbols to achieve higher resolution, the hardware requirements might be a huge issue.

To tackle the problem of heavy computation costs, we propose the Multi-level Intensive Subset-Training (MIST) Algorithm as an efficient way to discover the finer temporal association rules with less symbols (breadth searching) first, and only perform a deeper searching with more symbols (depth searching) for the relevant rules recursively. This algorithm will continue to perform depth searching until there is insufficient data size or no more significant rules are needed to be diagnosed. The processing time and memory required will be greatly reduced because only the subset of the data is discretized with more symbols. The idea behind MIST algorithm is mainly based on the downward closure lemma, which states that the subsets of a frequent pattern are also frequent. See Algorithm 2 for the pseudo code and refer to Figure 5 for illustration.

Algorithm 2

1) Check for rules where the numbers of occurrences satisfy the minimum data size;

2) For each of the relevant rule, discretize with additional symbols;
3) Create new time-transaction database using the occurrences of the rule
4) Use Algorithm 1 to discover new finer rules;
5) Repeat Steps 1-4 as long as there are rules to be analyzed;
6) Return all new finer rules.

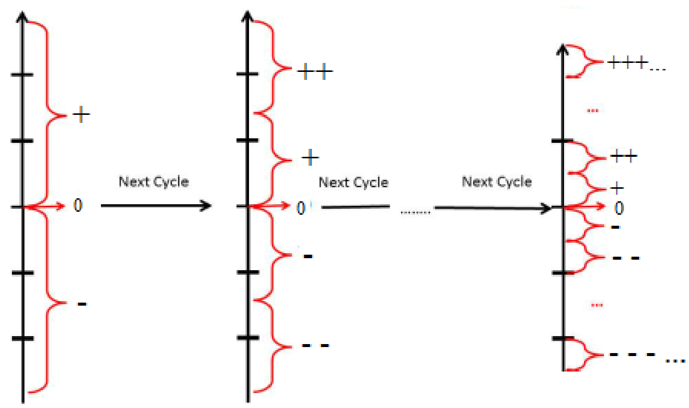

Figure 5 Overall view of MIST Algorithm: '+' and '-' are used as symbols for illustration purpose

4 Hypothesis Testing

Some temporal association rules might have low occurrences and, hence are rejected by the support threshold. However, these could be useful rules if their occurrences are not by chance.

To overcome this limitation, we need to check if the occurrences are sufficiently large enough for us to believe that the rules do not take place by chance. To find this out, the idea is to perform hypothesis testing where null hypothesis is set up as H0: $O_{rule} = O_{random}$ and the alternative hypothesis is formulated as H1: $O_{rule} > O_{random}$ at the a% significance level. O_{random} is the number of times that a particular rule gets triggered successfully in the random walk case and O_{rule} is the recorded occurrence of the rule as observed during learning from the training data. We assume that distribution is normal since we will use a sample size that is large enough [29]. So, Z-test is used. If H0 is accepted, it means the rule occurs by chance. Alternatively, if H0 is rejected, it means the rule does not occur by chance Table 2 shows the steps and Algorithm 3 shows the pseudo code.

Table 2 Explanation of the steps for hypothesis testing

#	Description
Step 1	H0: $O_{rule} = O_{random}$ H1: $O_{rule} > O_{random}$
Step 2	Select a significance level a%
Step 3	Use Z-test
Step 4	Compare the computed test statistic with critical value. If the computed value is within the rejection region, the null hypothesis will be rejected. Otherwise, the null hypothesis will not be rejected.

Algorithm 3

1) Generate normally-distributed pseudorandom numbers using randn in Matlab;
2) Use results from Step 1 to create time-series database to mimic random walk;
3) Follow the steps in Algorithm 1 to get the occurrences O_{random};
4) Repeat, say k times for Steps 1 – 3 to get different samples of O_{random};
5) Set the recorded occurrence of the rule we are interested in as O_{rule};

6) Conduct Z-test where H_0: $O_{rule} = O_{random}$ and H_1: $O_{rule} > O_{random}$ at the a% significance level
7) Repeat Steps 5 – 6 for the rest of the rules
8) Return the rules that pass the Z-test;

By adding MIST algorithm and hypothesis testing, we get the modified framework in the below figure.

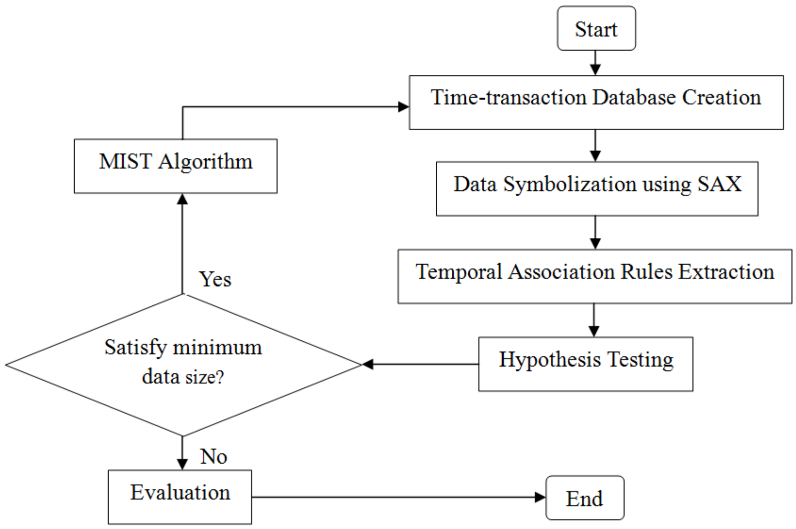

Figure 6 Flowchart of modified framework

5 Case Study for US Stocks

We will use the top four component stocks in terms of the highest daily volume from Dow Jones Industrial Average (DJIA) and DJIA itself will be used as a case study. Financial data are configured with 5 financial time-series: 4 series of stocks (daily adjusted close price in USD) and 1 series related to DJIA (daily close index measured in points). Data is taken for January 1994 to December 2013 from Yahoo Finance [21]. Data for January 1994 to December 2007 is used as training data while data for January 2008 to December 2013 is used as test data. Note that the adjusted close prices take into considerations all corporate actions, such as stock splits, dividends/distributions and rights offerings.

Table 3 Composition of database

Asset Name	Acronym
Cisco Systems, Inc.	CSCO
General Electric Company	GE
Intel Corporation	INTC
Microsoft Corporation	MSFT
Dow Jones Industrial Average	DJIA

Instead of using p_i as per Section 2, we use r_i which is rate of return (ROR) and formula is given below.

$$r_i = \frac{(P_t - P_{t+W})}{P_t} \times 100\% \tag{7}$$

where P_t represents the price at time t and P_{t+W} refers to the price after W number of days from time t. Here, we use W = 20 and IW = 3 and OW = 1.

After the stock prices database is converted into ROR database, the data can then be classified according to the range for ROR of each stock. One of the options is to classify ROR with a fixed range. For instance, a fixed range from 0% to 1% can be classified as weak positive trend with symbol “+”, while 1% to 3% as semi-strong positive trend with symbol “++” for all the stocks. However, this method does not work well because every stock should have its own range due to its nature of business or certain unique characteristics. Hence, the SAX algorithm is used. The time series ROR data is first separated into three main categories, which are (negative gain) “-“, (positive gain) “+”, and (no gain) “0”. Classification is then carried out by the SAX algorithm. Notice that the (no gain) category does not require to be symbolized again because its ROR is zero. This process is illustrated by Figure 7.

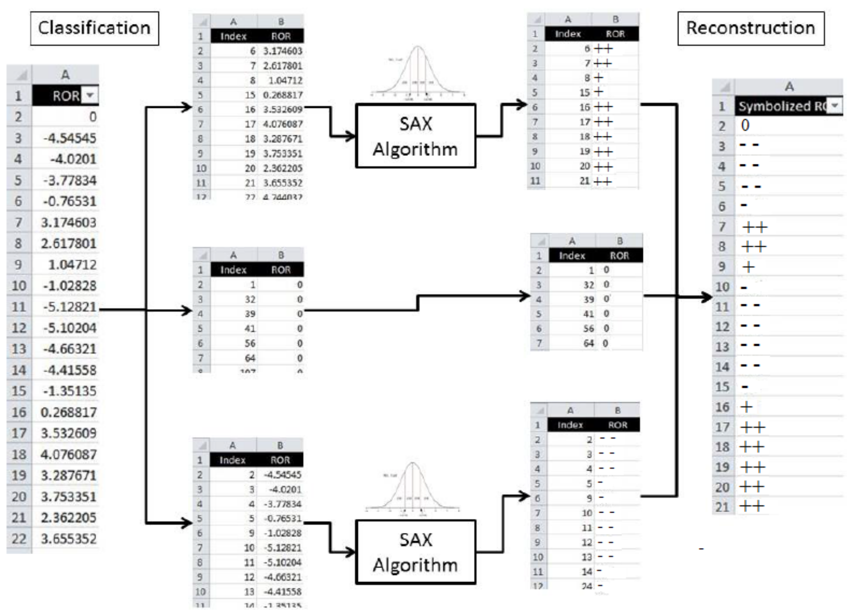

Figure 7 An overview of the classification process using SAX

To discover significant rules, we use MOWCALT algorithm, and select support threshold to be 10% and confidence threshold to be 50%. Also, the consequent will have two sets of measurements: one for when the rule holds and the other for when the rule fails. Let us define TM to be the mean for ROR in the first case and FM to be the mean for the ROR in the second case. So, the expected gain (EG) of the rule can be calculated using the following formula.

$$EG = C \times TM + (1 - C) \times FM \tag{8}$$

where C is the confidence of the rule as per defined in equation (4).

Next, we implement the algorithm described in Section 4 with k = 10 and a = 1% to carry out hypothesis testing. To get finer rules from the significant rules discovered using MOWCALT algorithm, we use the MIST algorithm and set data minimum size as 40. Lastly, we use multi-period portfolio optimization approach to simulate trading in the real world. We shall call this ‘Rule Learning’ model and will compare this against standard mean-variance model and equally-weighted model in terms of measurements such as terminal return and Sharpe ratio. This approach enables us to evaluate the usefulness of the rules.

Our trading strategy and some assumptions are as below.

- Trade according to the rule's consequent when all the antecedents are triggered
- Entry price is tomorrow's open price
- Close the position by the end of Output Window
- No limitation on short sell holding period
- No target profit (TP) and no stop loss (SL)
- Dividends, stock split and other corporate actions are ignored
- Trading costs (TC) are taken to be 0.18% of the value of the position

Table 4

Rule	Antecedents	Consequent	Support (%)	Confidence (%)	Expected Gain (%)	Total Trades	Success Rate (%)	Average Return per Trade (%)
1	{CSCO2+,DJA3+,INTC1+}	{CSCO+}	0.73	96	3.02	21	62	**-0.13**
2	{DJA2+,INTC1+,MSFT3+}	{CSCO+}	0.61	95	3.57	32	**47**	**-0.16**
3	{CSCO2+,GE1+,MSFT3+}	{CSCO+}	0.58	95	3.30	16	69	1.27
4	{CSCO2+,MSFT1+,MSFT3+}	{CSCO+}	0.58	95	2.69	42	60	0.43
5	{CSCO1+++,CSCO3+,MSFT2+}	{GE+}	0.58	95	1.57	5	**40**	**-2.95**
6	{DJA3+,GE1+,GE2+}	{GE+}	0.64	92	1.35	9	**44**	**-0.23**
7	{CSCO3+,GE2+,MSFT1+}	{INTC+}	0.67	96	2.60	10	70	1.89
8	{CSCO3+,DJA2+,MSFT1+}	{INTC+}	0.61	95	2.60	19	84	2.23
9	{CSCO3+,GE2+,MSFT1+}	{DJA+}	0.73	96	0.92	12	75	0.49
10	{GE2+,MSFT1+,MSFT3+}	{DJA+}	0.70	96	1.06	23	87	1.60
11	{DJA3+,GE1+,GE2+}	{DJA+}	0.70	96	1.11	10	50	**-0.39**
12	{DJA3+,GE1+,MSFT2+}	{DJA+}	0.64	96	1.06	15	53	**-0.27**
13	{DJA2+,MSFT1+,MSFT3+}	{DJA+}	0.61	95	1.31	15	93	0.95
14	{CSCO2+,DJA3+,GE1+}	{DJA+}	0.61	95	1.07	12	83	1.08
15	{DJA3+,GE1+,GE2+}	{DJA+}	0.61	95	1.17	11	55	**0.00**
16	{DJA3+,GE2+,MSFT1+}	{DJA+}	0.58	95	1.09	19	68	0.56
17	{CSCO2+,DJA3+,MSFT1+}	{DJA+}	0.58	95	1.06	35	86	1.35
18	{DJA3+,GE1+,MSFT2++}	{DJA+}	0.58	95	1.27	6	67	0.92

In addition, we will only select rules, which have antecedents that contain episodes for each of the three time steps. This will allow the price trends to be sufficiently described before the possible occurrence of the consequent. From here on, we will focus only on the finer rules and will refer to them simply as rules. For ease of presentation, we take the top rules in terms of confidence levels from each of the assets and focus on those that satisfy a certain support threshold. This gives us 18 rules and the simulation results are presented in Table 4.

The rules in Table 4 can be interpreted in the following manner. For example, rule 18: {DJA3+, GE1+, MSFT2++}$\Rightarrow${DJA+} means that when DJA3 in time step k+2 and GE1 in time step k show "+" positive trends, and MSFT2 in time step k+1 shows "++" positive trend, DJA in time step k+3 will show "+" positive trend with support value of 0.58% and confidence value of 95%. The average "Success Rate" here is 66% compared to the average "Confidence" value of 95%. This is expected because the time period used for testing is different from the time period used to discover rules. Using entry price as tomorrow's open price has also contributed to this phenomenon. Also, it is observed that the number of times that each rule is triggered is small compared to the number of occurrences for a typical significant rule. This is reasonable as these rules in Table 4 were generated using MIST algorithm, which discovers finer rules with more symbols from the subset of the corresponding significant rules.

6 Multi-period Portfolio Optimization

6.1 Introduction

In a single-period portfolio management, the aim is to optimize investment decisions for a fixed planning horizon with no flexibility to vary the portfolio composition. This means that the investor allocates his capital among various securities at the start of the investment period and observes the final outcome at the end of the investment period. One famous single-period model is the mean-variance framework suggested by Markowitz [22, 23]. In this model, risk is defined as the variance of a portfolio returns. Although the Markowitz model was first introduced over 50 years ago, it is still very popular among many portfolio managers. Also, it has been updated with refinements over time.

Let us now consider the multi-period case whereby investors can rebalance their portfolio at the end of each period until the terminal date is reached. Rebalancing can be done at only fixed times in a discrete-time model while investors in a continuous-time model can reallocate at any time. New information such as economic data and significant world events is available to investors for each period. Hence, investors are required to consider the new information and respond appropriately based on their portfolio objectives and constraints. In [24], Merton pioneered portfolio optimization for a multi-period continuous-time model, where stochastic control theory and dynamic programming are used for analyzing the appropriate partial differential equation of Hamilton-Jacobi-Bellman. For multi-period discrete-time model, the variance of wealth is non-separable in the sense of dynamic programming as noted in [25]. Since dynamic programming is not able to tackle the particular cost function in [25], an embedding technique is proposed to allow the problem to be transformed to an auxiliary problem, which can be solved to retrieve explicit, optimal solutions by dynamic programming. Later, it was shown in [26] that the problem can be solved by using convex analysis and no dynamic programming is involved.

We will proceed to describe the setting under which we will implement multi-period discrete-time model with the rules discovered from the proposed modified framework described earlier. In literature, the number of risky assets is typically fixed. We will allow this number to vary depending on which rules get triggered. Also, there are usually no limitations for weights in most of the current works. Here, we will state the limitation for each risky asset. The goal is to maximize the expected log of geometric growth rate of the portfolio, subjected to the weight constraints.

6.2 Log-optimal Portfolio

The general expression for wealth V_n at the end of n periods is given by the following.

$$V_n = R_n R_{n-1} \dots R_{k+1} R_k R_{k-1} \dots R_2 R_1 V_o \tag{9}$$

where V_0 is the initial wealth and R_k is a random return variable for k = 1,2…,n. Taking logarithm of both sides will give us the below.

$$\begin{aligned} \ln V_n &= \ln V_o + \ln R_n + \ln R_{n-1} \dots + \ln R_{k+1} + \ln R_k + \ln R_{k-1} \dots + \ln R_2 + \ln R_1 \\ &= \ln V_o + \sum_{k=1}^{n} \ln R_k \end{aligned} \tag{10}$$

Hence, we will select $U(V_n) = \ln V_n$ as the utility function. By maximizing the same utility function at each step, maximization of the expected final utility is ensured. This is because the separation property holds and the multi-period case will simplify to a series of single-period cases [27].

Each ith risky asset has price p_i governed by the standard geometric Brownian motion equation [27].

$$\frac{dp_i}{p_i} = \mu_i dt + dz_i \tag{11}$$

where μ_i is the drift term and z_i refers to a Wiener process with variance parameter σ_i^2. Thus, choosing the suitable weights for the risky assets to maximize the overall growth rate is equivalent to solving the problem below.

$$\max\left[\left(1-\sum_{i=1}^{m} w_i\right) r_f + \sum_{i=1}^{m}\left(\mu_i w_i - \frac{1}{2}\sum_{j=1}^{m} w_i \sigma_{ij} w_j\right)\right] \tag{12}$$

where r_f is the constant risk-free rate and σ_{ij} is the individual component in the covariance matrix. To solve this, set the derivative with respect to w_i equal to zero and we obtain the following.

$$\mu_i - r_f - \sum_{j=1}^{m} \sigma_{ij} w_j = 0 \tag{13}$$

Hence, when there is a risk-free asset, the log-optimal portfolio has weights for the risky assets that satisfy the below.

$$\sum_{j=1}^{m} \sigma_{ij} w_j = \mu_i - r_f \text{, } \textit{for i=1,2,...,m} \tag{14}$$

This is a system of m linear equations and m weights can be found easily.

6.3 Simulation to Evaluate Usefulness of Rules

The buy and sell position will be for any of the assets stated in Table 3, so the number of risky assets are 5. We will apply the following constraints: $20\% \le |w_i| \le 50\%$, where w_i is the weight for the ith asset, and for the risk-free asset, we will allow a weight of -100% to 100%. Here, m can be any number from 0 to 5 depending on the rules that get triggered. Expected gain of the rule will be the relevant μ_i. Covariance matrix will be calculated based on a look-back period of 220 days. The risk-free rate r_f will be calculated based on the historical data of 1 month US treasury bill [28].

Applying a confidence threshold of 70% to the pool of rules discovered in Section 5, we have 7,985 rules. For comparison purpose, we will also run simulation using the standard mean-variance model with covariance and returns estimated based on a look-back period of 220 days on a rolling basis. A portfolio with equal weightings will be simulated as well. Trading costs are taken 0.04%. The simulations will be conducted using data from January 2008 to December 2013. Results are presented in Table 5.

Table 5

Model	Terminal Return	Average Monthly Return (%)	Standard Deviation of Monthly Returns	Monthly Sharpe Ratio
Rule Learning	1.6804	0.9534	0.0721	0.1323
Standard Mean-Variance	1.1186	0.4023	0.0697	0.0577
Equally-weighted	1.0113	0.6819	0.1158	0.0589

It is observed that Rule Learning model gives the highest 'Terminal Return', 'Average Monthly Return' and 'Monthly Sharpe Ratio' compared to the other two models. This means that the rules are applicable in portfolio management in the real world.

7 Conclusion

We have proposed a modified framework that applies temporal data mining technique to financial time series. After using the MOWCATL algorithm to discover significant rules, we found finer rules within the subset of the respective significant rules by applying our proposed MIST algorithm. Later, we use hypothesis testing to filter for rules that do not occur by chance. From the simulation results, the rules are found to be useful in trading individual positions and for portfolio management. The right mixture of rules would enable investors to enhance portfolio performance as shown in our multi-period portfolio optimization model.

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Simulation of the Charge Motion near the Velocity of Light in Electric and Magnetic Fields

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ABSTRACT

A numerical simulation method for the charge motion near the velocity of light in electric and magnetic fields has been investigated using the relativistic mass by Einstein's special theory of relativity, and an electron acceleration for the Larmor motion in a static magnetic field perpendicularly applied a synchronized alternative electric field has been simulated by Java programing. The simulation results in a limitation of the electron velocity to a considerably lower value than the velocity of light in contradiction to the previous simulation in which the electron is easily accelerated over the velocity of light due to the use of the invariant mass.

Keywords: Simulation of charge motion near the velocity of light in electric and magnetic fields, Java programming, simulation of charge motion with the relativistic mass.

1 Introduction

The authors previously proposed a Java simulation for the rapid and accurate image learning of the charge motion in electric and magnetic fields using the Runge-Kutta method [1, 2]. In this simulation, we use an invariant mass of the charge resulting in an easy acceleration of the electron over the velocity of light for the Larmor motion in a static magnetic field applied a synchronized alternative electric field perpendicularly to the magnetic field. It is not accurate to use the invariant mass for the particle moving with a large momentum, and the relativistic mass by Einstein's special theory of relativity should be used in such a case. That is, the particle moving with a velocity near the velocity of light behaves effectively as a particle with a larger mass that increases with the increase in the velocity. This change of the effective mass causes a slip off of the above Larmor motion from the initial synchronization, resulting in a limitation of the acceleration and velocity.

In this paper, the numerical simulation method for the charge motion near the velocity of light in electric and magnetic fields has been investigated using the relativistic mass. Furthermore, the acceleration for the Larmor motion of electron in a static magnetic field applied the synchronized electric field has been simulated in comparison with the previous simulation used the invariant mass.

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2 Numerical Method for Charge Motion near the Velocity of Light in Electric and Magnetic Fields

2.1 Equations for the Charge Motion near the Velocity of Light in Electric and Magnetic Fields

The particle moving with a very large velocity $\boldsymbol{v} = (v_x, v_y, v_z)$ has a large momentum and thus behaves effectively as a particle with a large relativistic mass, m^*, according to Einstein's special theory of relativity. In this situation, the acceleration $\boldsymbol{a}$ under a force $\boldsymbol{F}$, $\boldsymbol{a} = d\boldsymbol{v}/dt$, is obtained as in Reference [3]:

$$\boldsymbol{a} = \frac{\boldsymbol{F} - (\boldsymbol{F} \bullet \boldsymbol{v})\boldsymbol{v}/c^2}{m\gamma}. \tag{1}$$

Here, m is the invariant mass, c is the velocity of light, and

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}. \tag{2}$$

Here, v is the magnitude of $\boldsymbol{v}$. Equation (1) represents the well-known Lorentz's longitudinal and transverse masses, that is, $m^*_L = m\gamma$□when $\boldsymbol{F}$ is parallel to $\boldsymbol{v}$ and $m^*_T = m\gamma$ when $\boldsymbol{F}$ is perpendicular to $\boldsymbol{v}$□

We consider a motion of the charge q under the electric field, $\boldsymbol{E} = (E_x, E_y, E_z)$, and the magnetic field, $\boldsymbol{B} = (B_x, B_y, B_z)$. In this case, the acceleration is given as

$$\boldsymbol{a} = \frac{q\boldsymbol{E}}{m\gamma^3} + \frac{q\boldsymbol{v}\times\boldsymbol{B}}{m\gamma}. \tag{3}$$

Because the electric force is effective on the parallel component of $\boldsymbol{v}$ and the electro-magnetic force is perpendicular to $\boldsymbol{v}$. We have the equation for the displacement of the charge, $\boldsymbol{r} = (x, y, z)$:

$$\frac{d\boldsymbol{r}}{dt} = \boldsymbol{v}. \tag{4}$$

By decomposing the vector equations (3) and (4) into the scalar equations in the x, y, and z directions, we obtain

$$\frac{dv_x}{dt} = \frac{q}{m\gamma^3}E_x + \frac{q}{m\gamma}(v_y B_z - v_z B_y), \tag{5}$$

$$\frac{dv_y}{dt} = \frac{q}{m\gamma^3}E_y + \frac{q}{m\gamma}(v_z B_x - v_x B_z), \tag{6}$$

$$\frac{dv_z}{dt} = \frac{q}{m\gamma^3}E_z + \frac{q}{m\gamma}(v_x B_y - v_y B_x), \tag{7}$$

$$\frac{dx}{dt}=v_x, \tag{8}$$

$$\frac{dy}{dt}=v_y, \tag{9}$$

$$\frac{dz}{dt}=v_z. \tag{10}$$

We can obtain the charge motion in the *x*, *y*, and *z* directions by solving six ordinary differential equations, (5) - (10).

2.2 Numerical Method

The ordinary differential equations can be solved numerically using the fourth-order Runge-Kutta method. The first-order increment functions for the differential equations (5) – (10) at the known variables (t, v_x, v_y, v_z, x, y, z) are given as

$$k_1^{(1)}=\frac{q}{m}\left(1-(v_x^2+v_y^2+v_z^2)/c^2\right)^{1.5}E_x(t)+\frac{q}{m}\sqrt{1-(v_x^2+v_y^2+v_z^2)/c^2}\left(v_yB_z(t)-v_zB_y(t)\right), \tag{11}$$

$$k_2^{(1)}=\frac{q}{m}\left(1-(v_x^2+v_y^2+v_z^2)/c^2\right)^{1.5}E_y(t)+\frac{q}{m}\sqrt{1-(v_x^2+v_y^2+v_z^2)/c^2}\left(v_zB_x(t)-v_xB_z(t)\right), \tag{12}$$

$$k_3^{(1)}=\frac{q}{m}\left(1-(v_x^2+v_y^2+v_z^2)/c^2\right)^{1.5}E_z(t)+\frac{q}{m}\sqrt{1-(v_x^2+v_y^2+v_z^2)/c^2}\left(v_xB_y(t)-v_yB_x(t)\right), \tag{13}$$

$$k_4^{(1)}=v_x, \tag{14}$$

$$k_5^{(1)}=v_y, \tag{15}$$

$$k_6^{(1)}=v_z. \tag{16}$$

The second-order increment functions are given as

$$k_1^{(2)}=\frac{q}{m}\left(1-[(v_x+\frac{hk_1^{(1)}}{2})^2+(v_y+\frac{hk_2^{(1)}}{2})^2+(v_z+\frac{hk_3^{(1)}}{2})^2]/c^2\right)^{1.5}E_x(t+\frac{h}{2})$$

$$+\frac{q}{m}\sqrt{1-[(v_x+\frac{hk_1^{(1)}}{2})^2+(v_y+\frac{hk_2^{(1)}}{2})^2+(v_z+\frac{hk_3^{(1)}}{2})^2]/c^2}\left((v_y+\frac{hk_2^{(1)}}{2})B_z(t+\frac{h}{2})-(v_z+\frac{hk_3^{(1)}}{2})B_y(t+\frac{h}{2})\right), \tag{17}$$

$$k_2^{(2)} = \frac{q}{m}\left(1\text{-}[(v_x+\frac{hk_1^{(1)}}{2})^2+(v_y+\frac{hk_2^{(1)}}{2})^2+(v_z+\frac{hk_3^{(1)}}{2})^2]/c^2\right)^{1.5} E_y(t+\frac{h}{2})$$

$$+\frac{q}{m}\sqrt{1\text{-}[(v_x+\frac{hk_1^{(1)}}{2})^2+(v_y+\frac{hk_2^{(1)}}{2})^2+(v_z+\frac{hk_3^{(1)}}{2})^2]/c^2}\left((v_z+\frac{hk_3^{(1)}}{2})B_x(t+\frac{h}{2})\text{-}(v_x+\frac{hk_1^{(1)}}{2})B_z(t+\frac{h}{2})\right), \quad (18)$$

$$k_3^{(2)} = \frac{q}{m}\left(1\text{-}[(v_x+\frac{hk_1^{(1)}}{2})^2+(v_y+\frac{hk_2^{(1)}}{2})^2+(v_z+\frac{hk_3^{(1)}}{2})^2]/c^2\right)^{1.5} E_z(t+\frac{h}{2})$$

$$+\frac{q}{m}\sqrt{1\text{-}[(v_x+\frac{hk_1^{(1)}}{2})^2+(v_y+\frac{hk_2^{(1)}}{2})^2+(v_z+\frac{hk_3^{(1)}}{2})^2]/c^2}\left((v_x+\frac{hk_1^{(1)}}{2})B_y(t+\frac{h}{2})\text{-}(v_y+\frac{hk_2^{(1)}}{2})B_x(t+\frac{h}{2})\right), \quad (19)$$

$$k_4^{(2)} = v_x+\frac{hk_1^{(1)}}{2}, \quad (20)$$

$$k_5^{(2)} = v_y+\frac{hk_2^{(1)}}{2}, \quad (21)$$

$$k_6^{(2)} = v_z+\frac{hk_3^{(1)}}{2}. \quad (22)$$

The third-order increment functions are given as

$$k_1^{(3)} = \frac{q}{m}\left(1\text{-}[(v_x+\frac{hk_1^{(2)}}{2})^2+(v_y+\frac{hk_2^{(2)}}{2})^2+(v_z+\frac{hk_3^{(2)}}{2})^2]/c^2\right)^{1.5} E_x(t+\frac{h}{2})$$

$$+\frac{q}{m}\sqrt{1\text{-}[(v_x+\frac{hk_1^{(2)}}{2})^2+(v_y+\frac{hk_2^{(2)}}{2})^2+(v_z+\frac{hk_3^{(2)}}{2})^2]/c^2}\left((v_y+\frac{hk_2^{(2)}}{2})B_z(t+\frac{h}{2})\text{-}(v_z+\frac{hk_3^{(2)}}{2})B_y(t+\frac{h}{2})\right), \quad (23)$$

$$k_6^{(3)} = v_z+\frac{hk_3^{(2)}}{2}. \quad (24)$$

The fourth-order increment functions are given as

$$k_1^{(4)} = \frac{q}{m}\left(1\text{-}[(v_x+hk_1^{(3)})^2+(v_y+hk_2^{(3)})^2+(v_z+hk_3^{(3)})^2]/c^2\right)^{1.5} E_x(t+h)$$

$$+\frac{q}{m}\sqrt{1\text{-}[(v_x+hk_1^{(3)})^2+(v_y+hk_2^{(3)})^2+(v_z+hk_3^{(3)})^2]/c^2}\left((v_y+hk_2^{(3)})B_z(t+h)\text{-}(v_z+hk_3^{(3)})B_y(t+h)\right), \quad (25)$$

$$k_6^{(4)} = v_z + hk_3^{(3)}. \tag{26}$$

Here, h is the increment of t. The variables at $t + h$ are given as

$$v_x(t+h) = v_x(t) + \frac{1}{6}(k_1^{(1)} + 2k_1^{(2)} + 2k_1^{(3)} + k_1^{(4)})\square \tag{27}$$

$$v_y(t+h) = v_y(t) + \frac{1}{6}(k_2^{(1)} + 2k_2^{(2)} + 2k_2^{(3)} + k_2^{(4)})\square \tag{28}$$

$$v_z(t+h) = v_z(t) + \frac{1}{6}(k_3^{(1)} + 2k_3^{(2)} + 2k_3^{(3)} + k_3^{(4)})\square \tag{29}$$

$$x(t+h) = x(t) + \frac{1}{6}(k_4^{(1)} + 2k_4^{(2)} + 2k_4^{(3)} + k_5^{(4)})\square \tag{30}$$

$$y(t+h) = y(t) + \frac{1}{6}(k_5^{(1)} + 2k_5^{(2)} + 2k_5^{(3)} + k_5^{(4)})\square \tag{31}$$

$$z(t+h) = z(t) + \frac{1}{6}(k_6^{(1)} + 2k_6^{(2)} + 2k_6^{(3)} + k_6^{(4)})\square \tag{32}$$

If the variables at a t are given, then the numerical values at $t + h$ can be obtained from equations (27) – (32), and then the values at $t + 2h$, at $t + 3h$, etc. are obtained by repeating the calculations.

3 Result of the Java Simulation

3.1 Conditions for the Simulation

We consider a simple charge motion injected along the x - direction with the initial velocity $\boldsymbol{v} = (v_0, 0, 0)$ at $\boldsymbol{r} = (0, 0, 0)$ in the alternating electric field, $\boldsymbol{E} = (E_x, 0, 0)$, and the static magnetic field, $\boldsymbol{B} = (0, 0, B_z)$. We use $E_x = E_0\sin(2\pi ft)$, and $B_z = B_0$. Here, E_0 and B_0 are constants, and f is the electric frequency. In this case, the charge moves in the $x - y$ plane and $v^2 = v_x^2 + v_y^2$. If we synchronize the electric field frequency to the frequency f_L of the Larmor motion, $f_L = qB_0/2\pi m^*$, then the charge is continuously accelerated by absorbing energy from the electric field, and its velocity increases with time. The increment functions from equations (11) – (26) for above conditions are given as

$$k_1^{(1)} = \frac{q}{m}\left(1 - (v_x^2 + v_y^2)/c^2\right)^{1.5} E_0 \sin(2\pi ft) + \frac{q}{m}\sqrt{1 - (v_x^2 + v_y^2)/c^2}\, v_y B_0, \tag{33}$$

$$k_2^{(1)} = -\frac{q}{m}\sqrt{1 - (v_x^2 + v_y^2)/c^2}\, v_x B_0, \tag{34}$$

$$k_4^{(1)} = v_x, \tag{35}$$

$$k_5^{(1)} = v_y, \tag{36}$$

$$k_1^{(2)} = \frac{q}{m}\left(1 - \left[(v_x + \frac{hk_1^{(1)}}{2})^2 + (v_y + \frac{hk_2^{(1)}}{2})^2\right] / c^2\right)^{1.5} E_0 \sin[2\pi f(t + \frac{h}{2})]$$

$$+\frac{q}{m}\sqrt{1 - \left[(v_x + \frac{hk_1^{(1)}}{2})^2 + (v_y + \frac{hk_2^{(1)}}{2})^2\right] / c^2}\left(v_y + \frac{hk_2^{(1)}}{2}\right)B_0, \tag{37}$$

$$k_2^{(2)} = -\frac{q}{m}\sqrt{1 - [(v_x + \frac{hk_1^{(1)}}{2})^2 + (v_y + \frac{hk_2^{(1)}}{2})^2] / c^2}\left(v_x + \frac{hk_1^{(1)}}{2}\right)B_0, \tag{38}$$

$$k_4^{(2)} = v_x + \frac{hk_1^{(1)}}{2}, \tag{39}$$

$$k_5^{(2)} = v_y + \frac{hk_2^{(1)}}{2}, \tag{40}$$

$$k_1^{(3)} = \frac{q}{m}\left(1 - \left[(v_x + \frac{hk_1^{(2)}}{2})^2 + (v_y + \frac{hk_2^{(2)}}{2})^2\right] / c^2\right)^{1.5} E_0 \sin[2\pi f(t + \frac{h}{2})]$$

$$+\frac{q}{m}\sqrt{1 - \left[(v_x + \frac{hk_1^{(2)}}{2})^2 + (v_y + \frac{hk_2^{(2)}}{2})^2\right] / c^2}\left(v_y + \frac{hk_2^{(2)}}{2}\right)B_0, \tag{41}$$

$$k_2^{(3)} = -\frac{q}{m}\sqrt{1 - \left[(v_x + \frac{hk_1^{(2)}}{2})^2 + (v_y + \frac{hk_2^{(2)}}{2})^2\right] / c^2}\left(v_x + \frac{hk_1^{(2)}}{2}\right)B_0, \tag{42}$$

$$k_4^{(3)} = v_x + \frac{hk_1^{(2)}}{2}, \tag{43}$$

$$k_5^{(3)} = v_y + \frac{hk_2^{(2)}}{2}, \tag{44}$$

$$k_1^{(4)} = \frac{q}{m}\left(1 - \left[(v_x + hk_1^{(3)})^2 + (v_y + hk_2^{(3)})^2\right] / c^2\right)^{1.5} E_0 \sin[2\pi f(t + h)]$$

$$+\frac{q}{m}\sqrt{1 - \left[(v_x + hk_1^{(3)})^2 + (v_y + hk_2^{(3)})^2\right] / c^2}\left(v_y + hk_2^{(3)}\right)B_0, \tag{45}$$

$$k_2^{(4)} = -\frac{q}{m}\sqrt{1-[(v_x+hk_1^{(3)})^2+(v_y+hk_2^{(3)})^2]/c^2}\left(v_x+hk_1^{(3)}\right)B_0, \tag{46}$$

$$k_4^{(4)} = v_x + hk_1^{(3)}, \tag{47}$$

$$k_5^{(4)} = v_y + hk_2^{(3)}. \tag{48}$$

The details of the Java programing for simulating the charge motion using the increment functions are described in Reference [1]. The numerical calculations are performed using the double precision method and using the time increment $h < 0.2m/qB_0$ to obtain an accurate simulation [2].

3.2 Electron Velocity Previously Simulated Using the Invariant Mass

A typical simulation reported previously [2] using the invariant mass for the electron motion with the velocity (v_x: blue line, v_y: red line) accelerated by the synchronized electric field is shown in Figure 1, in which, we use E_0 = 90 V/m, f = 279.92 MHz, and B_0 = 0.01 T, for the electron injected along the x-direction with the initial velocity of 116.8 km/s, consistent with the thermal velocity. In the simulation, the velocity of the electron results in a velocity greater than the velocity of light at the time of 38.5 μs after the injection. In fact, the electron motion near the velocity of light cannot be represented by the classic resonance using the invariant mass, as mentioned in section 2.1. Nevertheless, it is a fact that, if the synchronization is maintained, the electron is accelerated continuously, resulting in a velocity near the velocity of light.

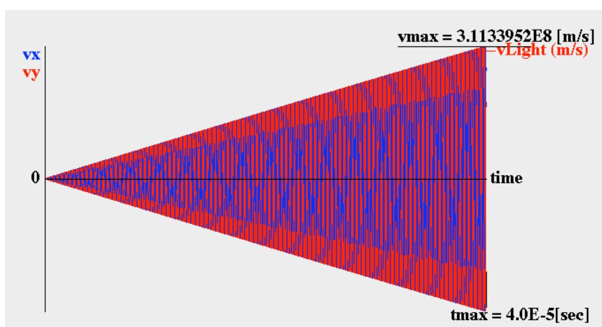

Figure 1: A typical acceleration of the electron by the synchronized electric field to the frequency of the Larmor motion, simulated using the invariant mass. The electron is injected with the initial velocity v_x = 116.8 km/s into the fields at E_0 = 90 V/m, f = 279.92 MHz, and B_0 = 0.01 T. In the simulation, the electron is accelerated up to the velocity of light at t = 38.5 μs after the injection. The velocity of light is shown as vLight in the figure.

3.3 Electron Velocity Simulated Using the Relativistic Mass

In the above simulations, we used the classical model using the invariant mass for the electron motion. As mentioned in section 2.1, a particle moving with a very large velocity behaves effectively as a particle with the relativistic mass. The relativistic mass increases with the increase in the velocity; as a result, the effective Larmor frequency decreases with the increase of the velocity. Therefore, if the value of v/c becomes an effective value to 1 with the increase of the velocity, the applied alternative electric field slips off the synchronization to the frequency of Larmor motion, and the deceleration of the electron becomes superior to the acceleration, resulting in the limitation of the electron velocity. The simulated result using the relativistic mass, that is, using Equations (33) – (48) under the same conditions as those in Figure 1, except for the mass, is shown in Figure 2. The maximum velocity is limited to 1.48 x 10^7 m/sec, that is, the deceleration of the electron becomes dominant at that velocity due to the decrease in the effective Larmor frequency.

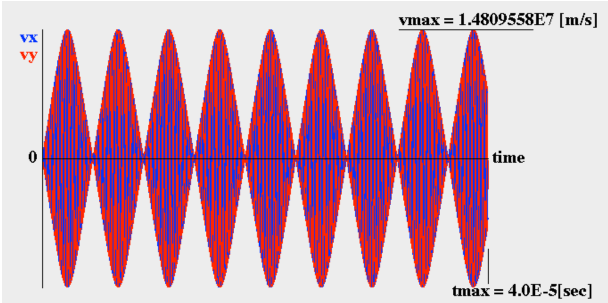

Figure 2: The electron velocity simulated using the relativistic mass under the same conditions as those in Figure 1, except for the mass. The maximum velocity of the electron is limited to 1.48 x 10^7 m/sec.

4 Discussion

The very precise synchronization of the alternative electric field to the frequency of Larmor motion, e.g., six digits of the frequency, is required for the acceleration of the electron near the light velocity, as shown in Figure 1. Therefore, in the use of the relativistic mass, the decrease in the frequency of Larmor motion due to the increase in the velocity is effective at a relatively low velocity, and the electron velocity is limited at the velocity, as shown in Figure 2. In this case, the electron is better accelerated using an initially lower electric frequency than the initially synchronized frequency, taking into consideration of the increase in the relativistic mass, as shown in Figure 3. Even in the case of accounting for the relativistic mass, if the frequency is less than 279.6 MHz, then the alternative electric field slips off the Larmor motion rapidly, and the deceleration becomes immediately dominant, resulting in very little acceleration, as shown in Figure 4. That is, the electron is actually accelerated up to approximately 2.34 x 10^7 m/s at most under these conditions.

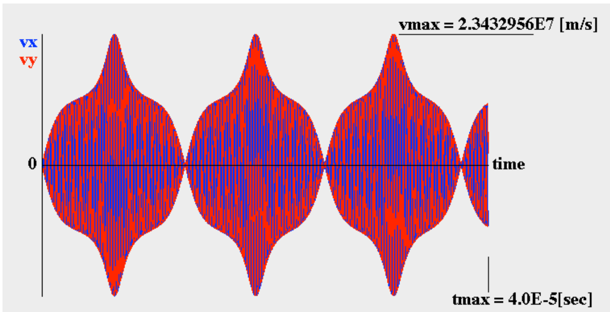

Figure 3: The electron velocity simulated using a slightly lower frequency under the same conditions as those in Figure 2, except the frequency, *f* = 279.6 MHz. The maximum velocity of the electron is limited at 2.34 x 10^7 m/sec, which is 1.6 times larger than the maximum velocity simulated using the initially synchronized frequency, *f* = 279.92 MHz.

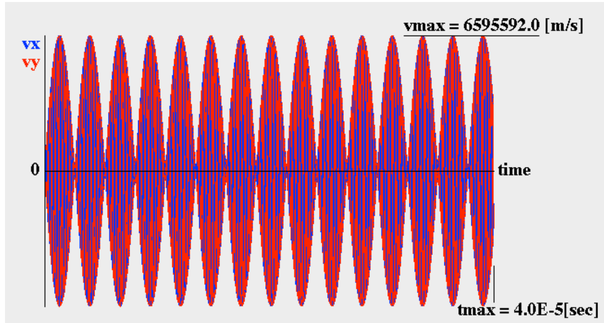

Figure 4: The electron velocity simulated using a slightly lower frequency than 279.6 MHz under the same conditions as those in Figure 3, except the frequency, *f* = 279.5 MHz. The maximum velocity of the electron is limited to a very low velocity, 6.6 x 10^6 m/sec, due the deceleration becoming dominant very rapidly.

5 Conclusion

The numerical simulation of the charge motion near the velocity of light, that is, the charge motion using the relativistic mass by Einstein's special theory of relativity, in electric and magnetic fields was investigated using the fourth-order Runge-Kutta method. The results are summarized as follows:

1. The charge motion can be solved numerically using the incremental functions, Equations (11) – (32).
2. The Larmor motion of the electron in a static magnetic field perpendicularly applied a synchronized electric field was simulated using a Java programing based on the above-described method, resulting in the limitation of the electron velocity at a considerably lower value than the light velocity in contradiction to the previous simulation in which the electron is easily accelerated over the velocity of light due to the use of the invariant mass.

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