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Effects of Strengths of Steel and Concrete, Eccentricity and Bar Size on the Optimization of Eccentrically Loaded Footings

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ABSTRACT

This paper aims to explore effects of the yield strength of steel, compressive strength of concrete, eccentricity of the axial load and steel bar size on the optimization of reinforced concrete isolated footings. The optimization tool adopted in this paper is genetic algorithms. Based on the ACI Building Code, constraints are built by considering the wide-beam and punching shears, bending moment, upper and lower limits of reinforcement, allowable soil pressure, development length for deformed bars and clear distance between parallel deformed bars. Design variables consist of the width, length and thickness of the footing and the number of bars in the long and short directions, all of which are integers. The objective is to minimize the cost of steel and concrete used in the footing. By changing one of the four factors: the yield strength of steel, compressive strength of concrete, eccentricity and bar size while fixing the other three, this paper finds that the highest yield strength of steel, the lowest compressive strength of concrete, the smallest eccentricity and No. 6 bar, respectively, will lead to the optimal results. In addition, when the size of the reinforcement gets larger, the optimal footing have a tendency to become square and thicker.

Keywords: Genetic Algorithms; Optimal Design; Reinforced Concrete Footings; Eccentricity

1 Introduction

A footing that serves the purpose of transmitting the load from the superstructure to the supporting soil is a very important element in an architectural structure. A conventional way to design a footing is to find its suitable dimensions and amount of reinforcement according to the provisions of a building code. The design results are usually not the most economical. In order to achieve this goal, the optimization techniques can be applied. There have been a number of optimization studies of reinforced concrete footings published over the past few years, such as optimization of combined footings using modified complex method of box [1], optimization of concentrically loaded reinforced concrete footing using an analytical model [2], and optimization of concentrically loaded reinforced concrete footings using genetic algorithms [3].

The fundamental techniques of genetic algorithms are designed to imitate processes in natural evolution. Genetic algorithms are the most effective methods in a search space for which little is known and which is uneven and has many hills and valleys with potential candidate solutions. The idea of genetic algorithms was inspired by the evolution theory of "survival of the fittest," and formally introduced in 1970s by Professor John Holland at the University of Michigan, who in 1975 published the ground-breaking book "Adaptation in Natural and Artificial System" [4] that led to many important discoveries. In 1989, Goldberg

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described in more detail the theory of genetic algorithms and its applications [5]. From then on, the continuing improvements of computational techniques have made genetic algorithms more attractive and popular. Genetic algorithms have successfully been applied to many fields, for example, engineering, economics, chemistry, manufacturing, mathematics and physics. Especially in the aspect of engineering structures, there are a lot of applications, such as reliability analysis of structures [6], global optimization of grillages [7], global optimization of trusses with a modified genetic algorithm [8], optimization of pile groups using hybrid genetic algorithms [9], prediction of concrete faced rock fill dams settlements [10], optimization of grid shell topology and nodal positions [11], optimizations of constrained layered damped (CLD) laminated structures [12], calibration of a hydrological model to predict stream flows [13] and optimal design of short columns [14].The fact that they are successfully applied to many problems which are difficult to solve by using conventional optimization techniques prove that genetic algorithms are a powerful, robust optimization technique.

Most optimization approaches have been focused on and developed for continuous variables, while the design variables are usually integers for problems in architectural structures. Due to their abilities to solve discrete optimization problems, genetic algorithms provided by the MATLAB Global Optimization Toolbox [15] are used in this paper to carry out the optimization of eccentrically loaded reinforced concrete isolated footings and explore effects of the yield strength of steel, compressive strength of concrete, bar size and eccentricity on the optimal results. Based on the provisions of the ACI Building Code Requirements for Structural Concrete and Commentary [16], the constraints of genetic algorithms are constructed, considering the wide-beam and punching shears, bending moment, upper and lower limits of reinforcement, allowable soil pressure, development length for deformed bars, clear distance between deformed bars. The design variables are the depth, width and length of the footing, the number of bending reinforcement in each direction of the footing; the objective is to find the minimum cost of concrete and steel.

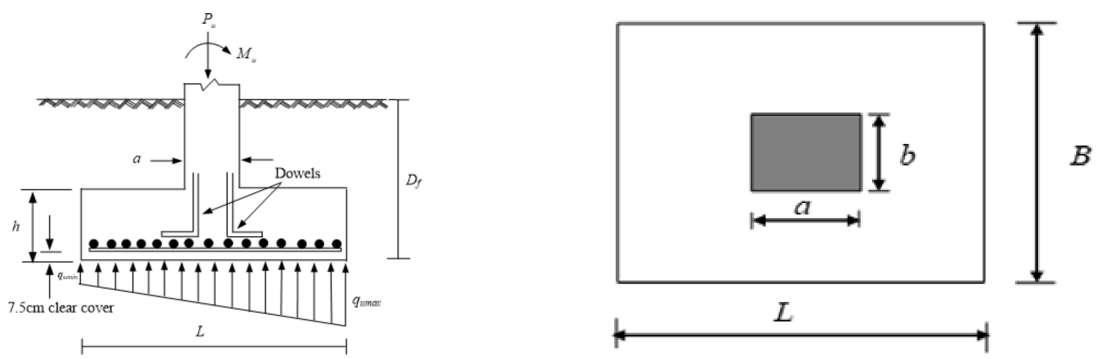
2 Genetic Algorithms

Genetic algorithms are basically a heuristic process for mimicking the survival of the fittest among individuals over a sequence of generations for solving an optimization problem. There is a population of individuals in each generation. Each individual made up of design variables represents a candidate solution to a given problem. The individuals are similar to chromosomes and the design variables to genes. A fitness value is assigned to each solution to measure its competitiveness. The individuals with higher fitness values are more likely to be selected to form the next generation. If the current population can no more produce individuals significantly better than those in the previous few generations, the algorithm is said to converge and the optimal solution are found.

The most common type of genetic algorithms works through the following process of natural selection: (1) Randomly create an initial population of individuals; (2) Score each individual of the current population by computing the value of the fitness function; (3) Scale the raw fitness scores to convert them into a range that is suitable for the selection function; (4) Select a specified number of individuals with lower fitness values, called parents, by using the selection function; (5) Choose a few elite individuals with the lowest fitness values from the current population. These elite individuals are then just passed to the next population; (6) Produce children from the parents. Children are produced either by combining portions of good individuals (i.e., crossover), which aims to create even better individuals or making random changes to a single individual (i.e., mutation), whose purpose is to maintain diversity within the population and inhibit premature convergence; (7) Replace the current population with the crossover and mutation children and elites to form the next generation and the process repeats form steps (2) to (7). The algorithm stops when one of the stopping criteria is met, such as the number of generation, the weighted average change in the fitness function value over some generations less than a specified tolerance, no improvement in the best fitness value for an interval of time, etc. Genetic algorithms can solve both constrained and unconstrained optimization problems. The constraints built for genetic algorithms can be linear or nonlinear in the form of equality or inequality with bounds on the variables. Each individual made up of the design variables can be real-coded or binary-coded. In this paper, the constraints consist of nonlinear and linear inequalities and all the design variables are integers.

3 Design Considerations in Eccentrically Loaded Footings

Both the concentric compressive force P_u and bending moment M_u, are considered to act on the reinforced concrete isolated footing whose layout is shown in Fig. 1. The rectangular footing has width B, length L and thickness h, and the column size is $a \times b$. The soil bearing pressure distribution on the footing is trapezoidal due to the combined effects of axial load and bending, as shown in Fig. 1(a). All the constraints required to design the isolated footing comply with the ultimate-strength design of ACI 318-11 Code, considering wide-beam and punching shears, bending moment, the development length for deformed bars, clear distance between parallel deformed bars and the upper and lower limits of reinforcement. The units of force and length in the following formulas are kgf (=9.81N) and cm, respectively. The factored load $P_u = 1.2P_D + 1.6P_L$,



(a) (b)

Figure 1 The reinforced concrete footing subjected to the concentric factored load Pu and bending moment Mu: (a) elevation and (b) plan

Figure 1 The reinforced concrete footing subjected to the concentric factored load P_u and bending moment M_u: (a) elevation and (b) plan.

where P_D and P_L are the dead and live loads, respectively. The eccentricity e is defined as M_u / P_u.

3.1 Factored Shears

To have enough shear capacity, there are two kinds of actions that need to be considered: wide-beam action and two-way action.

3.1.1 Wide-beam Action

The maximum and minimum soil pressures on the footing, as shown in Fig. 1, are

$$q_{u\max} = \frac{P_u}{LB}(1+\frac{6e}{L}) \quad (1)$$

and

$$q_{u\min} = \frac{P_u}{LB}(1-\frac{6e}{L}) \quad (2)$$

respectively, where $e \le \frac{L}{6}$ is the eccentricity. The plane of the critical section is assumed to extend in a plane across the entire width and lies at a distance *d* from the face of the column, as shown in Fig. 2(a). The nominal shear strength of this section is

$$V_{c1} = 0.53\sqrt{f_c'}Bd \quad (3)$$

and

$$V_{c2} = 0.53\sqrt{f_c'}Ld \quad (4)$$

respectively in the long and short directions, where *d* is the average effective depth of the footing. Let q_{udL} denote the soil pressure in the long direction of the footing at a distance *d* from the right face of the column and ϕ =0.75 be the strength reduction factor for shear. The constraints for the wide-beam shear are

$$V_{uL} = q_{u\,avgL}(\frac{L-a}{2}-d)B \le \phi V_{c1} \quad (5)$$

and

$$V_{uB} = q_{u\,avgB}(\frac{B-b}{2}-d)L \le \phi V_{c2} \quad (6)$$

respectively for the long and short directions of the footing, where

$$q_{uavgL} = \frac{q_{u\max}+q_{udL}}{2} \quad (7)$$

and

$$q_{uavgB} = \frac{q_{u\max}+q_{u\min}}{2} \quad (8)$$

3.1.2 Two-way Action

The critical section occurs at a distance *d*/2 from the face of the column, as shown in Fig. 2 (b). The maximum allowable nominal shear strength is the smallest of the following three equations

$$V_c = (0.53 + \frac{1.06}{\beta_c})\sqrt{f_c'}b_0 d \ ,$$

$$V_c = (0.53 + \frac{0.265\alpha_s d}{b_0})\sqrt{f_c'}b_0 d \quad (9)$$

$$V_c = 1.06\sqrt{f_c'}b_0 d$$

where β_c = long side *a*/short side *b* of the concentrated load or reaction area, b_0 = perimeter of the critical section *CDEF* and α_s = 40, 30 and 20 for interior, edge and corner columns, respectively. In this paper, interior columns are considered; therefore, α_s = 40. Let q_{uL1} and q_{uL2} denote the soil pressures at a distance *d*/2 from the left and right faces of the column, respectively. The constraint for the punching shear is

$$V_u = P_u - \frac{q_{uL1} + q_{uL2}}{2}(a+d)(b+d) \le \phi V_{c,\min} \quad (10)$$

where $V_{c,min}$ is the smallest of Eqs. (9).

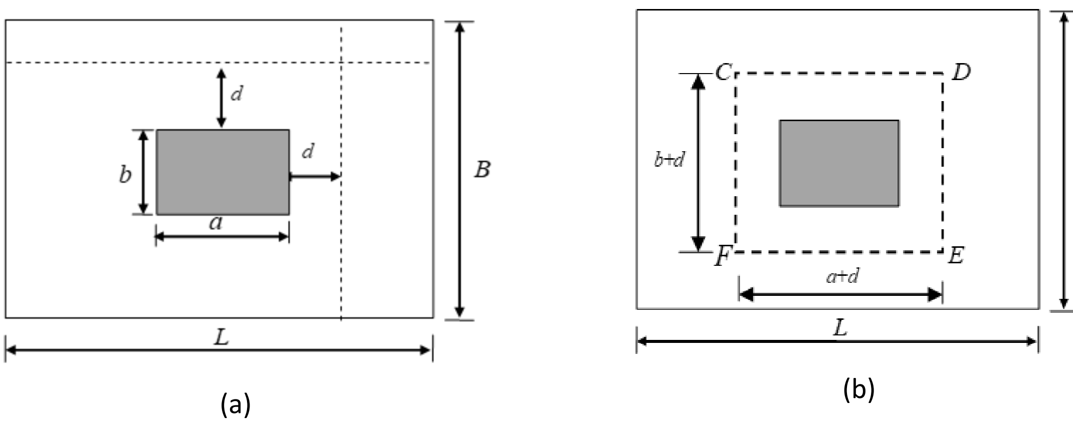


Figure 2 Critical sections: (a) wide-beam action and (b) two-way action.

3.2 Factored Moments

Suppose that N_L and N_B are the number of steel bars required in the long and short directions of the footing, respectively, and A_b is the cross-sectional area of the flexural reinforcement. The critical section for moment is taken at the face of the column. Let q_{ul3} denote the soil pressure at the right face of the column. The constraints for the factored moments are

$$M_{uL} = \frac{(q_{uL3} + q_{u\max})B}{2}(\frac{L-a}{2})k \le \phi_{mL} N_L A_b f_y (d - \frac{N_L A_b f_y}{2(0.85)f_c' B}) \quad (11)$$

and

$$M_{uB} = \frac{(q_{u\max} + q_{u\min})L}{2}\frac{(B-b)^2}{8} \le \phi_{mB} N_B A_b f_y (d - \frac{N_B A_b f_y}{2(0.85)f_c' L}) \quad (12)$$

where ϕ_{mL} and ϕ_{mB} are the strength reduction factors for moment and *k* is distance from the face of the column to the centroid of the trapezoid. Let ε_t be the tensile strain of the reinforcement, then

$$0.65 \le \phi_{mL} \ or \ \phi_{mB} = 0.65 + 0.25 \frac{\varepsilon_t - \varepsilon_y}{0.005 - \varepsilon_y} \le 0.9 \tag{13}$$

3.3 Upper and Lower Limits of Reinforcement

To prevent sudden failure with little or no warning when the beam cracks or fails in a brittle manner, the ACI code limits the minimum and maximum amount of steel to be

$$A_{sL,\min} \le N_L A_b \le A_{sL,\max} = \frac{0.85 f_c' \beta B d}{f_y} (\frac{3}{7}) \tag{14}$$

and

$$A_{sB,\min} \le N_B A_b \le A_{sB,\max} = \frac{0.85 f_c' \beta L d}{f_y} (\frac{3}{7}) \tag{15}$$

respectively in the long and short directions, where β is the stress block depth factor,

$$A_{sL,\min} \ge \max\ (\frac{0.8\sqrt{f_c'}}{f_y} Bd,\ \frac{14Bd}{f_y}) \tag{16}$$

and

$$A_{sB,\min} \ge \max\ (\frac{0.8\sqrt{f_c'}}{f_y} Ld,\ \frac{14Ld}{f_y}) \tag{17}$$

The formula for $A_{sL,max}$ in Eq. (14) and $A_{sB,max}$ in Eq. (15) is derived based on the requirement that the tensile strain must be greater than or equal to 0.004. In addition, both the steel ratios $N_L A_b/(Bh)$ and $N_B A_b/(Lh)$ must exceed the minimum value required for temperature and shrinkage: 0.0018 for grade 60 deformed bars and 0.002 for grade 40 or 50 deformed bars.

3.4 Allowable Bearing Capacity of Soil

Bearing capacity is the capacity of soil to support the loads applied to the ground. Usually, only the service loads need to be considered, i.e., P_D and P_L without load factors. Assume that the allowable soil pressure under the base of the footing is q_a. The gross soil pressure must not exceed the allowable soil pressure, that is,

$$\frac{P_D + P_L}{BL} + h w_c + \gamma_s (D_f - h) \le q_a \tag{18}$$

where D_f is the distance from the base of the footing to the ground surface, as shown in Fig. 1, w_c is the weight of concrete and γ_s is the unit weight of soil over the footing.

3.5 Development Length for Deformed Bars

The ACI Code specifies that the equation for the development of deformed bars in tension be expressed by

$$L_d = \frac{0.15 d_b f_y \psi_t \psi_e \lambda}{\sqrt{f'_c}} \geq 30\ cm \tag{19}$$

or $$L_d = \frac{0.19 d_b f_y \psi_t \psi_e \lambda}{\sqrt{f'_c}} \geq 30\ cm \tag{20}$$

for No. 6 and smaller bars or No. 7 and larger bars, respectively, with clear spacing not less than $2d_b$ and clear cover not less than d_b, where d_b is the bar diameter, and ψ_t and ψ_e are the bar location and coating factors, respectively. In this paper ψ_t and ψ_e are assumed to be 1.0 and λ=1 for normal weight concrete. The critical section for development length of the bars in tension is the same as the critical section in flexure, that is, at the face of the column. Hence,

$$0.5(L\text{-}a) \text{ - concrete cover} \geq L_d \tag{21}$$

and

$$0.5(B\text{-}b) \text{ - concrete cover} \geq L_d \tag{22}$$

respectively in the long and short directions. The equation for the development length of bars in compression is

$$L_{dc} = \max\ \left(\frac{0.075 d_b f_y}{\sqrt{f'_c}},\ 0.0043 d_b f_y\right) \tag{23}$$

The dowel bars stressed to f_y are required to transfer the axial compression force in the column into the footing, as shown in Fig. 1; hence, there should be minimum extension of the dowels into the footing. Therefore, the thickness h of the footing must satisfy the following constraints:

$$h - \text{concrete cover - } 2d_b \text{ (footing bars) - } d_b \text{ (dowels)} \geq L_{dc} \tag{24}$$

In addition, depth of footing above bottom reinforcement shall not be less than 15 cm for footings on soil and a practical minimum thickness h should not be less than 25 cm.

3.6 Distribution and Minimum Clear Distance of Steel Bars

The total steel area $N_B A_b$ in the short direction determined from Eqs. (12) and (15) should be uniformly distributed over the central band of the footing, whose width is B, as shown in Fig. 3. The ratio of the reinforcement in the central band to the total reinforcement is equal to $2/(L/B+1)$. The reinforcement that is not placed in the central band is uniformly spaced at each side of the central band. In the long direction, the total steel area $N_L A_b$ determined from Eqs. (11) and (14) is uniformly distributed across the entire width of the footing. The clear distance s between parallel steel bars in both the long and short directions must satisfy

$$Min\ (3h,\ 45\ cm) \geq s \geq Max\ (2d_b,\ 2.5\ cm) \tag{25}$$

Instead of d_b, the minimum clear distance $2d_b$ used in Eq. (25) is due to the requirement of Eqs. 19 and 20.

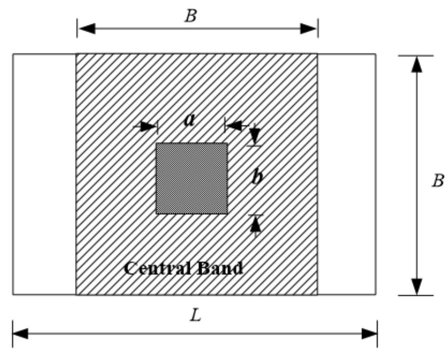


Figure 3 The central band of the footing.

4 Numerical Results

The given design conditions for the eccentrically loaded footings are as follows: the dead load P_D=100 ton, the live load P_L=80 ton, the distance from the footing bottom to the ground surface D_f =1.5m, the unit weight of concrete γ_c=2.4 ton/m^3, the unit weight of soil over the footing γ_s= 2 ton/ m^3 and the allowable soil pressure at the base of the footing q_a=25 ton/m^2. The size of the column transferring the axial load and eccentric moment to the footing is assumed to be 0.40 m×0.40 m. The concrete cover for the reinforcement of the footing is assumed to be 7.5 cm. In order to explore their effects on the optimization of eccentrically loaded footings, the yield strength of steel f_y, compressive strength of concrete f_c', size of the flexural reinforcement or eccentricity e is varied, with the other three fixed. In Taiwan, the unit price of concrete is 1950 NT$/m, 2150 NT$/m^3, 2350 NT$/m^{3}and 2450 NT$/m^3 for f_c' =210 kgf/cm^2 (3000 psi), 280 kgf/cm^2 (4000 psi), 350 kgf/cm^2 (5000 psi) and 420 kgf/cm^2 (6000 psi), respectively; the unit price of steel is 14400 NT$/ton. Design variables are the thickness h, width B and length L of the footing, and the number of steel bars in the long direction N_L and short direction N_B. In this paper, there are six kinds of bar sizes: Nos. 4 to 9; four kinds of f_c': 210 kgf/cm^2, 280 kgf/cm^2, 350 kgf/cm^2 and 420 kgf/cm^2; three kinds of f_y: 2800 kgf/cm^2 (40 ksi), 3500 kgf/cm^2 (50 ksi) and 4200 kgf/cm^2 (60 ksi); and seven kinds of eccentricity: 0 cm, 10 cm, 20 cm, 30 cm, 40cm, 50 cm and 60 cm. The fitness function is the total cost in New Taiwan Dollars of the footing reinforcement and concrete. All the constraints are built according to the formulas discussed in Sec. 3. The population size is set to be 100, crossover rate 0.8, and elite number 5. Furthermore, all the individuals are encoded as integers; "Rank" is used as the scaling function that scales the fitness values based on the rank of each individual; "Roulette" is the selection function to choose parents for the next generation; "Two-point crossover" is used as the crossover method to form a new child for the next generation; The "Adaptive Feasible Function" is selected as the mutation function. The results are discussed as follows.

4.1 Optimal Results by Fixing f_y = 4200 kgf/cm^2, f_c' =210 kgf/cm^2 and e = 10 cm

The size of reinforcement is varied, ranging from No. 4 to No. 9. The optimal results are listed in Table 1, where No. 6 reinforcement has the minimum cost. The lowest cost can be seen clearly in Fig. 4. In addition, when the size of the reinforcement becomes larger, the optimal footing grows square and thicker, as shown in Fig. 5.

4.2 Optimal Results by Fixing f_c' =210 kgf/cm^2, No. 6 bar and e = 10 cm

There are three kinds of yield strengths of steel: f_y = 4200 kgf/cm^2, 3500 kgf/cm^2 and 2800 kgf/cm^2. The optimal results are listed in Table 2, where f_y = 4200 kgf/cm^2 has the minimum cost. Besides, when f_y changes, the optimal thickness remains the same.

4.3 Optimal Results by Fixing f_y = 4200 kgf/cm^2, No. 6 bar and e = 10 cm

There are four kinds of compressive strengths of concrete: f_c' =210 kgf/cm^2, 280 kgf/cm^2, 350 kgf/cm^2 and 420 kgf/cm^2. The results are listed in Table 3, where f_c' =210 kgf/cm^2 has the minimum cost. Besides, when f_c' becomes larger, the thickness of the footing turns to be smaller.

4.4 Optimal Results by Fixing f_y = 4200 kgf/cm^2, f_c' =210 kgf/cm^2 and No. 6 bar

There are seven kinds of eccentricity explored: e= 0 cm, 10 cm, 20 cm, 30 cm, 40 cm, 50 cm and 60 cm. The results are listed in Table 4, which shows the smaller the eccentricity is, the less cost the footing becomes. Aside from that, when the eccentricity becomes bigger, the optimal footing turns to be thicker.

Table 1 Optimal results by fixing f_y = 4200 kgf/cm^2, f_c' =210 kgf/cm^2 and e = 10 cm.

Bar size	h (cm)	B (cm)	L (cm)	N_B	N_L	Cost (NT$)
No. 4	62	270	307	43	38	13,188
No. 5	62	266	311	28	24	13,180
No. 6	62	261	317	20	16	13,166
No. 7	63	300	300	14	14	14,547
No. 8	71	335	335	14	14	20,672
No. 9	79	372	372	14	14	28,628

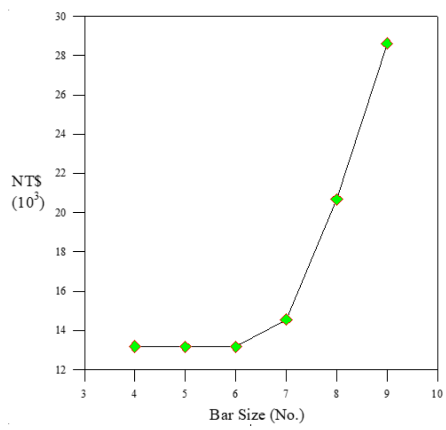


Figure 4 The optimal prices for different bar sizes

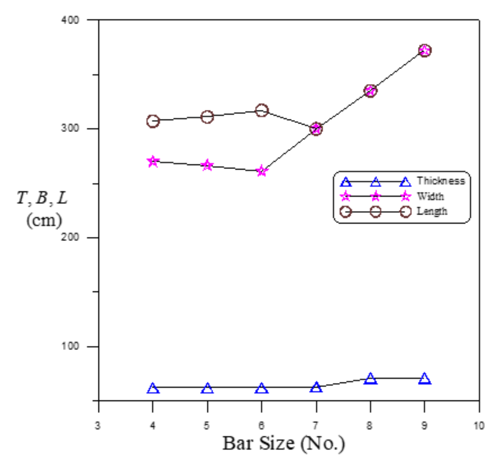


Figure 5 The Optimal Results of thickness, width and length for different bar sizes

Table 2 Optimal results by fixing f_c' =210 kgf/cm^2, No. 6 bar and e = 10 cm.

f_y (kgf/cm^2)	h (cm)	B (cm)	L (cm)	N_B	N_L	Cost (NT$)
2800	62	282	294	27	26	14,715
3500	62	257	322	24	19	13,781
4200	62	261	317	20	16	13,166

Table 3 Optimal results by fixing f_y = 4200 kgf/cm^2, No. 6 bar and e = 10 cm.

f_c' (kgf/cm^2)	h (cm)	B (cm)	L (cm)	N_B	N_L	Cost (NT$)
210	62	261	317	20	16	13,166
280	58	275	301	17	16	13,240
350	55	281	294	17	17	13,683
420	52	285	290	17	17	13,535

Table 4 Optimal results by fixing fy = 4200 kgf/cm2, fc' =210 kgf/cm2 and No. 6 bar.

e (cm)	h (cm)	B (cm)	L (cm)	N_B	N_L	Cost (NT$)
0	62	261	317	20	16	13,166
10	62	261	317	20	16	13,166
20	62	275	301	19	17	13,187
30	64	284	292	19	19	13,715
40	67	286	290	20	20	14,378
50	72	277	300	22	21	15,478
60	86	231	361	33	21	18,724

5 Conclusions

This paper explores effects of the yield strength of steel, compressive strength of concrete, bar size and eccentricity of the axial load transmitted to the footing on the optimization of reinforced concrete isolated footings. Genetic algorithms are used to optimally design the eccentrically loaded reinforced concrete footings. From the numerical results, the principal conclusions may be summarized as follows:

(1) The steel yield strength of 4200 kgf/cm^2, the concrete compressive strength of 210 kgf/cm^2, the smallest eccentricity and No. 6 bar, respectively, will have the optimal results if one of them is varied and the other three are fixed.
(2) When the size of the reinforcement is getting larger, the optimal footing have a tendency to become square and thicker.
(3) When f_y changes, the optimal thickness of the footing remains the same.
(4) When f_c' becomes larger, the optimal footing is getting thinner.
(5) When the eccentricity becomes bigger, the optimal footing grows thicker.

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