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Analytical Modeling of the Dynamics of Random Processes During Combat Use of a Military Tetrasystem

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Abstract: A military system consisting of four autonomous subsystems (a tetrasystem) is considered: air, land, sea, and drone. During combat, each subsystem is subject to a stream of random events involving losses and restorations. The dynamics of random processes is studied using a continuous-time Markov chain with sixteen asymmetric possible states. The corresponding mathematical model of the random processes is constructed in the form of sixteenth-order Kolmogorov differential equations. Formulas are found for the sixteen roots of the characteristic Kolmogorov equation, expressed in terms of the intensities of the tetrasystem's loss and restoration flows. The analytical solution to the Kolmogorov differential equations for the tetrasystem is represented in the form of ordered matrices and sixteenth-order determinants, which allows for a compact description of a large volume of initial data, overcomes limitations associated with the problem's dimensionality, and ensures adaptability to computer technologies, including the problem of verification.

Keywords: tetrasystem, Markovian peocess, Kolmogorov equation, characteristic equation, state probabilities, reliability of systems

INTRODUCTION

Comprehensive scientific research conducted by authoritative scientists [1] shows that the main factors causing negative changes in the planet's climate are caused by human activity and are related to emissions of gases: carbon dioxide, greenhouse gases, freon, etc. With the growth of birth rates and population, the consumption of the planet's resources is accelerating globally and outpacing the Earth's ability to regenerate. This leads to catastrophic consequences in the social sphere, provoking risks of social collapse, conflicts, wars, migration, famine, etc. The theoretical foundations for solving such global problems are laid out in the works of P. Teilhard de Chardin, E. Leroy, and V. I. Vernadsky on the evolution of the biosphere into the noosphere. The practical implementation of these ideas is possible with the growing role of science, the unification of the world's scientists under the auspices of the UN, the development of artificial intelligence (AI), computational experiments, analytical modeling, improving the level of education, intelligence quotient (IQ) of the population, government and political figures, regulation of the number of ruminants, and the use of coal, oil, gas, and other natural resources.

In connection with the consideration of the global climate problem, a number of urgent tasks arise in modeling, forecasting, and managing the dynamics of random processes in the fields of energy, ecology, sociology, biology, medicine, economics, technology, etc. [2, 3, 4]. Markov chains with discrete states and continuous time serve as a universal dynamic scheme for such tasks, and the corresponding Kolmogorov differential equations serve as mathematical models [5, 6, 7].

The purpose of this work is to develop a universal mathematical model of Markov random processes in complex systems, including military tetrasystems in combat conditions, and to develop accurate, analytical methods for solving Kolmogorov's equations to assess the reliability of a military tetrasystem over time depending on the initial conditions, loss and recovery rates of four autonomous military subsystems.

METHODS

Problem Statement

We consider a military system consisting of four autonomous subsystems (tetrasystem) designated in the following order:

1. Air forces (A);
2. Ground forces (G);
3. Naval forces (M);
4. Drone forces (D).

Each subsystem is affected by a stream of random events that transition the subsystem from a combat-ready state, denoted by the number 1, to a non-combat-ready state, denoted by the number 0, and vice versa. The intensity of the loss flows λ_i and recovery flows μ_i of the subsystems (i = 1, 2, 3, 4) is assumed to be statistically known on average over a given time interval [0, T]. The state of the military tetrasystem at the initial moment of time is given.

It is necessary to determine the probabilities of possible states of the military tetrasystem at any current moment in time and in the future, depending on the initial state and the intensity of loss and recovery flows of four autonomous subsystems.

Solution Algorithm

The general algorithm for solving similar problems for two-system and three-system configurations is given in [8,9] and is implemented in this work for a military four-system configuration.

The possible asymmetric states of the four-system configuration are represented by the following matrix:

$$S = \begin{Vmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{Vmatrix}$$

Based on the constructed state matrix S, a matrix of loss and recovery flow intensities of the tetrasystem $R^{(4)}$ is formed:

$$R^{(4)} = \begin{Vmatrix} R_{11}^{(4)} & R_{12}^{(4)} & R_{13}^{(4)} & R_{14}^{(4)} \\ R_{21}^{(4)} & R_{22}^{(4)} & R_{23}^{(4)} & R_{24}^{(4)} \\ R_{31}^{(4)} & R_{32}^{(4)} & R_{33}^{(4)} & R_{34}^{(4)} \\ R_{41}^{(4)} & R_{42}^{(4)} & R_{43}^{(4)} & R_{44}^{(4)} \end{Vmatrix},$$

where:

$$R_{11}^{(4)} = \begin{Vmatrix} -(\lambda_1+\lambda_2+\lambda_3+\lambda_4) & 0 & \mu_1 & 0 \\ 0 & -(\mu_1+\mu_2+\mu_3+\mu_4) & 0 & \lambda_1 \\ \lambda_1 & 0 & -(\mu_1+\lambda_2+\lambda_3+\lambda_4) & 0 \\ 0 & \mu_1 & 0 & -(\mu_1+\mu_2+\mu_3+\mu_4) \end{Vmatrix}$$

$$R_{22}^{(4)} = \begin{Vmatrix} -(\lambda_1+\mu_2+\lambda_3+\lambda_4) & 0 & 0 & 0 \\ 0 & -(\mu_1+\lambda_2+\mu_3+\mu_4) & 0 & 0 \\ 0 & 0 & -(\lambda_1+\lambda_2+\mu_3+\lambda_4) & 0 \\ 0 & 0 & 0 & -(\mu_1+\mu_2+\lambda_3+\mu_4) \end{Vmatrix}$$

$$R_{33}^{(4)} = \begin{Vmatrix} -(\lambda_1+\lambda_2+\lambda_3+\mu_4) & 0 & 0 & \mu_3 \\ 0 & -(\mu_1+\mu_2+\mu_3+\lambda_4) & \lambda_3 & 0 \\ 0 & \mu_3 & -(\mu_1+\mu_2+\lambda_3+\lambda_4) & 0 \\ \lambda_3 & 0 & 0 & -(\lambda_1+\lambda_2+\mu_3+\mu_4) \end{Vmatrix}$$

$$R_{44}^{(4)} = \begin{Vmatrix} -(\mu_1+\lambda_2+\mu_3+\lambda_4) & 0 & 0 & 0 \\ 0 & -(\lambda_1+\mu_2+\lambda_3+\mu_4) & 0 & 0 \\ 0 & 0 & -(\mu_1+\lambda_2+\lambda_3+\mu_4) & 0 \\ 0 & 0 & 0 & -(\lambda_1+\mu_2+\mu_3+\lambda_4) \end{Vmatrix}$$

$$R_{12}^{(4)} = \begin{Vmatrix} \mu_2 & 0 & \mu_3 & 0 \\ 0 & \lambda_2 & 0 & \lambda_3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{Vmatrix}, \ R_{21}^{(4)} = \begin{Vmatrix} \lambda_2 & 0 & 0 & 0 \\ 0 & \mu_2 & 0 & 0 \\ \lambda_3 & 0 & 0 & 0 \\ 0 & \mu_3 & 0 & 0 \end{Vmatrix}$$

$$R_{13}^{(4)} = \begin{Vmatrix} \mu_4 & 0 & 0 & 0 \\ 0 & \lambda_4 & 0 & 0 \\ 0 & 0 & \mu_2 & 0 \\ 0 & 0 & 0 & \lambda_2 \end{Vmatrix}, \ R_{31}^{(4)} = \begin{Vmatrix} \lambda_4 & 0 & 0 & 0 \\ 0 & \mu_4 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 \\ 0 & 0 & 0 & \mu_2 \end{Vmatrix}$$

$$R_{14}^{(4)} = \begin{Vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \mu_3 & 0 & \mu_4 & 0 \\ 0 & \lambda_3 & 0 & \lambda_4 \end{Vmatrix}, \ R_{41}^{(4)} = \begin{Vmatrix} 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \mu_3 \\ 0 & 0 & \lambda_4 & 0 \\ 0 & 0 & 0 & \mu_4 \end{Vmatrix}$$

$$R_{23}^{(4)} = \begin{Vmatrix} 0 & 0 & \mu_1 & 0 \\ 0 & 0 & 0 & \lambda_1 \\ 0 & 0 & 0 & \mu_4 \\ 0 & 0 & \lambda_4 & 0 \end{Vmatrix}, \ R_{32}^{(4)} = \begin{Vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \lambda_1 & 0 & 0 & \mu_4 \\ 0 & \mu_1 & \lambda_4 & 0 \end{Vmatrix}$$

$$R_{24}^{(4)} = \begin{Vmatrix} 0 & \mu_4 & 0 & \mu_3 \\ \lambda_4 & 0 & \lambda_3 & 0 \\ \mu_1 & 0 & 0 & \mu_2 \\ 0 & \lambda_1 & \lambda_2 & 0 \end{Vmatrix}, \ R_{42}^{(4)} = \begin{Vmatrix} 0 & \mu_4 & \lambda_1 & 0 \\ \lambda_4 & 0 & 0 & \mu_1 \\ 0 & \mu_3 & 0 & \mu_2 \\ \lambda_3 & 0 & \lambda_2 & 0 \end{Vmatrix}$$

$$R_{34}^{(4)} = \begin{Vmatrix} 0 & \mu_2 & \mu_1 & 0 \\ \lambda_2 & 0 & 0 & \lambda_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{Vmatrix}, \ R_{43}^{(4)} = \begin{Vmatrix} 0 & \mu_2 & 0 & 0 \\ \lambda_2 & 0 & 0 & 0 \\ \lambda_1 & 0 & 0 & 0 \\ 0 & \mu_1 & 0 & 0 \end{Vmatrix}.$$

Kolmogorov equations for a tetrasystem are formulated as follows:

$$\frac{d}{dt}\left[P_i(t)\right] = R^{(4)} \cdot \left[P_i(t)\right], \ (i = 1, 2, 3, \ldots, 16)$$

where P_i is the probability of the *i*-th state.

The characteristic determinant of the Kolmogorov equations of the sixteenth order is formed:

$$\Delta^{(4)}\left(\nu\right)=\det(R^{(4)}-\nu E^{(16)}),$$

where $E^{(16)}$ is a unit matrix of order 16.

A characteristic (algebraic) equation of degree 16 is formed:

$$\sum_{j=0}^{16} a_{16-j}^{(16)} \cdot \nu^{16-j} = 0$$

where $a_{16}^{(16)} = \det\left(-E^{(16)}\right)$, $a_{15}^{(16)} = -Sp\left(R^{(4)}\right)$,..., $a_0^{(16)} = \det\left(R^{(4)}\right)$.

Note that the matrix $R^{(4)}$ is special, i.e. $\det(R^{(4)})=0$.

A set of sixteen roots of the characteristic equation is established depending on the intensities of failure and recovery flows of the tetra system:

$$\nu_1 = 0,\ \nu_2 = -(\lambda_1+\mu_1)-(\lambda_2+\mu_2)-(\lambda_3+\mu_3)-(\lambda_4+\mu_4),$$

$$\nu_3 = -(\lambda_1+\mu_1),\ \nu_4 = -(\lambda_2+\mu_2)-(\lambda_3+\mu_3)-(\lambda_4+\mu_4),$$

$$\nu_5 = -(\lambda_2+\mu_2),\ \nu_6 = -(\lambda_1+\mu_1)-(\lambda_3+\mu_3)-(\lambda_4+\mu_4),$$

$$\nu_7 = -(\lambda_3+\mu_3),\ \nu_8 = -(\lambda_1+\mu_1)-(\lambda_2+\mu_2)-(\lambda_4+\mu_4),$$

$$\nu_9 = -(\lambda_4+\mu_4),\ \nu_{10} = -(\lambda_1+\mu_1)-(\lambda_2+\mu_2)-(\lambda_3+\mu_3),$$

$$\nu_{11} = -(\lambda_1+\mu_1)-(\lambda_2+\mu_2),\ \nu_{12} = -(\lambda_3+\mu_3)-(\lambda_4+\mu_4),$$

$$\nu_{13} = -(\lambda_1+\mu_1)-(\lambda_3+\mu_3),\ \nu_{14} = -(\lambda_2+\mu_2)-(\lambda_4+\mu_4),$$

$$\nu_{15} = -(\lambda_1+\mu_1)-(\lambda_4+\mu_4),\ \nu_{16} = -(\lambda_2+\mu_2)-(\lambda_3+\mu_3).$$

The distribution of the set of roots of the characteristic equation in the complex plane has the property of symmetry about the center $M^{(16)}\left(\nu_k\right)$:

$$M^{(16)}\left(\nu_k\right) = -\frac{a_{15}^{(16)}}{16},$$

where:

$$a_{15}^{(16)} = 8\left[(\lambda_1+\mu_1)+(\lambda_2+\mu_2)+(\lambda_3+\mu_3)+(\lambda_4+\mu_4)\right].$$

A resolving square matrix of the sixteenth order $\left[\Delta_i^{(4)}\left(\nu_k\right)\right]$ is formed according to the characteristic determinant depending on the roots ν_k and states of the tetrasystem, using the column matrix of initial conditions $\left[P_i\left(0\right)\right]$. A resolving column matrix of exponents is formed depending on the roots and time:

$$\left[\frac{e^{\nu_k \cdot t}}{\Pi_k^{(4)}}\right],$$

where $\Pi_k^{(4)} = \prod_{\substack{s=1 \\ s\neq k}}^{16} \left(\nu_k - \nu_s\right)$.

The desired solution to Kolmogorov's equations is determined in the form of a column matrix of probabilities of tetrasystem states $\left[P_i(t)\right]$ at the time interval under consideration as the product of a resolving square matrix and a column matrix:

$$\left[P_i(t)\right] = \left[\Delta_i^{(4)}\left(\nu_k\right)\right]\left[\frac{e^{\nu_k \cdot t}}{\Pi_k^{(4)}}\right] \quad \left(i,k = 1,2,3,\ldots,16\right)$$

states of the tetrasystem, in the symmetric structure of the distribution of roots of Kolmogorov's characteristic equation, in the ordered matrix form of the description of Kolmogorov's differential equations and the dynamics of random Markov processes.

CONCLUSIONS

A universal mathematical model of a tetrasystem has been constructed for analytical modeling of the dynamics of random Markov processes in a wide range of applications: energy, military, social, environmental, medical, technical, etc. We succeeded in establishing formulas for the roots of the characteristic equation of the sixteenth degree, expressed in terms of the intensities of failure and recovery flows of four autonomous subsystems. The analytical solution of the Kolmogorov equations of the tetrasystem is presented in the form of determinants and matrices of the sixteenth order, adapted for use with standard math software.

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