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Periodic Solutions for a Delayed Malware Mutation Model

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ABSTRACT

This paper investigates the existence of periodic solutions for a malware mutation model with time delays. We extend the result in the literature from a two-delay model to a five-delay model. Using the method of mathematical analysis, the original system has been linearized. The instability of the unique positive equilibrium point and the boundedness of the solutions will force this system to generate a periodic solution. Two sufficient conditions to guarantee the periodic oscillation of the solutions are provided, and computer simulations are given to support the present criteria.

Keywords: malware mutation model, delay, stability, periodic solution.

INTRODUCTION

Recently, Liu et al. [1] provided a malware mutation model in a wireless rechargeable sensor network with charging delay as follows:

$$\begin{cases} S'(t) = A - \beta_1 S(t)I_1(t) - \beta_2 S(t)I_2(t) - c_1 S(t) - dS(t) + \alpha R(t), \\ I_1'(t) = \beta_1 S(t)I_1(t) - (c_1 + d + \varepsilon + \gamma_1)I_1(t), \\ I_2'(t) = \beta_2 S(t)I_2(t) - (c_2 + d + \gamma_2)I_2(t) + \varepsilon I_1(t), \\ R'(t) = \gamma_1 I_1(t) + \gamma_2 I_2(t) + \mu L(t-\xi) - (c_1 + d + \alpha)R(t), \\ L'(t) = c_1\big(R(t) + S(t)\big) + c_2\big(I_2(t) + I_1(t)\big) - \mu L(t-\xi) - dL(t), \end{cases} \tag{1}$$

where $c_1, c_2, \alpha, \varepsilon, \gamma_1, \gamma_2$, and μ are the state conversion ratios, ξ is time delay, A is the quantity of the injected susceptible node, β_1, β_2 are the conversion ratios of susceptible nodes to the infectious nodes and the virus mutation nodes, respectively. $S(t), I_1(t)$, $I_2(t)$ and $L(t)$ represent the numbers of the susceptible, the infectious, the mutated malware, the recovered, and the low-energy nodes at time t, respectively. The authors investigated the bifurcation phenomenon of the model (1) with the time delay ξ. Since the recovered nodes need a certain period to convert into susceptible nodes again due to their partial or temporary immunity. Therefore, Wang et al. [2] extended model (1) to the following two-delay model:

$$\begin{cases} S'(t) = A - \beta_1 S(t)I_1(t) - \beta_2 S(t)I_2(t) - c_1 S(t) - dS(t) + \alpha R(t-\xi_2), \\ I_1'(t) = \beta_1 S(t)I_1(t) - (c_1 + d + \varepsilon + \gamma_1)I_1(t), \\ I_2'(t) = \beta_2 S(t)I_2(t) - (c_2 + d + \gamma_2)I_2(t) + \varepsilon I_1(t), \\ R'(t) = \gamma_1 I_1(t) + \gamma_2 I_2(t) + \mu L(t-\xi_1) - (c_1 + d + \alpha)R(t-\xi_2), \\ L'(t) = c_1\big(R(t) + S(t)\big) + c_2\big(I_2(t) + I_1(t)\big) - \mu L(t-\xi_1) - dL(t). \end{cases} \tag{2}$$

The stability and Hopf bifurcation analysis of the model (2) were concerned. Not only do the recovered nodes require a certain time delay to convert into susceptible nodes, but the susceptible, infectious, mutated malware nodes also have time delays. Many researchers have studied various susceptible and infectious delayed systems [3-12]. Therefore, in this paper, we extend model (2) to a five-delayed system:

$$\begin{cases} S'(t) = A - \beta_1 S(t) I_1(t) - \beta_2 S(t) I_2(t) - c_1 S(t) - dS(t-\tau_1) + \alpha R(t-\tau_4), \\ I_1'(t) = \beta_1 S(t) I_1(t) - (c_1 + d + \varepsilon) I_1(t) - \gamma_1 I_1(t-\tau_2), \\ I_2'(t) = \beta_2 S(t) I_2(t) - (c_2 + d + \gamma_2) I_2(t) + \varepsilon I_1(t-\tau_2), \\ R'(t) = \gamma_1 I_1(t) + \gamma_2 I_2(t-\tau_3) + \mu L(t-\tau_5) - (c_1 + d) R(t) - \alpha R(t-\tau_4), \\ L'(t) = c_1\big(R(t) + S(t)\big) + c_2 I_2(t) + c_2 I_1(t-\tau_2) - \mu L(t-\tau_5) - dL(t), \end{cases} \tag{3}$$

where $\tau_i\,(i = 1, 2, \cdots, 5\,)$ are positive constants. Our goal is to investigate the existence of periodic solutions of the system (3). Obviously, bifurcation can arouse a periodic solution. However, the bifurcation method is still not easy to deal with a five-delay system if the delays are different real numbers, as in our simulation. Therefore, in this paper, we shall use the mathematical analysis method to discuss the existence of periodic solutions in model (3).

PRELIMINARIES

For model (3), we first have the following lemma:

Lemma 1: All solutions of the system (3) subject to a non-negative initial condition are bounded.

Proof: It is known that time delays do not affect the boundedness of the solutions. Therefore, set time delays are zeros in system (3). Then we have

$$S'(t) + I_1'(t) + I_2'(t) + R'(t) + L'(t) = A - d\big(S(t) + I_1(t) + I_2(t) + R(t) + L(t)\big). \tag{4}$$

Let $M(t) = S(t) + I_1(t) + I_2(t) + R(t) + L(t)$. From (4) we have

$$M'(t) = -dM(t) + A. \tag{5}$$

Noting that $-d < 0$. From (5) we have $M(t) = M(0)e^{-dt} + \frac{A}{d}$. When $t \to +\infty$, $M(t) =\to \frac{A}{d}$. This means that all solutions of system (3) are bounded.

Noting that $A > 0$ in system (3), system (3) has a positive equilibrium point. Suppose that $(S^*, I_1^*, I_2^*, R^*, L^*)^T$ is a positive equilibrium point of the system (3), make the change of the variables $S(t) \to S(t) - S^*, I_1(t) \to I_1(t) - I_1^*, I_2(t) \to I_2(t) - I_2^*, R(t) \to R(t) - R^*, L(t) \to L(t) - L^*$, noting that

$$\begin{cases} A-\beta_1 S^* I_1^* - \beta_2 S^* I_2^* - c_1 S^* - dS^* + \alpha R^* = 0, \\ \beta_1 S^* I_1^* - (c_1 + d + \varepsilon) I_1^* - \gamma_1 I_1^* = 0, \\ \beta_2 S^* I_2^* - (c_2 + d + \gamma_2) I_2^* + \varepsilon I_1^* = 0, \\ \gamma_1 I_1^* + \gamma_2 I_2^* + \mu L^* - (c_1 + d) R^* - \alpha R^* = 0, \\ c_1 (R^* + S^*) + c_2 I_2^* + c_2 I_1^* - \mu L^* - dL^* = 0, \end{cases} \tag{6}$$

We have the following system

$$\begin{cases} S'(t) = -\beta_1 I_1^* S(t) - \beta_1 S^* I_1(t) - \beta_2 I_2^* S(t) - \beta_2 S^* I_2(t) - c_1 S(t) \\ -dS(t-\tau_1) + \alpha R(t-\tau_4) - \beta_1 S(t) I_1(t) - \beta_2 S(t) I_2(t), \\ I_1'(t) = -\beta_1 I_1^* S(t) + \beta_1 S^* I_1(t) - (c_1 + d + \varepsilon) I_1(t) - \gamma_1 I_1(t-\tau_2) + \beta_1 S(t) I_1(t), \\ I_2'(t) = \beta_2 I_2^* S(t) + \beta_2 S^* I_2(t) - (c_2 + d + \gamma_2) I_2(t) + \varepsilon I_1(t-\tau_2) + \beta_2 S(t) I_2(t), \\ R'(t) = \gamma_1 I_1(t) + \gamma_2 I_2(t-\tau_3) + \mu L(t-\tau_5) - (c_1 + d) R(t) - \alpha R(t-\tau_4), \\ L'(t) = c_1 \big(R(t) + S(t)\big) + c_2 I_2(t) + c_2 I_1(t - \tau\tau_2) - \mu L(t-\tau_5) - dL(t). \end{cases} \tag{7}$$

System (7) can be expressed in the following matrix form:

$$U'(t) = AU(t) + BU(t-\tau) + f\big(U(t)\big), \tag{8}$$

where $U(t) = (S(t), I_1(t), I_2(t), R(t), L(t))^T$, $U(t-\tau) = (S(t-\tau_1), I_1(t-\tau_2), I_2(t-\tau_3), R(t-\tau_4), L(t-\tau_5))^T$, A and Bboth are 5×5 matrices,

$$A = (a_{ij})_{5\times 5} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 \\ a_{31} & 0 & a_{33} & 0 & 0 \\ 0 & a_{42} & 0 & a_{44} & 0 \\ a_{51} & 0 & a_{53} & a_{54} & a_{55} \end{pmatrix},$$

$$B = (b_{ij})_{5\times 5} = \begin{pmatrix} b_{11} & 0 & 0 & b_{14} & 0 \\ 0 & b_{22} & 0 & 0 & 0 \\ 0 & b_{32} & 0 & 0 & 0 \\ 0 & 0 & b_{43} & b_{44} & b_{45} \\ 0 & b_{52} & 0 & 0 & b_{55} \end{pmatrix},$$

where $a_{11} = -\beta_1 I_1^*, a_{12} = -\beta_1 S^*, a_{13} = -\beta_2 S^*, a_{21} = -\beta_1 I_1^*, a_{22} = \beta_1 S^* - (c_1 + d + \varepsilon), a_{31} = \beta_2 I_2^*, a_{33} = \beta_2 S^* - (c_2 + d + \gamma_2), a_{42} = \gamma_1, a_{44} = -(c_1 + d),\ a_{51} = c_1,\ a_{53} = c_2,\ a_{54} = c_1,\ a_{55} = -d;\ b_{11} = -d,\ b_{14} = \alpha,\ b_{22} = -\gamma_1, b_{32} = \varepsilon, b_{43} = \gamma_2, b_{44} = -\alpha, b_{45} = \mu, b_{52} = c_2, b_{55} = -\mu.$ $f\big(U(t)\big) = (-\beta_1 S(t) I_1(t) - \beta_2 S(t) I_2(t), \beta_1 S(t) I_1(t),\ \beta_2 S(t) I_2(t), 0, 0)^T$. The linearized system of (8) is as follows:

$$U'(t) = AU(t) + BU(t-\tau). \tag{9}$$

Then we have

Lemma 2: If matrix $P = A + B$ is a nonsingular matrix for selected parameters, then there exists a unique positive equilibrium point $U^* = (S^*, I_1^*, I_2^*, R^*, L^*)^T$ of the system (3).

Proof: Obviously, the zero equilibrium point of the system (9) corresponds to the positive equilibrium point of the system (3). If Z^* is an equilibrium point of the system (9), then we have

$$AZ^* + BZ^* = PZ^* = \mathbf{0} \tag{10}$$

According to Cramer's Rule of linear algebraic theory, the system (9) has a unique trivial solution since $P = A + B$ is a nonsingular matrix, namely, $Z^* = \mathbf{0}$, implying that the system (3) has a unique positive equilibrium point. The proof is completed.

Based on Lemma 1 and Lemma 2, in the following, we provide two theorems to guarantee the existence of periodic oscillatory solutions.

THE EXISTENCE OF PERIODIC SOLUTIONS

Theorem 1: Assume that zero is the unique equilibrium point of the system (9) for selecting parameter values. Let $\omega_1, \omega_2, \cdots, \omega_5$ be characteristic values of matrix A and $\theta_1, \theta_2, \cdots, \theta_5$ be characteristic values of matrix B. If there exists one characteristic value, say ω_1, such that $\omega_1 > 0$, or $Re(\omega_1) > 0$ and $Re(\omega_1) > \max\{\theta_1, \theta_2, \cdots, \theta_5\}$. Then the unique trivial solution of the system (9) is unstable, implying that there exists a periodic oscillatory solution in the system (3).

Proof: According to the basic differential equation theory, if there exists one characteristic value, say ω_1, such that $\omega_1 > 0$, or $Re(\omega_1) > 0$ and $Re(\omega_1) > \max\{\theta_1, \theta_2, \cdots, \theta_5\}$, then the trivial solution $U(t)$ of the system (9) is unstable [13]. Indeed, the characteristic equation associated with the system (9) can be written as follows:

$$\prod_{i=1}^{5}\left(\lambda - \omega_i - \theta_i\, e^{-\lambda\tau_i}\right) = 0. \tag{11}$$

Therefore, there is a characteristic equation from the system (11) as follows:

$$\lambda - \omega_1 - \theta_1\, e^{-\lambda\tau_1} = 0. \tag{12}$$

If $Re(\omega_1) > 0$ and $Re(\omega_1) > \max\{\theta_1, \theta_2, \cdots, \theta_5\}$, this means that equation (12) has a positive real part characteristic value. Thus, the trivial solution of the system (9) is unstable. Meanwhile, the nonlinear term of the system (8) is a higher-order infinitesimal as $S(t) \to 0, I_1(t) \to 0, I_2(t) \to 0$. Therefore, the instability of the trivial solution of the system (9) ensures the instability of the trivial solution of the system (8). This means that the unique equilibrium point $U^* = (S^*, I_1^*, I_2^*, R^*, L^*)^T$ of the system (3) is unstable. The instability of the unique equilibrium point together with the boundedness of the solutions will force the system (3) to generate a limit cycle, namely, there exists a periodic solution of the system (3) [14,15]. The proof is completed.

For simplify, setting $\sigma = \max_{1\le j\le 5}\{a_{ij}\}, \upsilon = \max\{|b_{11}|, |b_{43}|, |b_{14}| + |b_{44}|, |b_{45}| + |b_{55}|, |b_{22}| + |b_{32}| + |b_{52}|, \}$. Then we have

Theorem 2: Assume that the conditions of Lemma 1 and Lemma 2 hold. If the following inequality is satisfied

$$\sigma + \upsilon > 0. \tag{13}$$

Then the trivial solution of the system (9) is unstable, implying that the system (3) has a periodic solution.

Proof: To prove the instability of the trivial solution of the system (9), let $M(t) = S(t) + I_1(t) + I_2(t) + R(t) + L(t)$. So $M(t) > 0$, and

$$M(t) \le \sigma M(t) + \upsilon M(t-\tau). \tag{14}$$

Specifically, consider a scalar delayed differential equation

$$N(t) = \sigma N(t) + \upsilon N(t-\tau). \tag{15}$$

We have $M(t) \le N(t)$. Now we prove that the trivial solution of equation (15) is unstable. Indeed, the characteristic equation associated with equation (15) is the following

$$\lambda = \sigma + \upsilon e^{-\lambda\tau}. \tag{16}$$

We claim that there exists a positive root of (16) under the condition (13). Let $\varphi(\lambda) = \lambda - \sigma - \upsilon e^{-\lambda\tau}$. Thus, $\varphi(\lambda)$ is a continuous function of λ. When $\lambda = 0$ we have $\varphi(0) = -\sigma - \upsilon = -(\sigma + \upsilon) < 0$, since $\sigma + \upsilon > 0$. On the other hand, there exists a suitably large λ, say $\lambda_0 (> 0)$ such that $\varphi(\lambda_0) = \lambda_0 - \sigma - \upsilon e^{-\lambda_0\tau} > 0$, since $e^{-\lambda_0\tau} \to 0$ as $\lambda_0 \to .+\infty$. Based on the Intermediate Value Theorem, there exists a λ, say $\lambda_* \in (0, \lambda_0)$ such that $\varphi(\lambda_*) = 0$. In other words, λ_* is a positive characteristic root of the equation (15). So the trivial solution of the equation (15) is unstable, implying that the unique equilibrium point $U^* = (S^*, I_1^*, I_2^*, R^*, L^*)^T$ of the system (3) is unstable. Similar to Theorem 1, there exists a periodic solution of the system (3). The proof is completed.

SIMULATION RESULTS

This simulation is based on the system (3). We first select the parameters as follows:
$A = 20, \beta_1 = 0.76, \beta_2 = 0.35, c_1 = 0.76, c_2 = 0.48, d = 0.08, \varepsilon = 0.54, \alpha = 0.62, \mu = 0.55, \gamma_1 = 0.25, \gamma_2 = 0.38$, then the unique positive equilibrium point is $(S^*, I_1^*, I_2^*, R^*, L^*)^T = (5.0132, 6.4912, 6.3924, 13.8543, 25.4678)^T$. We see that $a_{11} = -7.9189, a_{12} = -3.8093, a_{13} = -1.7542, a_{21} = 4.8674, a_{22} = 3.8093, a_{31} = 2.2365, a_{33} = 0.8174, a_{42} =$

$0.25, a_{44} = -0.67, a_{51} = 0.75, a_{53} = 0.48, a_{54} = 0.75, a_{55} = -0.08$. Thus, the five eigenvalues of matrix A are $(1.4983, 0.74, -0.08, -1.1631, -4.7181)^T$, the five eigenvalues of matrix B are $(0, -0.08, -0.20, -0.25, -0.55)^T$. Noting that the matrix A has an eigenvalue 1.4983, we see that the conditions of Theorem 1 are satisfied. When time delays are selected as $\tau_1 = 2.12, \tau_2 = 2.15, \tau_3 = 2.18, \tau_4 = 2.22, \tau_5 = 2.25$, and $\tau_1 = 2.25, \tau_2 = 2.28, \tau_3 = 2.32, \tau_4 = 2.35, \tau_5 = 2.38$, respectively, the system (3) has a periodic solution, see Fig.1 and Fig.2. Then we change the value from $A = 20$ to $A = 15$ and $A = 25$, respectively, the other parameter values are the same as in Fig.1, we see that the positive equilibrium points are changed to $(S^*, I_1^*, I_2^*, R^*, L^*)^T = (4.2285, 4.9162, 4.8135, 10.3722, 17.6232)^T$, and $(S^*, I_1^*, I_2^*, R^*, L^*)^T = (6.3036, 7.6716, 8.3348, 15.8652, 32.1633)^T$, respectively, the oscillatory behavior of the solutions is maintained (see Fig.3 - Fig.6). Then we select another set of parameters as $A = 20, \beta_1 = 0.20, \beta_2 = 0.09, c_1 = 0.28, c_2 = 0.18, d = 0.20, \varepsilon = 0.45, \alpha = 0.75, \mu = 0.16, \gamma_1 = 0.34, \gamma_2 = 0.43$, the unique positive equilibrium point is $(S^*, I_1^*, I_2^*, R^*, L^*)^T = (5.8992, 5.7852, 9.7524, 7.1576, 17.7493)^T$. We see that $a_{11} = -2.3145, a_{12} = -1.1798, a_{13} = -0.5309, a_{21} = 1.1573, a_{22} = 1.1798, a_{31} = 0.8775, a_{33} = -0.2791$ $a_{42} = 0.34, a_{44} = -0.08, a_{51} = 0.28, a_{53} = 0.18, a_{54} = 0.18, a_{55} = -0.20$. Thus, $\sigma = 0.6145, \upsilon = 1.5$ and $\sigma + \upsilon > 0$. The conditions of Theorem 2 are satisfied. When time delays are selected as $\tau_1 = 3.12, \tau_2 = 3.15, \tau_3 = 3.18, \tau_4 = 3.22, \tau_5 = 3.25$, and $\tau_1 = 3.62, \tau_2 = 3.65, \tau_3 = 3.68, \tau_4 = 3.72, \tau_5 = 3.75$, respectively, the system (3) has a periodic solution (see Fig.7 and Fig.8). Then we change the value of A from A = 20 to A = 15 and $A = 25$, respectively, the other parameter values are the same as in Fig.7, we see that the positive equilibrium points are changed to $(S^*, I_1^*, I_2^*, R^*, L^*)^T = (6.3458, 3.8982, 6.7092, 7.2274, 13.9426)^T$, and $(S^*, I_1^*, I_2^*, R^*, L^*)^T = (5.8598, 7.5418, 12.8452, 9.5545, 21.5346)^T$, respectively, the oscillatory behavior is still kept as in Fig.7 (see Fig.9 - Fig.12).

CONCLUSION

This paper discusses the oscillatory behavior of the solutions for a malware mutation model with time delays. Based on the method of mathematical analysis, we provided two sufficient conditions to guarantee the oscillation of the solutions. Computer simulations are provided to indicate the effectiveness of the criteria. Based on our simulation, the construction of the solutions of the system (3) is very complex. The value of A will affect the equilibrium point and oscillatory frequency.

Competing Interests

The author has declared that no competing interests exist.

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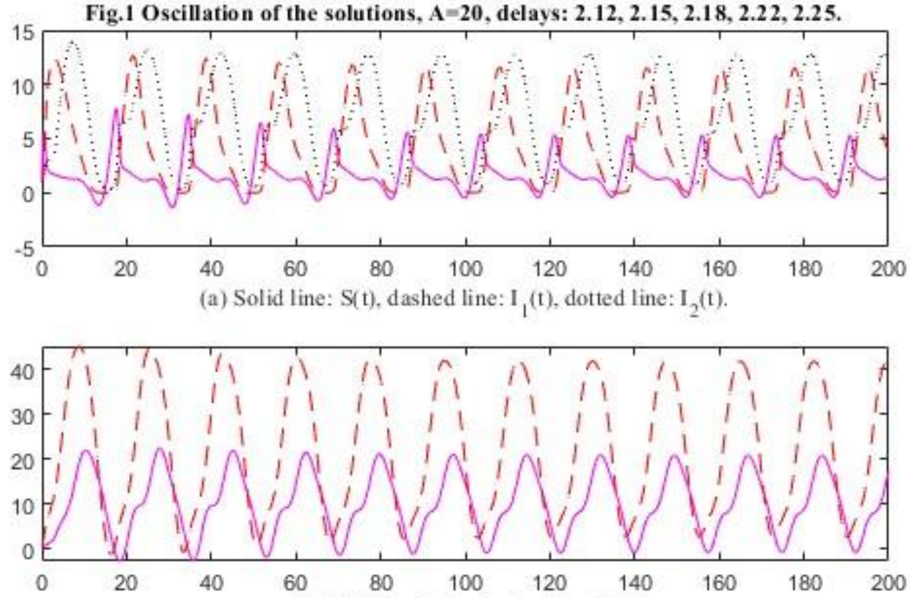
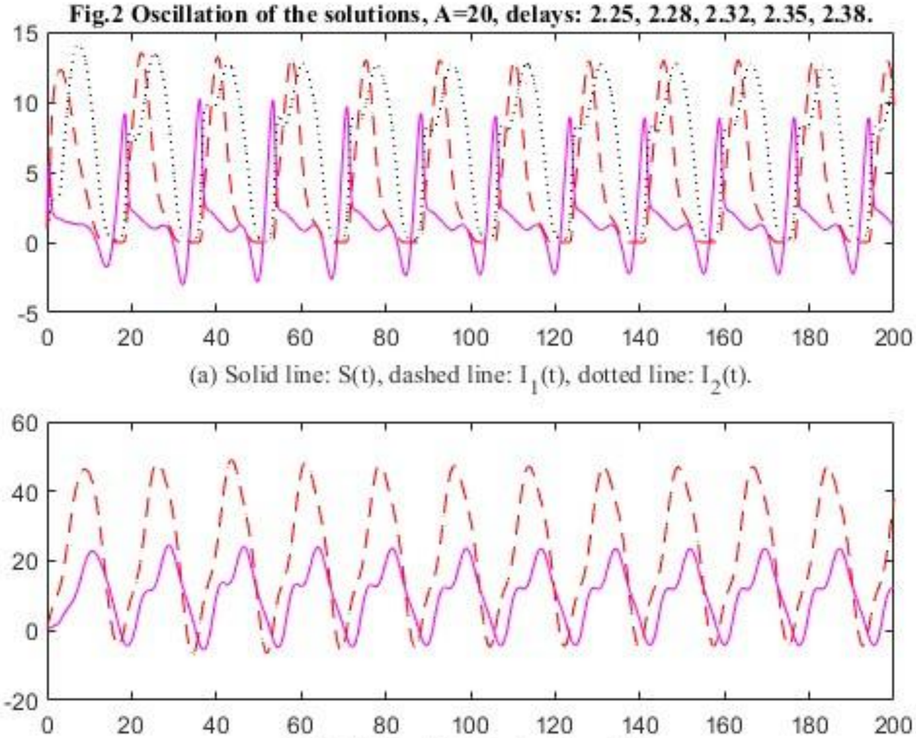
Fig.1 Oscillation of the solutions, A=20, delays: 2.12, 2.15, 2.18, 2.22, 2.25.



(a) Solid line: S(t), dashed line: $I_1(t)$, dotted line: $I_2(t)$.

(b) Solid line: R(t), dashed line: L(t).

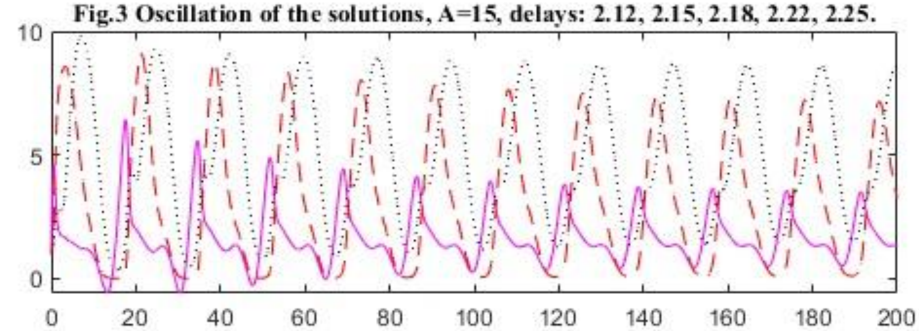
Fig.2 Oscillation of the solutions, A=20, delays: 2.25, 2.28, 2.32, 2.35, 2.38.



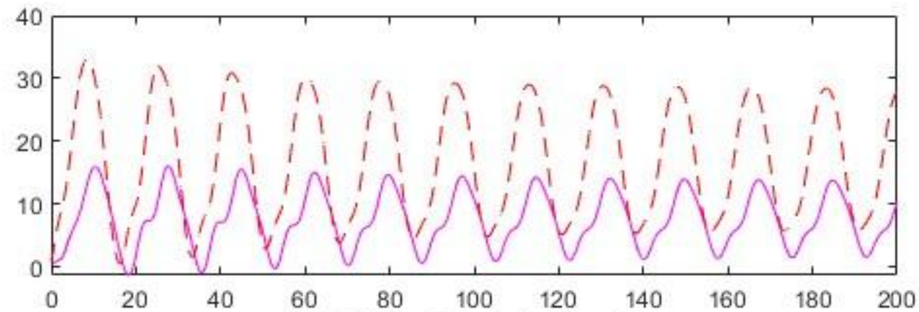
(a) Solid line: S(t), dashed line: $I_1(t)$, dotted line: $I_2(t)$.

(b) Solid line: R(t), dashed line: L(t).

Fig.3 Oscillation of the solutions, A=15, delays: 2.12, 2.15, 2.18, 2.22, 2.25.

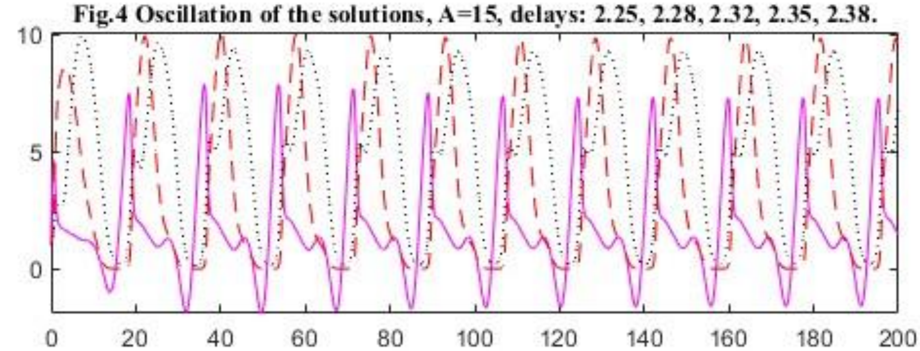


(a) Solid line: S(t), dashed line: $I_1(t)$, dotted line: $I_2(t)$.

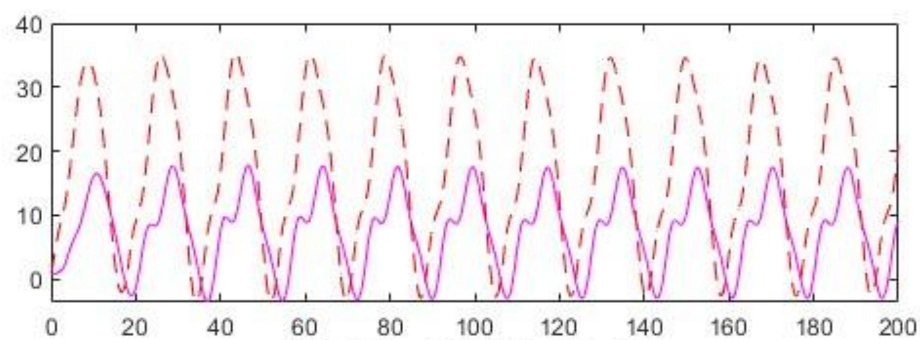


(b) Solid line: R(t), dashed line: L(t).

Fig.4 Oscillation of the solutions, A=15, delays: 2.25, 2.28, 2.32, 2.35, 2.38.

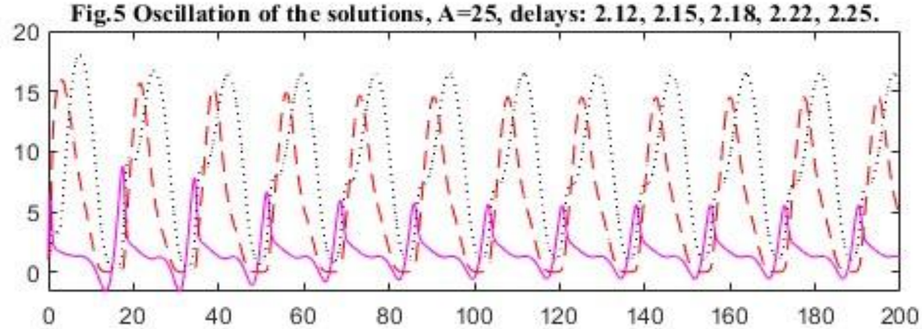


(a) Solid line: S(t), dashed line: $I_1(t)$, dotted line: $I_2(t)$.

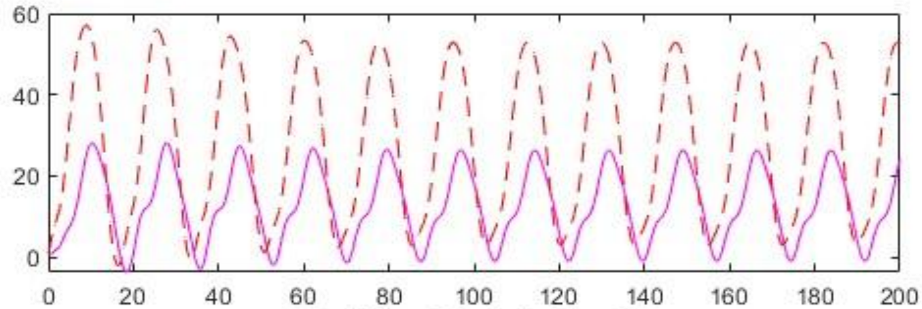


(b) Solid line:RS(t), dashed line: L(t).

Fig.5 Oscillation of the solutions, A=25, delays: 2.12, 2.15, 2.18, 2.22, 2.25.

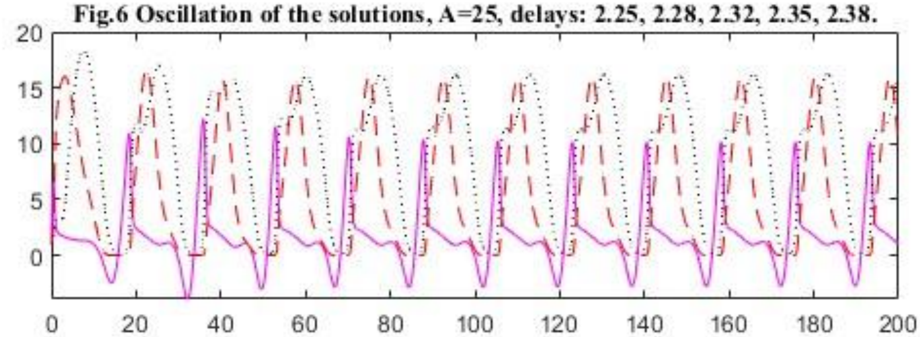


(a) Solid line: S(t), dashed line: $I_1(t)$, dotted line: $I_2(t)$.

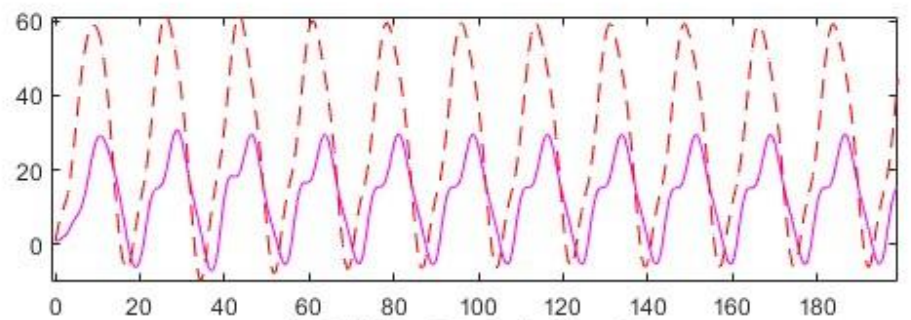


(a) Solid line: R(t), dashed line: L(t).

Fig.6 Oscillation of the solutions, A=25, delays: 2.25, 2.28, 2.32, 2.35, 2.38.

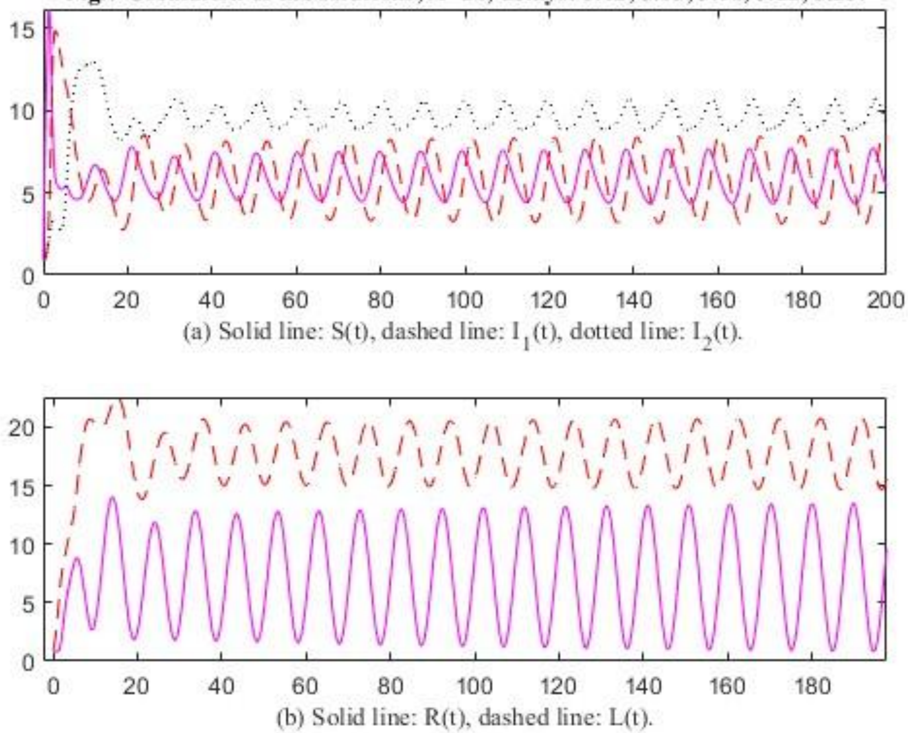


(a) Solid line: S(t), dashed line: $I_1(t)$, dotted line: $I_2(t)$.



(b) Solid line: R(t), dashed line: L(t).

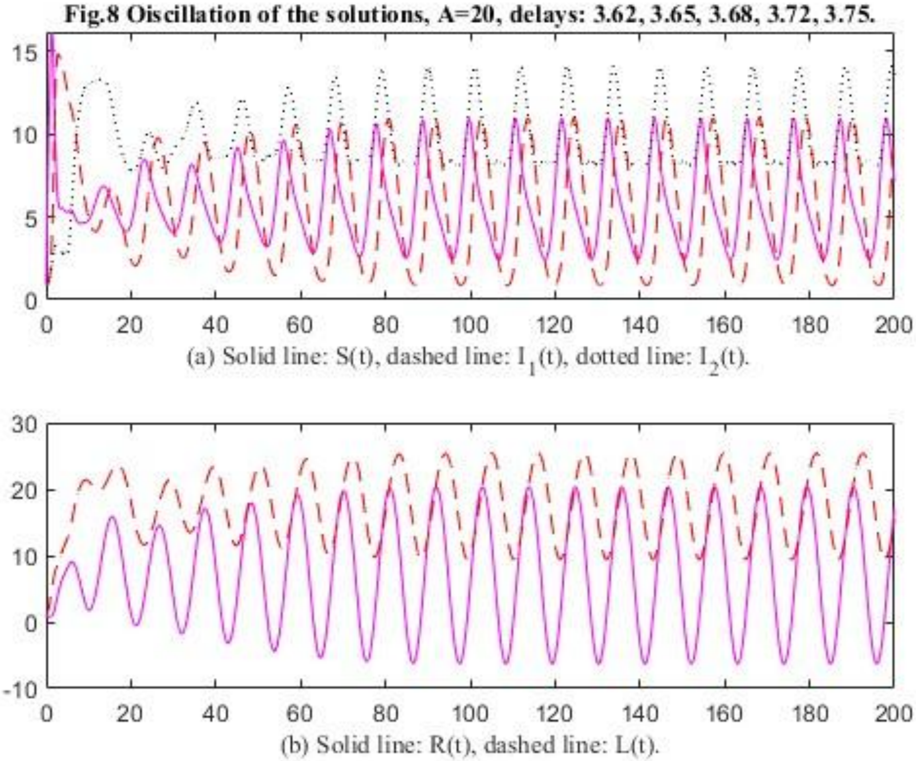
Fig.7 Oscillation of the solutions, A=20, delays: 3.12, 3.15, 3.18, 3.22, 3.25.



(a) Solid line: S(t), dashed line: $I_1(t)$, dotted line: $I_2(t)$.

(b) Solid line: R(t), dashed line: L(t).

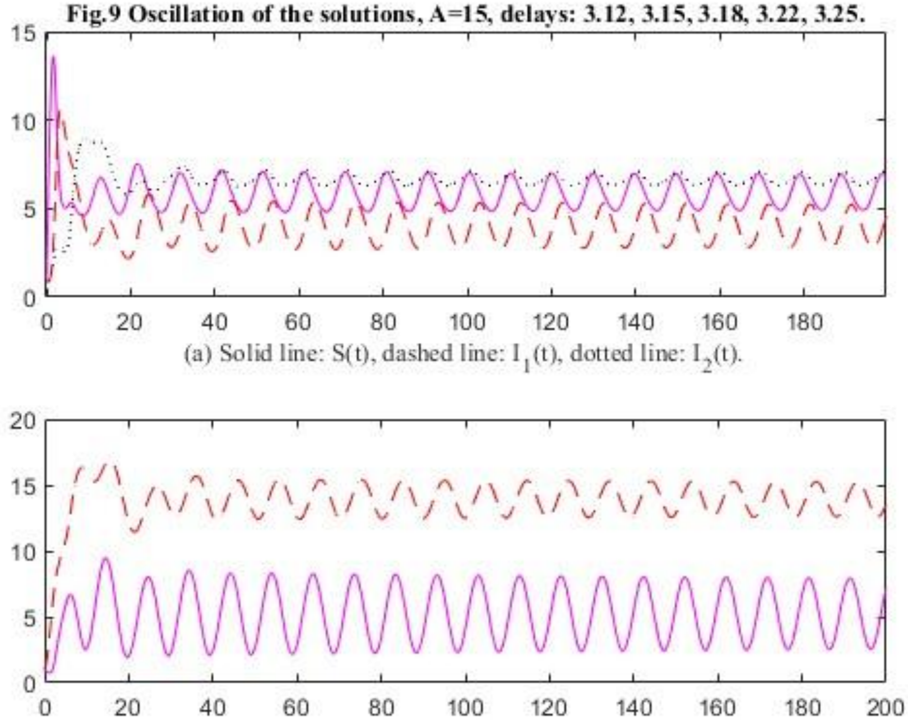
Fig.8 Oiscillation of the solutions, A=20, delays: 3.62, 3.65, 3.68, 3.72, 3.75.



(a) Solid line: S(t), dashed line: $I_1(t)$, dotted line: $I_2(t)$.

(b) Solid line: R(t), dashed line: L(t).

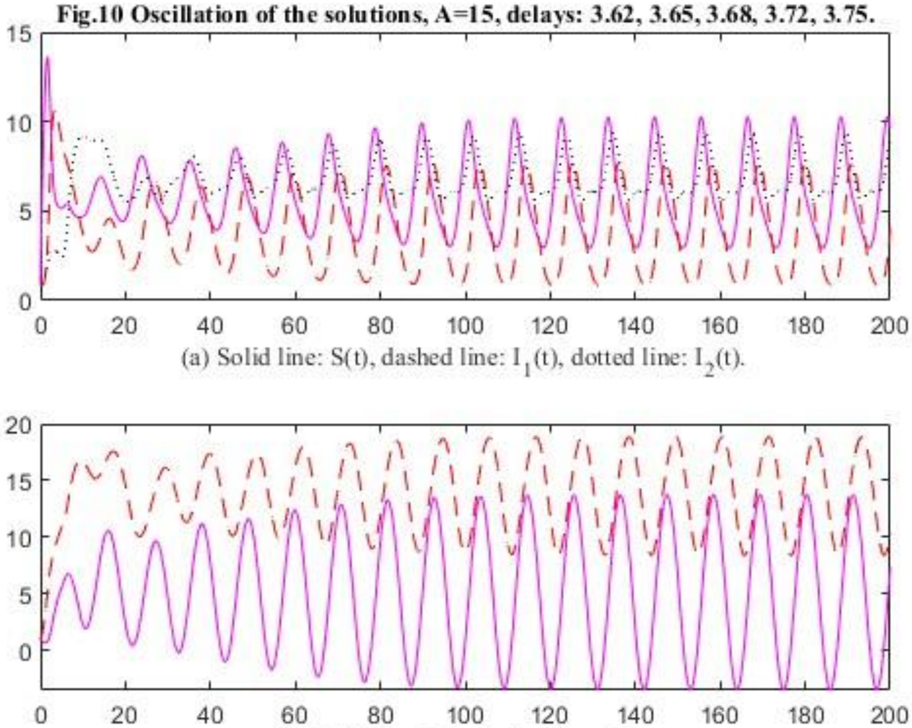
Fig.9 Oscillation of the solutions, A=15, delays: 3.12, 3.15, 3.18, 3.22, 3.25.



(a) Solid line: $S(t)$, dashed line: $I_1(t)$, dotted line: $I_2(t)$.

(b) Solid line: $R(t)$, dashed line: $L(t)$.

Fig.10 Oscillation of the solutions, A=15, delays: 3.62, 3.65, 3.68, 3.72, 3.75.



(a) Solid line: $S(t)$, dashed line: $I_1(t)$, dotted line: $I_2(t)$.

(b) Solid line: $R(t)$, dashed line: $L(t)$.

Fig.11 Oscillation of the solutions, A=25, delays: 3.12, 3.15, 3.18, 3.22, 3.25.

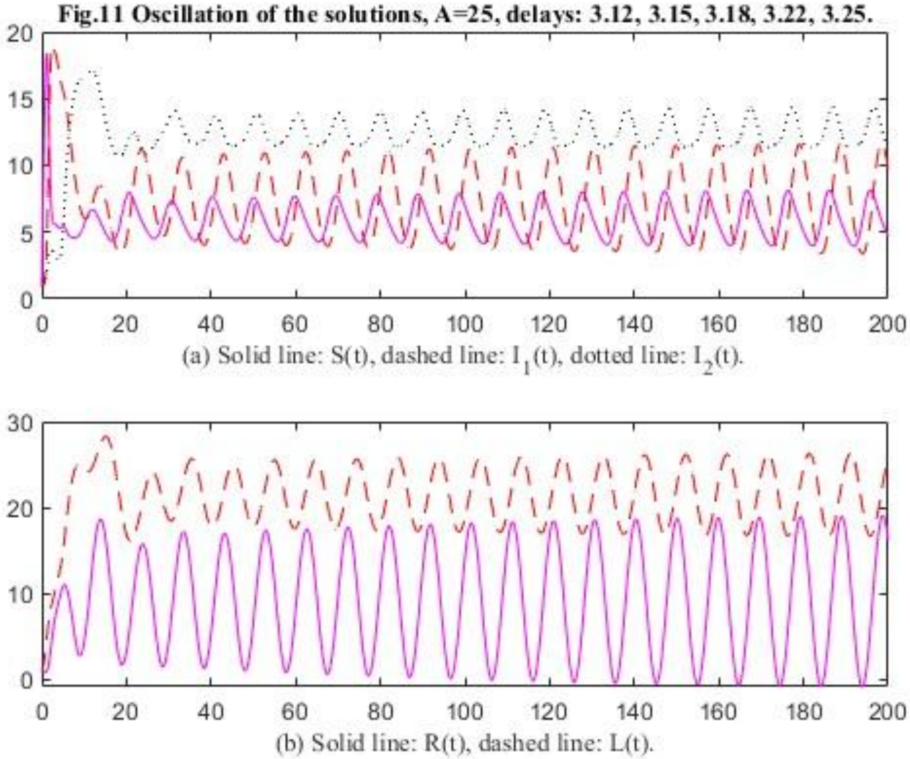


Fig.12 Oscillation of the solutions, A=25, delays: 3.62, 3.65, 3.68, 3.72, 3.75.