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A Coordinate Transformation Method based on the Random Variable for Showing the Optical Properties

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ABSTRACT

A coordinate transformation method of square projection circular with random various has been proposed for the optical properties of the cloaking material. The square produced randomly is projected to the circle and all the points in the perfectly matched layer are transformed. The perfectly matched layer is between the produced square and the border of the square. The first kind transformation has three steps. The first step is that the square grids are produced. The second step is the lattice grids are transformed. The third step is that the lattice grids and those transformed lattice grids are shown. The second kind transformation has four steps. The first step is that the ray came through the center point is assumed. The second step is that the 10 points each ray in the perfectly matched layer are produced randomly. The third step is the 10 points each ray are transformed. The fourth step is that the 10 points each ray and those transformed points are shown. The results show that the transformed results are nothing to do with the position of the randomly produced square. The transformation method is suit for the research of the optical properties in cloaking material.

***Keywords*:** Cloaking material; Coordinate transformation; Projection

1 Introduction

In the past few decades, the metal sandwich were designed with nano-scale to change the distribution of the conductivity and the magnetic permeability. These efforts promote the development of the transformation optics. In particular, Pendry et al[1] and Leonhardt[2] put forward the cloaking material independently since 2006 in Science. The research of the cloaking material has become the hot topic[3-5].

The electromagnetic metamaterial [4] which has cloaking region with micron scale and a high degree of symmetry, such as spherical symmetry, cylindrical symmetry or cubic symmetry. The coordinate transformation [6-7] is important for searching the cloaking regions of the electromagnetic metamaterial. Based on the coordinate transformation invariance for Maxwell's equations the perfectly matched layer [8-9] could be transformed.

2 Coordinate transformation

The model of the researched problem is shown in figure 1[10]. The centers of the square which the side length is 2 times of a and of the circle whose radius is b are superposition.

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The square which the side length is 2 times of *c* is selected randomly and the number of *c* is between *b* and *a*. The center point O of the square which the side length is 2 times of *c* is superposition with the centers of the square which the side length is 2 times of *a* and of the circle which the radius is *b*. The shadow region in figure 1 is between the squares of the side length is 2 times of *c* and of the side length is 2 times of *a*.

In figure 1 the ray is through the center point *O*. On the ray the point *A* is in the square which side length is 2 times of *a*, the point *C* is in the square which side length is 2 times of *c*, the point *B* is in the circle which radius is b, and the point *P* is randomly in the shadow region. The coordinates of *A*, *B*, *C*, *P* are as *A*(*a*, *ay*), *B*(*b*, *by*), *C*(*c*, *cy*), and *P*(*x*,*y*). The transformation is that the point *C*(*c*, *cy*) is transformed to point *B*(*b*, *by*) and the point *P*(*x*,*y*) is transformed to $P'(x', y')$.

The transformation matrix is as equation (1).

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \frac{a-bx}{a-c} & 0 \\ 0 & \frac{ay-by}{ay-cy} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} bx - c\frac{a-bx}{a-c} \\ by - cy\frac{ay-by}{ay-cy} \end{pmatrix} \tag{1}$$

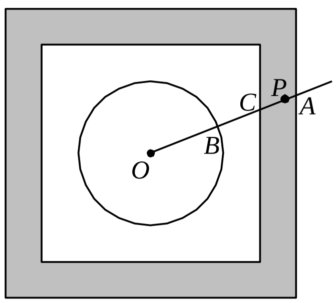


Figure 1 The model of the coordinate transformation

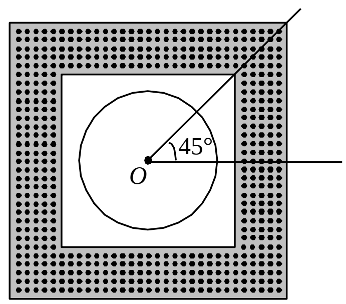


Figure 2 The square grid points

2.1 The transform of the square grid points

In figure 1 the position of the square whose side length is 2 times of c is random and the shadow region is random too. In figure 2 the length of the grid is (*a*-*c*)/6. The larger of the c is, the more the square grid points are. In the shadow region with 45 degrees circular angle the number of the grid point is n1, n2, n3, n4, n5 from left to right. The values of n1, n2, n3, n4, n5 are $\mathrm{round}\left[6 * \frac{\mathrm{a}}{\mathrm{a-c}}\right] - 5$, $\mathrm{round}\left[6 * \frac{\mathrm{a}}{\mathrm{a-c}}\right] - 4$, $\mathrm{round}\left[6 * \frac{\mathrm{a}}{\mathrm{a-c}}\right] - 3$, $\mathrm{round}\left[6 * \frac{\mathrm{a}}{\mathrm{a-c}}\right] - 2$, $\mathrm{round}\left[6 * \frac{\mathrm{a}}{\mathrm{a-c}}\right] - 1$ respectively. The flow diagram of the coordinate transformation is shown in figure 3. The coordinate transformation whose circular angle cover 0 to 360 degrees is by the rotation and the inversion based on the transformation results which is cover 0 to 45 degrees of the circular angle .

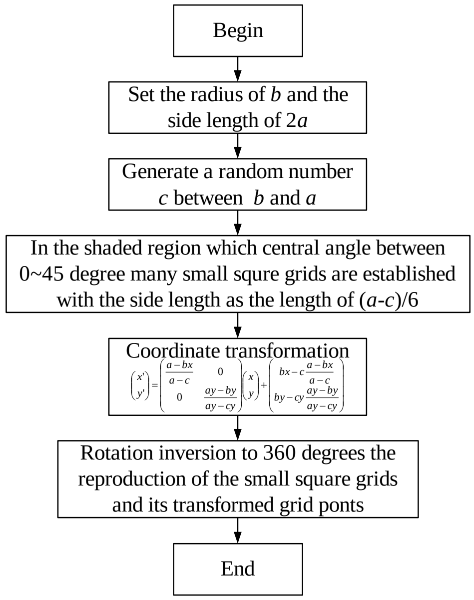
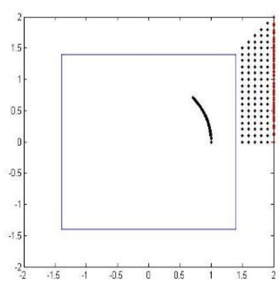
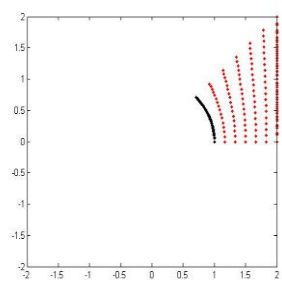


Figure 3 The flow diagram of the coordinate transformation of the square grid points

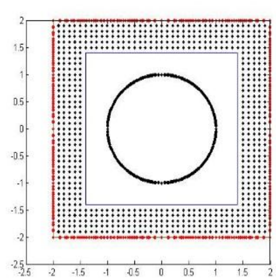
The programming is assumed *a*=2, *b*=1, and *c* is between 1 and 2. The figures of the square grid points and its transformed points are shown in figure 4. The figure 4(a) is the square grid points in 45 degrees for *c*=1.25. The figure 4(b) is the transformed square grid points in 45 degrees for *c*=1.25. The figure 4(c) is the square grid points in 360 degrees for *c*=1.25. The figure 4(d) is the transformed square grid points in 360 degrees for *c*=1.25. The figure 4(e) is the square grid points in 360 degrees for *c*=1.7. The figure 4(f) is the transformed square grid points in 360 degrees for *c*=1.7.



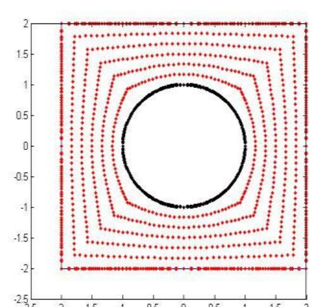
(a) The square grid points in 45 degrees for *c*=1.25



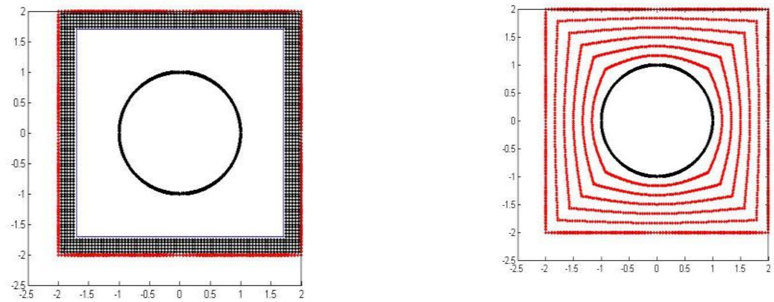
(b) The transformed square grid points in 45 degrees for *c*=1.25



(c) The square grid points in 360 degrees for *c*=1.25



(d) The transformed square grid points in 360 degrees for *c*=1.25



(e) The square grid points in 360 degrees for *c*=1.7 **(f) The transformed square grid points in 360 degrees for *c*=1.7**

Figure 4 The transform of the square grid points

The transformed results shown that the shape made of the transformed points is the same. The difference between the figure 4(d) and the figure 4(f) is the points are different on same shape curve. The number of the points in figure 4(f) is more than that of the points in figure 4(d) because the side length of the grid is small in figure 4(f).

2.2 The transformation of the random points on the ray

In figure 1 the ray is through the center point *O*. The 10 points are chosen randomly on the ray in the shadow region. The transformation is that the point *C* is transformed to point *B*. The point C is in the border of the produced square whose side length is 2 times of *c*. The point *B* is on the circle whose radium is *b*. The points *C* and *B* are on the ray. The point *P* has 10 positions corresponding to 10 random points is transformed to point *P*′. The transform is according to the equation (1) and it can be expressed as the equation (2) too.

$$\overline{OP'} = \frac{\overline{OA}}{\overline{OB}}\overline{CP} \tag{2}$$

The programming is assumed *a*=2, *b*=1, and *c* is between 1 and 2. The flow diagram is shown in figure 5.

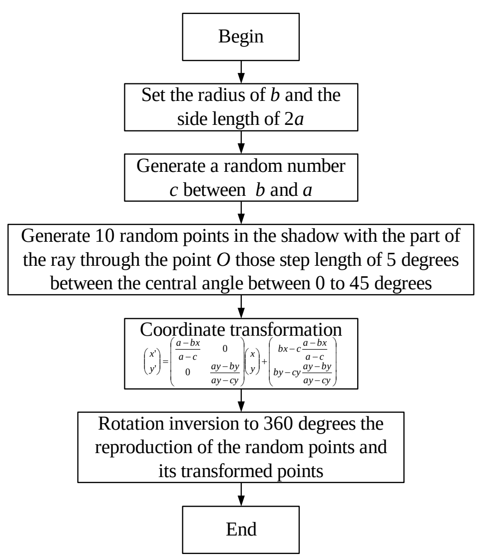
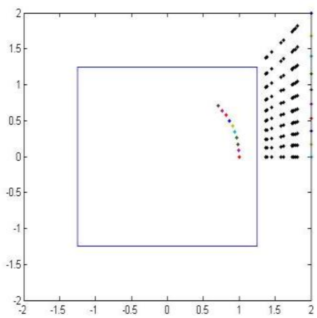
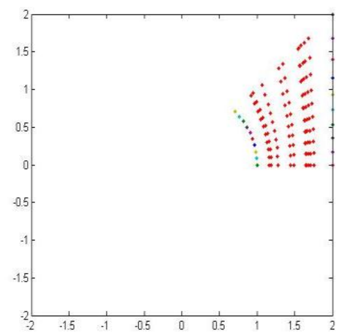


Figure 5 The flow diagram of the coordinate transformation of the random points on a ray

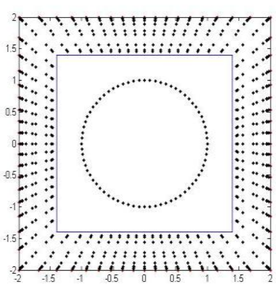
After the square which side length is 2 times of *c* is produced randomly there are four steps in the transform of the random points on a ray. The value of *c* is between *a* and *b*. The first step is to produce the ray which is through the center point *O*. The second step is that the 10 points each ray in the perfectly matched layer are produced randomly. Here the perfectly matched layer is the region between the squares of the side length is 2 times of *c* and of the side length is 2 times of *a*. The third step is the 10 points each ray are transformed according to the equation (1). The fourth step is that the 10 points each ray and those transformed points are shown as figure 6.



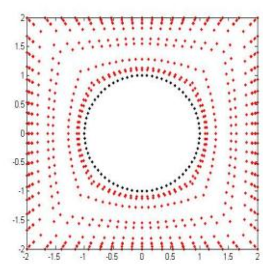
(a) 10 random points each ray in shadow region in 45 degree for c=1.5



(b) 10 transformed points each ray in shadow region in 45 degree for c=1.5



(c) 10 random points each ray in shadow region in 360 degree for c=1.78



(d) 10 transformed points each ray in shadow region in 360 degree for c=1.78

Figure 6 The transform of the random points on the ray

3 Conclusion

The perfectly matched layer is produced after the square whose side length is 2 times of randomly produced *c*. The value of *c* is between *a* and *b*. The outer square which side length is 2 times of *a* and the inner circle whose radium is *b*. The transform is that all the points in the square which side length is 2 times of *c* are transformed to the corresponding points in circle which radium is *b* and the points in the perfectly matched layer are transformed to corresponding points. The results are shown that the transformed square grid points are on the curves of the determined shape and the curves have nothing to do with the position of the square which side length is 2 times of *c* for the square grid points in perfectly matched layer. The random points those in the perfectly matched layer and on the ray which is through the center point *O* are transformed with simple programming. The coordinate transformation method is advantageous to the research of the optical properties in cloaking effect of electromagnetic metamaterial.

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REFERENCES

[1] J. B. Pendry, D. Schurig, D. R. Smith. Controlling electromagnetic fields [J]. Science. 2006, 312:1780-1782

[2] Ulf Leonhardt. Optical conformal mapping and dielectric invisibility devices [J]. Science. 2006, 312: 1777-1780

[3] Adrian Cho. High-tech materials could render objects invisible [J]. Science. 2006, 312: 1120

[4] Raimond Grimberg. Electromagnetic metamaterials [J]. Materials Science and Engineering B. 2013, 178: 1285-1295

[5] Jaeyub Hyun, Junghwan Kook, Semyung Wang. Efficient and stable model reduction scheme for the numerical simulation of broadband axoustic metamaterials [J]. Computers and Mathematics with Application. 2015, 69: 876-892

[6] D. Schurig, J. J. Mock, B. J. Justice, et al. Metamaterial electromagnetic cloak at microwave frequencies [J]. Science. 2006, 314: 977-980

[7] Ozlem Ozgun, Mustafa Kuzuoglu. Monte Carlo simulations of Helmholtz scattering from randomly positioned array of scatters by utilizing coordinate transformations in finite element method [J]. Wave Motion.2015, 56: 165-182

[8] Archana Rajput, Kumar vaibhav Srivastava. Dispersion study on scattering cross section of metamaterial cloak due to various cloaking parameters [J]. Optik. 2015, 126: 2362-2367

[9] ZiyuWang, WeiWei, NiHu, et al. Manipulation of elastic waves by zero index metamaterials [J]. Journal of Application Physics. 2014, 116; 204501,1-5

[10] Ozlem Ozgun, Mustafa Kuzuoglu. Electromagnetic metamorphosis: reshaping scatters via conformal anisotropic metamaterial coatings [J]. Microwave and Optical Technology Letters. 2007, 49: 2386-2392