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Research on Linear Fractional Town Traffic Flow Model Tactic

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ABSTRACT

Traffic flow is a worldwide problem. It has many influencing factors and it is the complex system. Fractional calculus is a powerful tool for dealing with complex systems. Fractional calculus is a direct way of extending traditional integer order calculus, which allows the order to be a fraction. Fractional order model achieves better results than the integer order model. A linear fractional order model based on Grunwald–Letnikov's definition for traffic flow is proposed in this paper. City road traffic flow system is composed of a large number of complex dynamic behaviors of traffic participant. It is a highly nonlinear and non-stationary complex system. Firstly, fractional order calculus is introduced. Then the linear fractional order traffic flow model is proposed based on fractional calculus. The fractional order parameters can be determined by a large number of data and mathematical statistics method. The proposed model was simulated and applied to actual Linghai town road traffic flow. The practicability and effectiveness of the method have been validated.

Keywords: Fractional Order Calculus; Grunwald–Letnikov's Definition; Linear; Traffic Flow; Model.

1 Introduction

Traffic flow theory is a scientific which uses physics and mathematical tools to describe the traffic characteristics. The earliest theory was formed in the 1930s. The probabilistic methods were adopted in the early theory of traffic flow basically. Now researchers are using a variety of methods to study the traffic flow. Reference [1] proposed a new stochastic model of traffic flow with state dependent headways. And the proposed model is consistent with the CTM in the mean dynamic sense. The flow of traffic was analyzed under the operation of variable speed limits in the reference [2]. A new traffic flow model called shockwave profile model was developed in reference [3]. The model is suitable for modeling traffic flow on congested arterials and is empirically validated using field data. The influence of vehicle-to-vehicle communication on traffic flow has been studied in reference [4]. A peak hour traffic scenario with open boundaries and an on-ramp was examined in that paper. Traffic flow of mixed modal share generates dissimilar road density with time in reference [5]. Scale of pollutant dispersion depends upon different traffic flow conditions and spatial distribution of concentrations changes with the traffic flow pattern. Optimal traffic flow management was achieved by delay control in reference [6]. Parallel computing platform accelerates the computation for traffic flow management. And decomposition methods are promising for a real-time traffic flow management platform. The slow-to-start effect in two-dimensional Biham–Middleton–Levine traffic flow model

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with traffic light periods was studied in reference [7]. And they have explained this via the evolution process from a designed regular initial configuration.

At present, non-equilibrium traffic flow is managed by high order continuous model. But some relevant parameters are difficult to accurately identify, especially the function of specific traffic behaviours and mechanism. These have a direct impact on the reliability of simulation results. Therefore, the establishment of a suitable traffic flow model has important theoretical significance and practical value. We should explore the traffic laws and establish a set of strict theoretical basis taking real-time, accuracy, reliability and mixed traffic flow into account. There is a wealth of nonlinear dynamic characteristics in the traffic flow system. And it is a highly nonlinear complex system involved human intervention. The natural world is essentially a fractional. Fractional model is suitable for processing with the human factors and fractional characteristics of complex systems. Fractional character is also suitable for dealing with nonlinear fluid mechanics characteristics.

It is well known that fractional order systems itself is an infinite dimensional filter due to the fractional order in the differentiator or integrator while the integer-order systems are with limited memory (finite dimensional). There has been a surge of interest in the possible engineering application of fractional order differentiation. Examples may be found in [8] and [9]. The significance of fractional order theory is that it is a generalization of classical integral order theory, which could lead to more adequate modeling and more robust control performance.

Fractional order systems could model various real materials more adequately than integer order ones and thus provide an excellent modeling tool in describing many actual dynamical processes[10]. Fractional model provided the scientific basis for prevention and treatment of satellite monitoring absorption rate [11]. The nematode movement can be simulated through fractional model [12]. It may be used for building "love" models using fractional-order system [13]. Reference [14] modeled iron meteorites crystallization by fractional theory. And there are some people pay close attention to unemployment rates by means of fractional calculus [15].

Linear fractional traffic flow model is proposed in this paper. Model parameters can be obtained by the corresponding actual data. The artificial and natural factors for the road traffic flow can be effectively dealt with fractional model. And it can improve the rationality and accuracy. Fractional traffic flow model can describe traffic flow more objective and accurate. Fractional traffic flow control strategy can be put forward on the basis of fractional traffic flow forecast.

The remaining part of this paper is organized as follows. In Section II, fractional order calculus is introduced. In Section III, the linear fractional model method is presented for traffic flow. In Section IV, simulation examples are presented to verify the feasibility. Finally, conclusions are drawn in the paper.

2 Fractional Order Calculus

Although the fractional order calculus is a 300-years old topic, the theory of fractional order derivative was developed mainly in the 19th century. Reference [16] provided a good source of references on fractional order calculus. Fractional order calculus is a generalization of integration and differentiation to a fractional, or non-integer order fundamental operator ${}_aD_t^{\alpha}$, where a and t are the lower/upper bounds of integration and α the order of the operation.

$$ {}_aD_t^\alpha = \begin{cases} \dfrac{d^\alpha}{dt^\alpha} & R(\alpha) > 0 \\ 1 & R(\alpha) = 0 \\ \int_a^t (d\tau)^{(-\alpha)} & R(\alpha) < 0 \end{cases} \tag{1} $$

which $R(\alpha)$ is the real part of α. Moreover, the fractional order can be a complex number as discussed in [17]. In this paper, we focus on the case where the fractional order is a real number.

Due to its importance in applications, the Grunwald–Letnikov's definition of the fractional order calculus is considered in this paper based on the generalization of the backward difference.

Definition 1: Introducing the positive integer *m*, $[m]$ is the integer part of *m*. Then this definition has the form

$$ \begin{aligned} {}_aD_t^\alpha f(t) &= \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{[(t-\alpha)/h]} (-1)^j \binom{\alpha}{j} f(t-jh) \\ &= \lim_{h\to 0} \frac{1}{\Gamma(\alpha)h^\alpha} \sum_{j=0}^{[(t-\alpha)/h]} \frac{\Gamma(\alpha+j)}{\Gamma(j+1)} f(t-jh) \end{aligned} \tag{2} $$

where

$$ \binom{\alpha}{j} = \frac{\alpha(\alpha-1)(\alpha-2)\cdots(\alpha-j+1)}{j!} = \frac{\alpha!}{j!(\alpha-j)!} \tag{3} $$

Firstly, the n-order derivative of continuous function $f(t)$ is

$$ f^{(n)}(t) = \lim_{h\to 0} h^{-n} \sum_{j=0}^{[(t-\alpha)/h]} (-1)^j \binom{n}{j} f(t-jh) \tag{4} $$

where

$$ \binom{n}{j} = \frac{n(n-1)(n-2)\cdots(n-j+1)}{j!} = \frac{n!}{j!(n-j)!} \tag{5} $$

When n is negative integer, there is

$$ \binom{-n}{j} = (-1)^n \binom{n}{j} \tag{6} $$

When n is any real number, there is the Grunwald–Letnikov's definition.

Compared with conventional calculus, fractional order calculus has the following qualities:

- If $f(t)$ is an analytic function of the variable t, the derivative $D_t^\alpha f(t)$ is an analytic function of t and α.
- The operation D_t^α has the same result with the usual derivative when $\alpha = n$ and $n \in Z^+$.
- The operation D_t^α has the same result with the usual n-fold integral when $\alpha = n$ and $n \in Z^-$.
- The operator of order $\alpha = 0$ is the identity operator.

- The operator should be linear:

$$D_t^{\alpha}[af(t)+bg(t)] = aD_t^{\alpha} f(t) + bD_t^{\alpha} g(t) \quad (7)$$

- For the fractional-order integrals of arbitrary order, it holds the additive law of exponents (semi-group property):

$$D_t^{\alpha}[D_t^{\beta} f(t)] = D_t^{\alpha+\beta} f(t) \quad (8)$$

- It can be seen that the fractional-order integral can be expressed as a causal convolution of the form：

$${}_aI_t^{\alpha} f(t) = \Phi_{\alpha}(t) * f(t), \qquad \alpha \in R^{+} \quad (9)$$

where $\Phi_{\alpha}(t) = \dfrac{t_{+}^{\alpha-1}}{\Gamma(\alpha)}$, $\alpha \in R^{+}$.

As in the case for conventional linear systems, for linear fractional-order systems, linear fractional-order differential equations are the fundamental governing equations. The linear fractional-order differential equation is defined as

$$\begin{aligned} & a_n D_t^{\beta_n} y(t) + a_{n-1} D_t^{\beta_{n-1}} y(t) + \cdots + a_1 D_t^{\beta_1} y(t) + a_0 D_t^{\beta_0} y(t) \\ & = b_m D_t^{\alpha_m} x(t) + b_{m-1} D_t^{\alpha_{m-1}} x(t) + \cdots + b_1 D_t^{\alpha_1} x(t) + b_0 D_t^{\alpha_0} x(t) \end{aligned} \quad (10)$$

Denoting the left hand side of the equation by

$$u(t) = b_m D_t^{\alpha_m} x(t) + b_{m-1} D_t^{\alpha_{m-1}} x(t) + \cdots + b_0 D_t^{\alpha_0} x(t) \quad (11)$$

The original fractional-order differential equation can be rewritten in the form

$$a_n D_t^{\beta_n} y(t) + a_{n-1} D_t^{\beta_{n-1}} y(t) + \cdots + a_0 D_t^{\beta_0} y(t) = u(t) \quad (12)$$

Substituting Grunwald–Letnikov's definition into the above equation, one may find that

$$\frac{\alpha_0}{h^{\beta_0}} \sum_{j=0}^{[(t-a)/h]} \omega_j^{(\beta_i)} y_{t-jh} + \cdots + \frac{\alpha_m}{h^{\beta_n}} \sum_{j=0}^{[(t-a)/h]} \omega_j^{(\beta_n)} y_{t-jh} = u(t) \quad (13)$$

where the binomial coefficients $\omega_j^{(\beta_i)}$ can still be evaluated recursively with

$$\omega_0^{(\beta_i)} = 1, \quad \omega_j^{(\beta_i)} = \left(1 - \frac{\beta_i + 1}{j}\right) \omega_{j-1}^{(\beta_i)}, \qquad j = 1, 2, \cdots \quad (14)$$

By slight rearrangement of the terms, the closed-form solution of the fractional order differential equation can be obtained as

$$y_t = \frac{1}{\sum_{i=0}^{n} \frac{\alpha_i}{h^{\beta_i}}} \left[u_t - \sum_{i=0}^{n} \frac{\alpha_i}{h^{\beta_i}} \sum_{j=1}^{[(t-a)/h]} \omega_j^{(\beta_i)} y_{t-jh} \right] \quad (15)$$

The linear fractional traffic flow model is considered in this paper. Model parameters can be obtained by the corresponding actual data. It aims to describe the traffic flow of the actual situation more accurately, so that fractional control systems can be used on traffic flow in the future. It can reduce the complexity while improving the scientific validity of the system model based on the linear fractional order calculus.

3 Linear Fractional Model

Accurate mathematical and computer simulation model is a very effective and vital tool for supporting a new road and transport infrastructure planning. The appropriate traffic flow model can be used to evaluate design alternatives, which can greatly reduce the costs. Reference [18] introduced a new hybrid approach which combines the complementary features and capabilities of both continuum mathematical models e.g. and knowledge-based models e.g. in order to describe effectively traffic flow in road networks. Convective instability is relevant for describing congested traffic flow in reference [19]. Only two out of five stability classes agree with observations. And synchronized traffic and jams can be described by “two-phase” models. A short-term traffic forecasting model which combines the support vector regression model with continuous ant colony optimization algorithms (SVRCACO) was studied in reference [20]. And the SVRCACO model is a promising alternative for forecasting traffic flow. A new lattice model of traffic flow with the consideration of the traffic interruption probability has been presented in reference [21]. The traffic interruption probability on the stability of traffic flow was explored. Quantitative criteria for validating models with respect to traffic instabilities were applied to a database of hundreds of congestions in reference [22]. The railway traffic flow was simulated when a train suddenly stops with malfunction in reference [23]. Two ways are proposed to alleviate the harmful influence caused by train failure. Increase of train operation interval can decrease trains’ delay. And decrease of maximum speed can reduce the go-and-stop wave. Mainstream traffic flow control (MTFC) is proposed as a novel and efficient motorway traffic management tool, and its possible implementation and principal impact on traffic flow efficiency was analyzed [24]. Variable speed limit was considered as one (out of several possible) way(s) for MTFC realization, either as a stand-alone measure or in combination with ramp metering. A novel non-linear modeling formalism was suggested for freeway traffic flow in reference [25]. A second-order macroscopic model has been transformed into a Linear Parameter Varying (LPV) model. A new driver’s forecast lattice model of traffic flow has been presented in reference [26]. The analytical and numerical results show that the driver’s forecast effect can improve the stability of traffic flow. A linear fractional model is introduced in this paper. Traffic flow of a location on urban highway is represented by the fractional model.

In this system the input is hours and the output is traffic flow. The order of fractional has the important meaning. They show man-made and natural factors. Combined other correlation coefficients fractional model is shown as the following:

$$\mathrm{a}_1 D^{\alpha_1} y(t) + \mathrm{a}_2 D^{\alpha_2} y(t) + \mathrm{a}_3 D^{\alpha_3} y(t) + \mathrm{a}_4 D^{\alpha_4} y(t) = bx(t) \tag{16}$$

where $x(t)$ expresses the hours, and $y(t)$ is the traffic. Fractional coefficient α_1 shows emergency situations. In general, there are many emergency situations. The value indicates the emergency of possibility. The larger the value of fractional coefficient shows more likely emergencies. And it is converse if the value is smaller. This fractional coefficient is decided by location and position. Fractional coefficient α_2 shows driver's state of mind. In Linghai traffic condition, psychological state of driver is good. So the value of this coefficient is relatively large. Fractional coefficient α_3 shows holidays. A position vehicle flow is related to data collection time. Whether it is holiday? Holidays are approaching? Fractional coefficient α_4 shows rain and snow status. When weather is bad, traffic will be reduced, which is well-recognized fact. Common coefficients are determined by some vehicle factors. Coefficient a_1 shows overtaking and changing lanes phenomenon. This coefficient represents the possibility of overtaking and diversion. The greater value shows that this possibility is greater. Coefficient a_2 expresses the interaction of individual vehicles. Coefficient a_3 expresses randomness of determination. Coefficient a_4 expresses vehicle inertia. Coefficient b expresses speed and distance between vehicles. The MATLAB function is written to implement the step response of the system.

4 Simulation Examples

Here Linghai town road traffic flow should be considered in this paper. This place actual traffic data has been gotten. Parameters of the above model can be determined by the data through other methods. So the linear fractional traffic flow mode is established, which can provide the basis for traffic management.

The corresponding data can be obtained by the perception of space equipment. The data should be dealt with through the quadratic approximation, the weighted minimum and fractional approximation methods. And then corresponding coefficients can be received. This fractional coefficient α_1 is decided by location and position. There are a lot of unexpected events at this location. From the point of statistical data, this is moderate attention orientation. So there is $\alpha_1 = 3.1$. In Linghai traffic condition, the psychological state of driver is not good. So the value of this coefficient is relatively small. It is $\alpha_2 = 0.9$. A position vehicle flow is related whether it is holiday. There are more festivals in September. In addition to the weekend, there is Mid-Autumn Festival, Teacher's Day and so on. So there is $\alpha_3 = 2.8$. The fractional coefficient α_4 shows Weather conditions. In general the weather is better in September. The inclement weather is less. So it is $\alpha_4 = 4.5$. Common coefficients show vehicle factors. The location is in the main ring road. It is not close to the export or import. In general, there is not much overtaking and lane changing. The value is not lager. $a_1 = 8$. The coefficient a_2 represents the interaction between vehicles. The Impact between vehicles is decided by many factors. Based on the existing data and through fractional analysis method, we get $a_2 = 7$. In all cases the determination randomness is quite lager. Which also shows that traffic flow model is not unique. So establishment uniform traffic flow model is not realistic. The actual location is considered in this part. There is the certain basis on the existing data. Here the coefficient is $a_3 = 4$. Coefficient a_4 expresses vehicle inertia. Vehicle inertia is great. So it is $a_4 = 6$. On the urban expressway, the relation of speed and distance between vehicles is very lager. The more value is taken, which shows that the relationship is very close. There is $b = 8$. Based on the above data and related methods, the fractional program evaluation model can be obtained:

$$8D^{3.1}y(t) + 7D^{0.9}y(t) + 4D^{2.8}y(t) + 6D^{4.5}y(t) = 8x(t) \tag{17}$$

For the fractional model, simulation model can be built by using the above algorithm. It is taken as the basis for traffic management that the curve can be gotten from the simulation model. According to the algorithm proposed in this paper, the corresponding traffic flow curve can be obtained with MATLAB from 15:00 to 22:00, as shown in Figure 1.

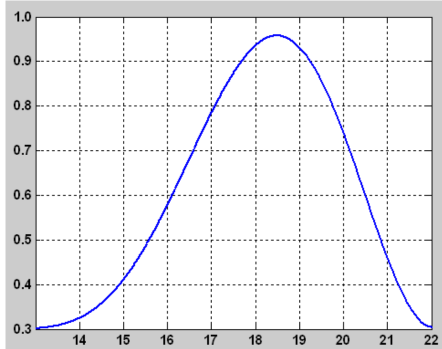

Figure 1. Output traffic flow curve

The result shows that peak traffic flow is gotten when it is six-half at the afternoon. Then the traffic flow falls. The traffic flow tends to the usual level of traffic more than nine-point in the evening. This result is consistent with the actual traffic flow. It is validated that the proposed method is very accurate and effective.

During the holidays, there have some changes in the corresponding data. Then the corresponding coefficient will change. According to the holiday period traffic flow data, the corresponding coefficients can be received. There is $\alpha_1 = 2.5$. The coefficient is $\alpha_2 = 0.7$. Because it is the New Year holiday period, there is $\alpha_3 = 3.2$. The fractional coefficient α_4 shows Weather conditions. Weather is changeable. So it is $\alpha_4 = 3.9$. Common coefficients show vehicle factors. There is $a_1 = 5$. The coefficient has $a_2 = 9$. Here the coefficient is $a_3 = 8$. Coefficient a_4 expresses vehicle inertia. So it is $a_4 = 4$. The relation of speed and distance between vehicles is very lager. The more value is taken, which shows that the relationship is very close. There is $b = 9$. Based on the above data and related methods, the fractional program evaluation model can be obtained:

$$5D^{2.5}y(t) + 9D^{0.7}y(t) + 8D^{3.2}y(t) + 4D^{3.9}y(t) = 9x(t) \tag{18}$$

For the fractional model, simulation model can be built by using the above algorithm. It is taken as the basis for traffic management that the curve can be gotten from the simulation model. According to the algorithm proposed in this paper, the corresponding traffic flow curve can be obtained with MATLAB from 7:00 to 20:00, as shown in Figure 2.

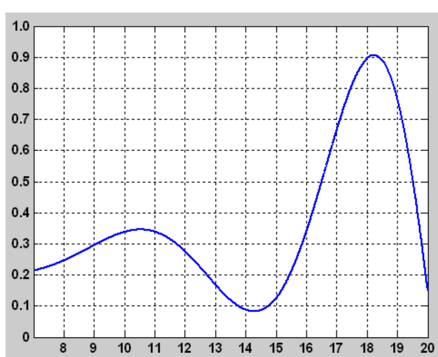

Figure 2. Output traffic flow curve

In the New Year holiday period, there are many friends eating and drinking at noon. So there is more traffic flow. At two and three point in the afternoon, most people are at rest. The peak traffic flow is gotten when it is six-point at afternoon. This result is consistent with the actual traffic flow. It is validated that the proposed method is very accurate and effective.

5 Conclusion

The traffic phenomenon is researched in traffic flow theory. General characteristics of transportation are described by measuring and modeling method. The mathematics and physics theory are applied in the various parameters of traffic flow and their qualitative and quantitative analysis. Traffic flow theory provides a theoretical basis and basic method for traffic engineering application. And the basic law of traffic flow should be interpreted. There are two functions of traffic flow model in traffic engineering application. The first function is to better understand the dynamics of traffic, especially to explain the formation and spread of congestion. Therefore, transportation scholars use identify the possible traffic bottleneck by using the traffic model. The second role is that the traffic flow model can be used as a simulation platform. Through this platform, various control strategies can be developed and assessed in order to improve traffic conditions. Traffic flow is a complex multi-factorial process, and it is not modeled by integer order model accurately. Fractional model is an effective method for analysis and processing of complex system.

Linear fractional modeling method of traffic flow is proposed in this paper. The coefficients of the model are obtained by a large number of related data. The method was applied to actual town transportation. And result indicates that this method is highly efficient for solving real-world problems. The validity of proposed method is validated by actual situation.

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