Ani Minarni

Direct Learning Models

Abden Rahim Aritonang

Mathematics Education Program, Post Graduate Program Universitas Negeri Medan, Indonesia

Waminton Rajagukguk

Mathematics Education Program, Post Graduate Program Universitas Negeri Medan, Indonesia

ABSTRACT

This research aims to analyze differences in the improvement of students' mathematical problem-solving ability (MPSA) through contextual learning model and direct learning model. Type of research is a quasi-experimental study. The research population was all students of upper secondary school (USS) at Sidikalang which consisted of nine classes by taking classes IX-4 and IX-5 as the sample. In this research students is categorized to master MPSA if they do MPSA test, they understand the problem, choosing a problem solving strategy, executing the strategy, and concluding the solution in line with the initial problem. Problems are designed contextually. Based on the results of this study it was found that there were differences in the improvement of MPSA between students taught through contextual learning model and through direct learning model and the students MPSA are categorized good. MPSA N-Gain of the taught through contextual learning models is higher than students taught through direct learning models.

Keywords: Mathematical Problem Solving Ability, Contextual Learning Model, Direct **Learning Model**

BACKGROUND

Mathematics is a universal science that provides the basis for the development of modern technology and provides a very important role in advancing various disciplines and human mindset. In school mathematics, the materials taught are basic sciences that develop rapidly both in content and application. Thus teaching mathematics in schools is a priority in education. Mathematics is also very important in everyday life. Mathematics with its various roles makes it a very important science and one of the roles of mathematics is as a means of thinking to deliver students to understand the mathematical concepts they are learning.

Mathematicians seek and use patterns (Ziegler, Günter, 2011) & Steen (1988) to formulate new conjectures; they resolve the truth or falsity of conjectures by mathematical proof. When mathematical structures are good models of real phenomena, then mathematical reasoning can

Ability of The Students at Upper Secondary School 3 Sidikalang Based on Contextual Learning Model And Direct Learning Models. Advances in Social Sciences Research Journal, 5(9) 284-293. Analysis of Differences in Improvement of Mathematical **Problem Solving Ability of The Students at Upper Secondary** School 3 Sidikalang Based on Contextual Learning Model And

SOCIETY FOR SCIENCE AND EDUCATION

provide insight or predictions about nature. Through the use of abstraction and logic, mathematics developed from counting, calculation, measurement, and the systematic study of the shapes and motions of physical objects. Practical mathematics has been a human activity from as far back as written records exist. The research required to solve mathematical problems can take years or even centuries of sustained inquiry. There are two visions of learning mathematics, namely: 1) directing mathematics learning to understanding concepts that are then needed to solve problems and other sciences, and 2) directing a broader future that is mathematics providing problem solving, and forming human good character, like systematic, critical, objective and open. These ability are needed in facing an ever-changing future.

In Indonesia, it is a common fact that mathematics for children is an unpopular subject, considered a difficult and complicated science. From various fields of study taught at school, mathematics is a field of study that is considered the most avoided by students elementary school until upper secondary school. This has an impact on students' mathematical achievements which are always low, for example in achieving national exams, and in participating in Trend in International Mathematics & Science Studies (TIMSS) (Wilson, 2007), also PISA (2012).

Mathematical problem solving ability (MPSA) becomes one of the abilities that must be developed in mathematics learning. NCTM (2000) states that problem solving is not just a goal of learning mathematics but also a major tool for doing or working in mathematics. Again, the fact that the Indonesian students MPSA was was also low (Wilson, 2007). There are many factors influenced MPSA achievement. One of these factors is learning approach used by the teacher in the classroom. Education experts found that conventional learning does not have enough role in increasing MPSA, thus it is necessary to try constructivism-based learning, such as problem-based learning, realistic mathematics education, inquiry, open-ended, and contextual learning, applied in the classroom. This learning is intended to make students construct their own understanding of the knowledge gained. Especially, contextual learning try to design problem proposed with the integration of context to make the student easier to understand the problem.

THEORETICAL FRAMEWORK

Mathematical problem solving ability of the students could be improved through learning in the classroom using certain learning model. Nihdayati, et.al (2018) used model eliciting activities to improved MPSA. Other learning model such as cooperative learning model has the possibility to enable the students to get such ability. The contextual teaching learning model is influenced by constructivism philosophy which was initiated by Mark Baldwin and later developed by Jean Piaget. The two of them is prominent education experts. The constructivism philosophy flowed from Giambastista's epistemological thinking. The view of constructivism philosophy about the nature of knowledge influences the concept of the nature of the teaching and learning process, that learning is not just memorizing, but constructing knowledge through experience. Knowledge is not the result of giving from other people such as teachers, but constructive results that are carried out by each individual.

The contextual learning model began in the United States when educators rejected dualism about thoughts, brain-motion, physical-psychic, concrete-abstract, theoretical-applicative and the like. Johnson (2007) writes in his book that the dualism is very unproductive, because the true meaning is the whole meaning that cannot be explained by reason of the specialization of the expertise of the book writers or curriculum development.

Contextual has a relationship with the context, atmosphere or real world situation, so that the contextual learning model can be interpreted as a learning that connects the achievement of knowledge through a process that links knowledge with the actual situation or situation as well as experiences that have been previously owned. From this definition it can be stated that the contextual learning model is a learning that is associated with students' knowledge in the real-life situations they already have. Contextual learning model is an approach that emphasizes the process of students full involvement to be able to find material that is learned and relate it to real life situations, it may also connected to the students culture, thus encouraging students to be able to apply it in their lives.

While, direct learning model is a learning model that is Teacher-centre. According to Arends (2008), Direct Learning model is one of the teaching models specifically designed to support student learning processes related to well structured declarative knowledge and procedural knowledge that can be taught with a step-by-step activity pattern. Direct learning can take the form of lectures, demonstrations, training or practice, and group work; used to convey lessons that are transformed directly by the teacher to students; requires careful planning and implementation on the part of the teacher. To be effective, direct learning requires that every detail of skills or content is carefully defined and demonstrated and the training schedule is carefully planned and implemented. So, there is the possibility to improve MPSA of the students at both learning model.

Mathematical Problem Solving Ability (MPSA)

NCTM (2000) details the problem solving abilities that students must develop including: 1) building new mathematical knowledge to solving problems, 2) solving problems that arise in mathematics and other contexts, 3) using and adapting variations of the right strategies to solve problems, 4) monitor and reflect on the process of problem solving.

According to Minarni (2017) problem solving ability is a skill that is very important to be obtained, because from birth, humans face the challenges of problems that force them to get solutions. Problem solving itself, according to Anderson (2001) is the process of applying knowledge in a variety of new and unusual situations. Napitupulu (2016) states that problem solving occupies a central position in mathematics. He continues that if mathematics is seen as a product then the problem solving is in its heart, various concepts, principles and procedures are sought and found with the aim that they can be utilized and lead to problem solving, while, if mathematics is seen as a process, then problem solving is also in the heart. Generally, the appearance of various mathematical objects begins and is triggered by problems that must be resolved or questions that demand answers.

According to Goos et.al. (2000), a person is considered a good problem solver if he is able to demonstrate the ability to solve problems faced by choosing and using various alternative strategies so as to overcome these problems. Some indicators of mathematical problem solving abilities in this study are modified from Polya (1987), that is; 1) Understanding, 2) Submitting strategies, 3) Completion, and 4) Summing up the results/solutions in line with initial problem.

RESEARCH QUESTION

1. Are there differences in the improvement of problem solving abilities between students taught through contextual learning models with students taught through direct learning models?

- 2. What are the category of mathematical problem solving abilities of the students taught through contextual learning models and students taught through direct learning models?
- 3. Does MPSA N-Gain of students' taught through contextual learning models is higher than students taught through direct learning models?

RESEARCH METHOD

This research is a quasi-experimental research which aims to improve students' mathematical problem solving ability (MPSA) through the Contextual Learning model and Direct Learning model. The research design used in this study is displayed in Table 1.

Tuble 1. Research Design									
Learning model	Pretest	Treatment	Posttest						
Contextual	01	X1	02						
Direct Learning	01	X2	02						

Table 1. Research Design

The population of this study was all students of upper secondary school (USS) Sidikalang which consisted of nine classes by taking classes IX-4 and IX-5 to become the sample randomly. The syntax of contextual learning models and direct learning models applied in this study are describe below.

Syntax of contextual learning model

- 1. Demonstrating introductory material.
- 2. Guiding students in finding linkages of subject matter from student learning experiences and with real life.
- 3. Raising questions that are stimulating the critical nature of students about the subject matter.
- 4. Divide students into several groups heterogeneously.
- 5. The teacher guides students in modeling.
- 6. Guiding students to conclude the material that has been learned.
- 7. Conduct actual assessments during learning (activity observation) and at the end of learning (portfolio).

Syntax of direct learning model

- 1. Teachers explain learning objectives, background information on the importance of lessons, prepare students for learning
- 2. Teachers demonstrate skills correctly, or present information step by step.
- 3. Teacher plans and gives initial training guidance.
- 4. Check whether students have successfully performed the task well, give feedback.
- 5. The teacher prepares the opportunity to undertake advanced training, with special attention to the application to more complex situations.

Like problem in student activity sheet (SAS), pretest and posttest are designed based on mathematical problem solving aspect. Aspects of mathematical problem solving required students to show their ability in understanding the problem; one indicator of understanding the problem is the ability to represent problem in other form that make the student can understand better the problem (Minarni, 2017). Other aspects of mathematical problem solving are proposed the strategy to solve the problem, execute the strategy and conclude the solution in line with initial problem. In contextual learning model classroom, math problems are designed based on the context of local culture and the problem arouse in students daily life. Through such designed, hope that the students familiar with the problem situation so they will

understand better. In direct learning class, math problems are designed with no connection with the students daily activity and the students culture. This is the key different treatment between the two classes.

In contextual learning classroom, the teacher guides the students to express much idea to approach the solution of the problem, while in direct learning model, the teacher demonstrate the strategy to solve the problem after explaining the topic of mathematic at every session. The teacher in direct instruction class act as the main source of information, while in contextual learning the source of information could also be searched from the internet.

Because of the existence of pretest as a covariate, data analysis used is Analysis of Covariance (ANACOVA). Some assumption to use ANACOVA such as linearity of the data, variance homogeneity, and the parallelism of regression equation of the data from both classes are done and presented in "data analysis & results" section of this report.

DATA ANALYSIS & RESULT

The purpose of his study was to analyze differences in the increase in MPSA of upper secondary students who taught through contextual learning models and students who taught through direct learning models. The data analyzed is the MPSA test results. The test results provide information about students' abilities before and after learning, both in contextual learning model classroom and in direct learning model classroom. The results of the Normality Test of pretest is presented in Table 2.

Learning model	Kolmogorov-Smirnov			Learning model Kolmogorov-Smirnov Shapiro-Wilk			lk			
	Statistic	Statistic Df Sig.			Df	Sig.				
CTL	.148	32	.071	.944	32	.095				
Direct Learning	.152	32	.059	.936 32		.059				

Table 2. Normality Test Results for Pretest Score

Table 2 showed that pretest data of the two classes (CTL and direct learning) are normally distributed. Next, data will be tested whether the variance pretest data of the classrooms is the same. The test equipment used for homogeneity is the Levene test. The results of the Normality Test of pretest data are presented in Table 3.

able 3.	ble 3. Levene's Test of Equality of Error Variances										
	F	df1	df2	Sig.							
	1.679	1	62	.200							

Table 3. Levene's Test of Equality of Error Variances^a

From Table 3: F count is 1.679 with a significance of 0.200. The significance is greater than the significance level of 0.05 so that the null hypothesis which states that there is no difference variance in the posttest score of CTL and direct learning model is accepted. The result of the Pretest Normality Test is presented in Table 4.

Table 4. Normanty Test Results for Postlest Score										
Learning model	Kolmogorov-Smirnov			ning model Kolmogorov-Smirnov Shapiro-Wilk			lk			
	Statistic Df Sig.			Statistic	Df	Sig.				
CTL	.082	32	.200*	.964	32	.354				
Direct Learning	.139	.139 32 .122			32	.038				

Table 4. Normality Test Results for Posttest Score

In Table 5: Posttest of experiment I and class II Experiment normally distributed. Furthermore, it will be tested whether the variance of experimental class I and experimental class II are

homogenous. The test equipment used for homogeneity is the Levene test. Homogeneity Tests for Posttest data is presented in Table 5.

Table 5. Leven	e's Test of	Equality o	f Error Va	riances ^a of	Posttest Score
	F	df1	df2	Sig	

F	df1	df2	Sig.
2.332	1	62	.132

From Table 5: F count is 2.332 with a significance of 0.132. The significance value is greater than the significance level of 0.05 so that the null hypothesis which states that there is no difference in variance of posttest score of the two classroom (CTL, direct learning model) is accepted.

Furthermore, it will be discussed how much improvement occurred between the CTL classroom and direct learning model. Student MPSA improvement in CTL classroom and direct learning model was calculated using the normalized gain or N-Gain formula. The results of the N-Gain MPSA for the classroom is presented in Table 6.

I able 6. Recapitulation of N-Gall Results of MPSAClass X_{min} X_{max} \overline{x} SI						
CTL	0.27	0.58	0.44	0.07		
Direct learning	0.05	0.47	0.33	0.09		

Table 6 Decenitulation of N-Cain Decults of MDSA

Table 6 above shows that the highest value of N-Gain in the CTL classroom was obtained 0.58 and in the direct learning classroom was obtained 0.47. While for the average value of N-Gain, CTL classroom was obtained 0.44 and in direct learning classroom was obtained 0.33. So the average N-Gain of CTL classroom is higher than the average N-Gain of direct learning classroom. Results of Calculation of the Coefficient of Class Regression Equations are presented in Table 7.

Table 7. R	esults of Calculation of Clas	ss Regression (Coefficient	S
	Coefficients	a		

	Coefficientsa									
		Unstand	lardized	Standardized						
		Coeffi	cients	Coefficients						
Model		В	Std. Error	Beta	Т	Sig.				
1	(Constant)	38.264	2.097		18.244	.000				
	Х	.378	.119	.502	3.175	.003				
a. Dep	a. Dependent Variable: Y									

Results of Calculation of the Coefficient Regression Equations of CTL classroom is presented in Table 8. Regression coefficient is the increasing of value for Y for every increasing value of X unit when sign of regression coefficient is positive. For example, if regression coefficient is 2, it means the value of Y increases two times of the increasing of the value of X. If sign of regression coefficient is negative, it means that every increasing value of X unit make the value of Y decreased as many as X times the coefficient of X.

Table 8. Calculation Results of Regression Equation Coefficients

	Coefficients								
Unstandardized Standardized									
		Coefficients		Coefficients					
Model		В	B Std. Error		Т	Sig.			
1	(Constant)	28.827	2.661		10.835	.000			
	Х	.554	.163	.527	3.400	.002			

Aritonang, A. R., Minarni, A., & Rajagukguk, W. (2018). Analysis of Differences in Improvement of Mathematical Problem Solving Ability of The Students at Upper Secondary School 3 Sidikalang Based on Contextual Learning Model And Direct Learning Models. *Advances in Social Sciences Research Journal*, 5(9) 284-293.

Based on data presented in Table 7 and Table 8, the MPSA test results: the regression equation for the experimental class I is Y1 = 38.26 + 0.37X1 and the regression equation for the experimental class II is Y2 = 28.82 + 0.55X2.

Independence Test and Linearity Test

Variance Analysis for MPSA Independence Test is presented in Table 9.

	Table 5. ANOVA for MF5A independence rest								
Model		Sum of Squares	Df	Mean Square	F	Sig.			
Ι	Regression	272.141	1	272.141	10.081	.003b			
	Residual	809.859	30	26.995					
	Total	1082.000	31						
a. Depe	ndent Variable	: Y							
b. Predi	ictors: (Consta	nt), X							

Table 9. ANOVA for MPSA Independence Test

From ANOVA or F test: F count is 10,081 with a significance level of 0.003. Because the probability is much smaller than 0.05, the regression model can be used with the regression equation Y1 = 38.26 + 0.37X1. Variance Analysis for the MPSA Regression Linearity Test is presented in Table 10.

	Table 10. ANOVA for MPSA Regression Linearity Test									
			Sum of		Mean					
			Squares	Df	Square	F	Sig.			
Y * X	Between Groups	(Combined)	728.000	18	40.444	1.485	.236			
		Linearity	272.141	1	272.14 1	9.994	.008			
		Deviation from Linearity	455.859	17	26.815	.985	.521			
	Within Groups		354.000	13	27.231					
	Total		1082.000	31						

Table 10. ANOVA for MPSA Regression Linearity Test

Based on Table 11: Sig. = 0.521 and for α = 5%, the value of α <Sig or 0.05 <0.521. Thus the regression of CTL classroom is linear. Analysis of Variance for MPSA Independence Test for MPSA data at direct learning classroom is presented in Table 12.

	Sum of Squares	Df Mean Square		F	Sig.			
Regression	381.873	1	381.873	11.560	.002b			
Residual	991.002	30	33.033					
Total	1372.875	31						
a. Dependent Variable: Y								
b. Predictors: (Constant), X								
	Residual Total endent Variabl	Regression381.873Residual991.002Total1372.875endent Variable: Y	Regression 381.873 1 Residual 991.002 30 Total 1372.875 31 endent Variable: Y Y	Regression 381.873 1 381.873 Residual 991.002 30 33.033 Total 1372.875 31	Regression381.8731381.87311.560Residual991.0023033.033Total1372.87531endent Variable: Y			

From ANOVA or F test: F count was 11.56 with a significance level of 0.002. Because the probability is much smaller than 0.05, the regression model can be used with the regression equation Y2 = 28.82 + 0.55X2. ANOVA for the MPSA data at direct learning model is presented in Table 11.

Table 11. ANOVA of Linearity of MPSA Data of Direct Learning Classroom								
Y * X	Between Groups	(Combined)	1089.625	17	64.096	3.168	.017	
		Linearity	381.873	1	381.873	18.875	.001	
		Deviation from Linearity	707.752	16	44.235	2.186	.074	
	Within Groups		283.250	14	20.232			
	Total		1372.875	31				

Table 11. ANOVA of Linearity of MPSA Data of Direct Learning Classroom

Based on Table 13: The value of Sig. = 0.74 and for α = 5%, the value of α <Sig is 0.05 <0.74. The regression model of MPSA data at direct learning model is linear. ANACOVA for the Similarities of Two MPSA Regression Models are presented in Table 12.

Tuble 12/11/11/00/11/01/the bining thes of 1 workes estion Flowers							
Α	В	SSTO(R)	SSR(R)	SSE(R)	\mathbf{F}^{*}	F _(0.95,2,60)	\mathbf{H}_{0}
33.40	0.47	3252.938	706.768	2546.169	17.210	3.15	Rejected

Table 12. ANACOVA for the Similarities of Two Regression Models

From the calculation results in Table 14: F = 17.21 for = 5%, F (1-0.05), (2, n-2) = F = 3.15. This means that H₀ is rejected and accepted H₁. This means that the two linear regression models are not significantly different. MPSA Covariance Analysis for Regression Model Alignment is presented in Table 13.

Table 13. ANACOVA for Regression Model Alignment								
Class	SSTx	SSTy	SPT	SSTx(adj)				
CTL	1906.21	1082	720.25	809.85				
Direct learning	1244.7	1372.87	689.44	990.39				
Total	3150.91	2454.87	140.69	1800.24				
А	В	F*	F	H0				
1800.24	1823.96	0.82	3.99	Accepted				

Table 13. ANACOVA for Regression Model Alignment

From the results of calculations in Table 13: $F_{table} = 0.82$ for = 5%, $F_{count} = 3.99$. Means H_o is accepted with a significant level of 5%. This means that both linear regression models for data of CTL classroom and direct learning are parallel. Because both regression models are not the same (not coinciding) and parallel, it can be concluded that there are differences MPSA between upper secondary school students at contextual learning model and direct learning model.

DISCUSSION

Based on the results of data analysis on the average pretest and posttest scores then calculated the increase (N-Gain) of both classes: Class taught through the contextual learning model obtain an average N-Gain score of 0.44 and for the class taught through the direct learning model, the average N-Gain score is 0.33. ANACOVA calculation results indicated that there are significant differences and the difference in the height of the two regression lines which affected by the regression constant. The height of the regression line describes the student MPSA, that is when X = 0 the regression equation for MPSA students in contextual learning class is Y = 38.26X and the regression equation for direct learning class is Y = 28.82X. This means that there is a difference in the improvement in MPSA between students taught through contextual learning models with students taught through direct learning models.

An interesting finding during the current research is that in contextual classroom, many students enjoy to engage in learning activity because in essence students like the culture they have and the problems they commonly see in their home and neighbors. Students saw that the

problems arose around their daily life could be solved by their village head and resolution of these problems usually satisfied various parties involved. Therefore, in the classroom they learn how to solve mathematical problems before them as if the village head and representatives of each group in the community are negotiating to solve the problem. Students seem excited in doing these mathematical tasks.

The reason why contextual classroom obtained higher N-gain is because contextual classroom required the teacher acts as a facilitator for students in solving problems. While, in the classroom that implements direct learning, students solve problems by following the example demonstrated by the teacher. Another thing, in contextual classes, mathematical problems are designed based on the problems often encountered by students in everyday life so that students can more quickly understand the mathematical problems they face. As for the class that implements direct learning, mathematical problems are almost unrelated to the problems encountered by students in their daily lives.

There is a difference in the improvement in PMSA because in contextual learning students are trained to solve context-related problems in accordance with the syntax of the learning. This can improve the ability of mathematical understanding as a major component of problem solving. Different from the direct learning class, in CTL classroom mathematical problems are not context related so students are not easy to understand problems. In addition, in contextual learning classroom, students are conditioned to construct the knowledge acquired so that students learn more meaningfully, not so in direct learning classes.

In contextual class, students are conditioned to learn to build their own knowledge because this learning is indeed constructivism-based learning. The students build knowledge along with their work in solving mathematical problems that teacher give in Students Activity Sheet (SAS). In direct learning classroom, there is no demand for students to build their own knowledge but students must be proficient in mechanistic calculations. Inferred from Lang & Evans (2006), this kind of learning is still needed if students background knowledge is low, teacher will deliver new material, time available is short.

Thus, it is clear that schools should begin to implement constructivism-based learning such as contextual learning model so that students can improve mathematical problem-solving abilities that are very much needed in the modern world as it is today. To apply contextual learning is not difficult because Indonesia is very rich in culture which can present its own problems in designing mathematical problems. Provided that the contextual learning syntax is actually carried out by the teacher, of course the learning can be interrupted by learning that has been carried out so far to complement the need for informative knowledge. The implementation will be even easier because the government has suggested to improve MPSA of upper secondary school students as documented in mathematics learning curriculum.

CONLUSION

Based on the results of data analysis and research findings, the conclusions which are answers to the research questions posed are consisted of :

- 1. There is the difference in the improvement of problem solving abilities between students taught through contextual learning models with students taught through direct learning models?
- 2. After implementing contextual teaching learning, tudents' mathematical problem solving abilities is categorize good.

3. MPSA N-Gain of the students who taught through contextual learning models is higher than it is the students taught through direct learning models.

In addition, although learning model based on constructivism is powerful, but direct learning model will still be needed at any time in accordance with the interests of comprehensive learning goals. Direct learning will not be shifted or ruled out.

References

Anderson ,L.W., *et al.*, (2001). *A Taxonomy For Learning, Teaching and Assessing.* New York: Addison Wesley Longman, Inc.

Goos, et.al. (2000). A Money Problem : A Source of Insight Into Problem Solving Action. Queensland : The University of Queensland

Johnson, D.W., Johnson, R. T., & Smith, K. A. (2013). Cooperative Learning: Improving University Instruction By Basing Practice On Validated Theory. *Journal on Excellence in University Teaching*.

Lang, H. R. & Evans, D.N. (2006). Model, Strategies, and Methods for Effective Teaching. New York: Pearson Education Inc.

Minarni, A. (2017). On Eight Grade Students Understanding in Solving Mathematical Problems. Asian Social Science. Vo. 13, No. 12, pp. 86-96. DOI. 10.5539/ass.v13n12p86.

Napitupulu, E.E. (2016). Cultivating Upper Secondary Students' Mathematical Reasoning Ability and Attitude Towards Mathematics through Problem-based Learning. *IndoMS Journal of Mathematics Education*, Vol. 7 No. 2. ISSN 2087-9995, e-ISSN 2407-0610.

NCTM. (2000). Principle and Standards for school Mathemstics. Reston, VA: NCTM

Nihdayati Pane, Edi Syahputra, and Mulyono, "Improving the Ability of Creative Thinking Mathematically and Self-Confidence Student through Application Model Eliciting Activities (MEAs) Review from Student Gender." *American Journal of Educational Research*, vol. 6, no. 4 (2018): 319-323. doi: 10.12691/education-6-4-4.

PISA 2012 Assessment and Analytical Framework: Mathematics, Reading, Science, Problem Solving, and Financial Literacy, OECD Publishing.

Polya, G. (1987). Mathematical Discovery: Combined Edition. New York : John Wiley Interscience.

Steen, L.A. (1988). *The Science of Patterns* Science, 240: 611–16. And summarized at Association for Supervision and Curriculum Development Archived October 28, 2010, at the Wayback Machine., www.ascd.org.

Wilson, et al. (2008). *TIMSS 2007 International Results in Mathematics*. TIMSS & PIRLS International Study Center Lynch School of Education, Boston College.

Ziegler, Günter M. (2011). "What Is Mathematics?". An Invitation to Mathematics: From Competitions to Research. *Springer. p. vii. ISBN 3-642-19532-6.*