

## Analysis of Eighth-Grade Students Failure in Solving Mathematical Problems

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### ABSTRACT

This research aimed to investigate the ability of eighth-grade students and types of error they made in solving mathematical problems. To obtain data, as many as 102 PJHS students are included in mathematical problem solving (MPS) test. The aspects of MPS ability used in this research consisted understanding the problem, proposed and used the strategy to solve the problem and conclude the solution in accordance to the initial problem. Results of the research: First, students MPS ability belong to the low category. The second one, types of error the students made in solving MPS consisted lack of algebra knowledge including algebra manipulation, lack of mathematical concept, and lack of informal and formal strategy including creating a picture. The third one, results of the interview reveal that the students view algebra as complicated material and does not have a definite pattern to conquer. Meanwhile, the students could not make a picture because they confuse in understanding words problems.

**Keywords:** mathematical problem solving

### INTRODUCTION

It is mentioned in NCSM (Wilson et al., 1997) that mathematical problem-solving ability (MPSA) is important because it is the main purpose of learning mathematics. In addition, the MPSA provides an important context that allows students to learn a variety of mathematical topics and enable students to learn new concepts (Kilpatrick, et.al, 2001). Problem-solving can be used by students to produce new mathematical knowledge because through solving problems students apply their knowledge, experience, and skills in new or unknown situations. Unfortunately, according to TIMSS report, Indonesian students achievement in mathematical problem solving is still low; that is 397 in 2007 and get score 386 in TIMSS 2011, while the standard average score is 500 (Provasnik, 2012), so it needs to be improved. Kind of problem from TIMSS related to problem-solving is attached here. This problem relating to ratio number where most students face difficulty. Here is kind of problem from TIMSS 2007.

#### Problem

There are 30 students in a class. The ratio of boys to girls in the class is 2:3. How many boys are there in the class?

Problems similar to TIMMS problem along with four other problems has been tested on students at several public junior high school students (PJHS) in Bandung. The results show that the percentage achievement of PJHS students in mathematical understanding and representation ability as components of MPS is only 32.46% (Minarni, Napitupulu, Husein,

2016). In addition, the ability of mathematical representation of grade VIII students in Medan and its surrounding areas is also low, that is 7 out of 20 or 35%, likewise, students' mathematical comprehension (understanding) skills, meanwhile, the ability of mathematical understanding is also the foundation for problem-solving (Minarni, 2017a).

The ability to solve mathematical problems is the ability to apply ideas/procedures/mathematical facts to solve mathematical problems. The aspects of MPSA including understanding the problem, choosing a strategy and using it to solve the problem, and summarizing the solution in accordance with the initial problem. A person may be able to understand the problem, but could not find a solution strategy, or someone may understand the problem and be able to use the problem-solving strategy but unable to deduce the solution as per the initial problem. When using a mathematical problem-solving strategy we will find algebraic forms, images, graphs, or mathematical equations, in which one student often experiences difficulties that lead to making mistakes.

Some students made mistakes in solving mathematical problems because of limited mathematical understanding ability (Minarni, 2017b), low ability in math understanding and representation (Minarni et.al, 2016). Fortunately, the mathematical problem-solving ability could be generated through the learning process (Minarni, 2017a). Problem-solving skills also increased by learning errors in solving the problems since learning can occur through mistakes. On the other side, teachers can reflect through errors that students make in solving problems. Then, knowledge of the types of mistakes made by students in solving problems is important to investigate deeply. The researcher eager to elaborate students MPSA as well as some kinds of mistakes made by the students in solving MPS.

### **THEORETICAL FRAMEWORK**

Problem-solving refers to cognitive processing directed at achieving a goal when the problem solver does not initially know a solution method. D'Zurilla (1988) defined problem-solving as a cognitive-affective behavioral process through which an individual (or group) attempts to identify, discover, or invent effective means of coping with problems encountered in everyday living". Schoenfeld (Grouws, 1992) states that problem-solving has used in multiple meanings (from knowledge about one's thought of processes to self-regulation during problem-solving). Problem-solving defined as a higher-order cognitive process (high order thinking skills-HOTS) and intellectual function that requires the modulation and control of more routine or fundamental skills (Backmann & Guthke, 1995), it has two major domains: mathematical problem solving and personal problem-solving. Both are seen in terms of some difficulty or barrier that is encountered (Berry & Broadbent, 1995).

Problem-solving must be the focus of school mathematics (NCTM, 1980). Inferred from NCTM (2000) that high order thinking consisted of mathematical reasoning, comprehension, representation, communication, connection, creative & critical thinking, and metacognition. Problem-solving is the heart of mathematics because various concepts, principles, and procedures are searched for and used in problem-solving (Lester & Kroll, 1990). Problem-solving is doing mathematics that produces mathematical objects.

Major cognitive processes in problem-solving are representing, planning, executing, and monitoring. Meanwhile, in mathematics, the process including understanding the problem proposed the strategy, execute the strategy, and looking back (Polya, 1987). The major kinds of knowledge required for mathematical problem solving are facts, mathematical ideas, concepts, procedures, strategies, and beliefs. Example of fact are math symbols, the example of math concept is all definition.

On the other hand, (mathematical) problem exists when someone has a goal but does not know how to achieve it. Problems can be classified as routine or non-routine, and as well defined or ill-defined from which appropriate solutions are to be made. Ill-defined problems are those that do not have clear goals, solution paths, or expected solutions. On the contrary, well-defined problems have specific goals, clearly defined solution paths, and clear expected solutions. These problems also allow for more initial planning than ill-defined problems (Altshuller, 1994).

There are two types of mathematical problem, real-world and abstract (pure) mathematical problems. The first type is used in mathematics education to teach students to connect real-world situation to the abstract language of mathematics. At school, to cultivate students' ability to think deeply, mathematical problems are designed in such a way that students are required to link their knowledge to the problems at hand, find useful knowledge to solve problems and think through the strategies and procedures required. Not all school math topics can be designed to be real-world mathematical problems. Here is an example of a question related to the topic of algebra, which could not be a simple real-world problem but grade eight students must master it.

<p><b>Problem</b></p> <p>Simplify the following algebra form:</p> $\frac{x-1}{x-5} + \frac{2(x^2-7x+17)}{x^2-7x+17}$
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Misconception that often occur in solving this problem is just adding the denominator and multiplying the nominator as follow:

$$\frac{x-1}{x-5} + \frac{2(x^2-7x+17)}{x^2-7x+17} = \frac{(x-1) + 2(x^2-7x+17)}{(x-5)(x^2-7x+17)}$$

The student has not understood yet the concept of adding fraction number.

A mathematician does solve a mathematical problem for their own sake, by doing so, results may be obtained that suitable for application outside the realm of mathematics. Many abstract problems can be solved routinely, others have been solved with great efforts, some have unsolved yet until now. The process of problem-solving includes understanding the problem, plan the strategy, execute the strategy and looking back (Polya, 1987). But, it is not claimed that these are MPS indicators. In line with Polya's idea, mathematical problems from TIMSS are matters designed so that students are required to understand the problem but are not asked to write down what is known and what the questions are asked. The students are required to use problem-solving strategies but not instructed to write strategies that he will use, as well as mathematical problems in Tong & Hiong (2006).

Mathematical problem solving underpinned by mathematical comprehension, representation, and mathematical reasoning. Mathematical comprehension consists of the ability to make a math connection and the ability in math representation (Carpenter & Lehrer, 1999). The difficulty in recalling knowledge in cognitive structures is a matter of understanding (Hiebert & Carpenter, 1992). Research on solving mathematical word problems suggests that students may perform better on a problem closed to real-life problem representation of the problem situation than on word problems (Hoogland et.al., 2018). Inferred from Minarni (2017) that the students with good math representation performed better in solving mathematical problems.

It is alleged that conventional learning approach with expository methods could not develop mathematical problem solving because the teacher is the center of the teaching-learning process. There is no time for the student to solve a challenging mathematical problem since too many topics should be acquired. Meanwhile, problem-solving could be increased through implemented joyful problem-based learning (Minarni & Napitupulu, 2017).

### Research aim

The objectives of the research are to investigate:

1. Mathematical problem solving ability of the students.
2. Types or error the students made in solving mathematical problems.
3. Cause of error the students made in solving mathematical problems.

## METHOD

### Research design

This research is stage I of developmental research. According to Thiagarajan, et.al. (1974), this stage is the design phase which is aimed to investigate students profile, one of the profile is student mathematical problem-solving ability. At this stage, an MPSA test is carried out to measure student MPSA. Then observations were made during the test, followed by interviews after the tests were completed. Interviews and observations are also used to reveal the learning approach used by the teacher in the classroom. Because there are allegations that certain learning cannot foster mathematical problem solving (Ronis, 2008).

### Subject of the Research

Developmental research stage I is not aimed to generalize the population, so the researcher used the subject of the research that is chosen purposively. Consideration of choosing this subject is data about the need to improve high order thinking of the students. The mathematical problem-solving ability of eighth-grade students at public junior high school (PJHS) in Medan City and District Deli Serdang is low. As many as 102 students are included in this research.

### Instrumentation

The instrument used in this study consist of two types. The first one is the instrument test used to measure students mathematical problem-solving ability (MPSA). Type of the test is essays test that consists of five items. Some examples of the problems are presented here.

#### Problem No. 1

In an arithmetic sequence, the fifth term is 35, the sum of the seventh and ninth term is 100. Determine sum of first and third term.

Problem number 1 used to measure the student ability in solving a mathematical problem related to a pattern of a sequence. The student should use their reasoning to determine the  $n$ -th term of a given sequence.

#### Problem No. 3

Mia types a book step-by step. The first day, she types  $\frac{1}{5}$  part of that book. In the second day, she types  $\frac{1}{2}$  part of the rest. If she types 30 pages on the first day, then determine the number of pages that have not typed yet.

Problem number 4 used to reveal the student ability in solving a mathematical problem related to algebraic factorization. Problem 2 and Problem 5 are not attached here. Problem 2 related to

the concept of line, problem 5 related to the concept of the Pythagorean rule in association with a beam.

**Problem No. 4**  
Simplify the following algebraic form:  
$$\frac{cx + cx - c - 1}{x - 1}$$

Problem number 4 used to reveal the student ability in solving a mathematical problem related to algebraic factorization. Problem 2 and Problem 5 are not attached here. Problem 2 related to the concept of line, problem 5 related to the concept of the Pythagorean rule in association with a beam.

Score for each item of MPSA test is four. The total score of five items is 20. Classification of MPSA in accordance with total score (X) presented in Table 1.

**Table 1 Classification of Students' MPSA Score (n=102)**

MPSA Category	Criteria
Excellent	$16 < X \leq 20$
Good	$12 < X \leq 16$
Enough	$8 < X \leq 12$
Low	$X < 8$

**Note: Ideal total score = 20**

To reveal the causes of mistakes made by students in solving mathematical problems, researchers conducted an interview. The content of the interview is designed to reveal the student's mistake on each question. Type of questions in the interview will vary because the answers and mistakes made by students in solving problems also varied.

### Method of Data Analysis

The techniques of analysis used were descriptive statistics such as means and standard deviation used to summarize MPSA data and summarize the result of observation and interview. These statistics are used to classify the MPSA test score and students failure in solving mathematical problems. Types of error may occur are:

Wrong in implemented math principle, for example,

1. Wrong in implemented math principle, for example:
  - a.  $5-3 \times 2 = (5-3) \times 2 = 2 \times 2 = 4$  : wrong in implemented the order of algebraic operation.
  - b.  $\frac{3}{4} + \frac{1}{2} = \frac{4}{6}$ . Used multiplication rule for adding up fraction.
2. An error occurred related to using strategy. For example, the student uses the elimination technique to solve a system of linear equation, but he did not eliminate one of the variable included in the system.
3. Fail to recall mathematical prior knowledge.

### FINDINGS AND DISCUSSIONS

This section consists of three parts: the first part deals with mathematical problem-solving ability. The second part presents the type of error/failure the students made in solving mathematical problems. The third part presents the cause of the failure students made in solving mathematical problems. Data in the first part gathered through the MPSA essays test.

Meanwhile, data for second and third part gathered through analysis students' worksheets and interview. Observation is used to reveals learning approach the teacher used in the classroom.

### Mathematical problem solving (MPS)

This result is obtained from the second sample that consist of 102 students, the result of the first sample that consists of 40 students is presented at an international seminar. The result of the MPSA test from the second sample is summarized in Table 2. It can be seen that the maximum MPSA average score belongs to PJHS 17, that is 8.29, even though this score is just categorized enough (sufficient), far below the ideal score. This school is located at the capital city of North Sumatera Province, so it is hypotheses that the students at this school are easier to get information and access source of knowledge at the internet than other students whose school located at rural area.

**Table 2 Statistic of Students MPSA**

School	N	Mean	Std. Deviation
PJHS-17	29	5.86	2.371
PJHS-4	17	8.29	3.531
PJHS-1	26	7.42	3.733
PJHS-2	29	4.28	.591
Total	101	6.22	3.065

**Note: Total ideal score = 20**

The students' MPSA average score at other schools belong to the low category. In addition, the standard deviation of students MPSA at each PJHS is large enough. It means students' ability is very diverse. Based on Table 1, the average score of each school is categorized low. MPSA average score based on each item test can be seen in Table 3. The result of observation and interview shows that learning the approach the teacher used in the classrooms is a conventional approach with the expository method. Then, low of students MPSA is predicted because of the learning approach used in the classroom. This prediction is in line with the statement of Ronis (2008).

The following table shows that Problem number 4 (item test 4) is the hardest one. No one doing this problem. The task in this problem is to simplify an algebraic form. This problem is more abstract than other problems. It showed that it is hard enough for the students to think formally, they still need concrete matter. So, in the next learning process, the teacher should give contextual problems that include mathematical representation, i.e.picture, to aid the student to solve the problems as proposed by Hoogland (2018).

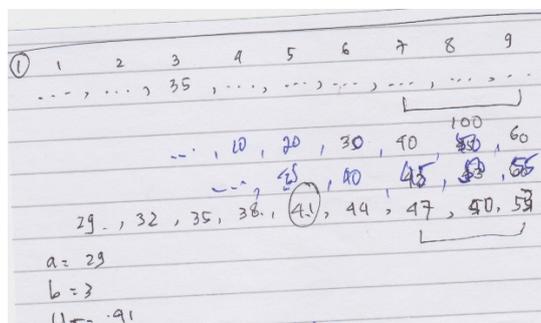
Descriptive statistic of students MPSA at each item score is presented in Table 3.

**Table 3 Average score of each item test**

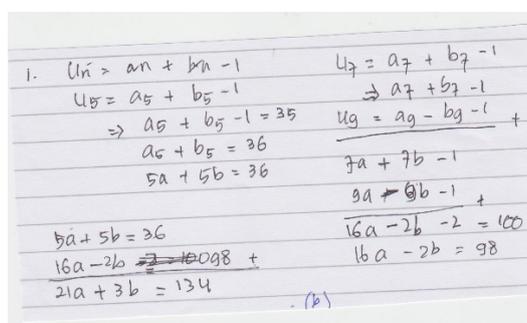
Item test	Mean	Std. Deviation
1	1,375	0,9439
2	1,263	0,7769
3	1,618	0,4079
4	0	0
5	2,170	0,6648
Average	1,285	0,5587

Table 3 showed that problem number 4 is the hardest problem for the students. This problem related to algebraic form and the students should make factorization to solve this problem. Obviously, the students have not proficient in factorizing algebraic form.

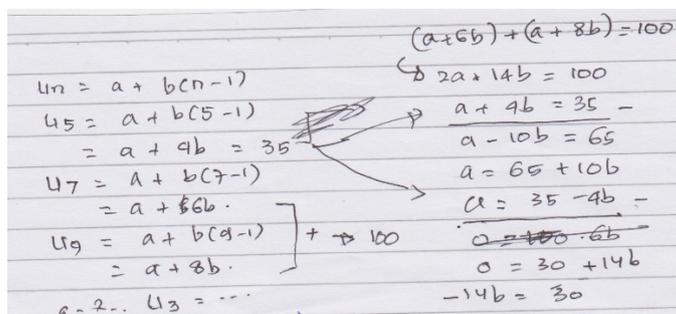
Example of student performance in doing Problem number 1 MPSA test is presented in Figure 1.



(a)



(b)



(c)

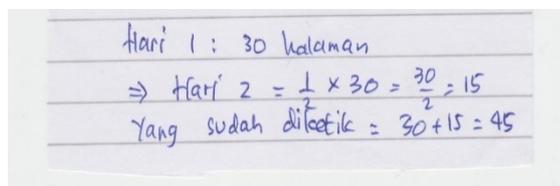
**Figure 1 Student Answer Sheet for Problem 1**

Figure 1 (a) shows that the student is able to think informally, i.e, guess the possible sequence where the 3rd, 7th, and 9th term of the sequence are known. At first, he guessed the 7th, and 9th term were 40 and 60 respectively, so the difference between the two terms was 10. Then, he lowered the difference to 5. Finally, he guessed the difference between the two terms becomes 3 and this guess leads to the correct solution. However, the guessing technique only applies if the number of terms is limited. If he asks to determine a 100th term of a sequence, does not know the formula for determining the nth term or does not remember how to determine the nth term of a sequence, the guessing technique will be time-consuming or boring. This student's performance shows that he has not been able to make generalizations. Figure 1(b) tells us that the student proposed a formula of the nth term as  $u_n = b^n - 1$ , but it is not right. So, he could not attain the solution. Actually,  $u_n = a + (n-1)b$ . From Figure 1(c), the

student exactly proposed a right formula to determine un, he could determine u5, u7, and u9. Unfortunately, he does not remember a substitution or elimination technique, or other technique to solve a system of linear equation of two random variables. Then, he does not get the solution. Overall, the majority of the students could not solve this problem because of does not know how to determine the nth term of a sequence based on the number pattern in the sequence. So, when they forgot the formula, they can do nothing to solve the problems at hand.

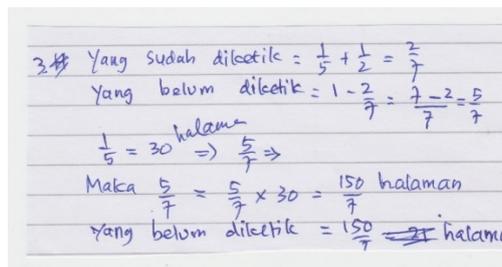
**Types of error in Solving MPS**

As a first step in investigating students' mistakes in solving mathematical problems is to analyze student performance in completing the MPSA test. We have seen student performance for problem 1 in Figure 1. The type or error student made in solving Problem 1 is the inability to recall mathematical prior knowledge such as techniques to solve a system of linear equation of two random variable. It also indicated as lack of mathematical connection, component of mathematical understanding (Carpenter & Lehrer, 1999). Other types of error the student made could be reviewed in Figure 2. This figure displays student performance in solving problem number 3.



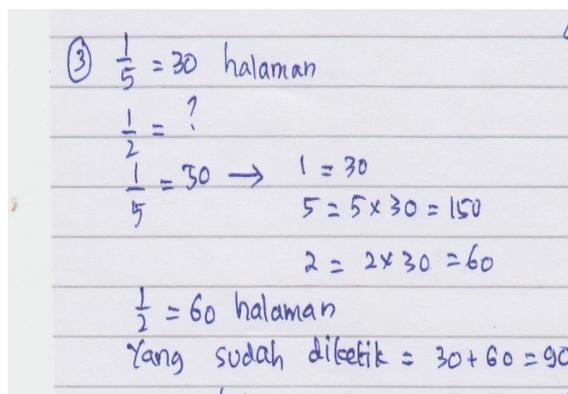
(a)

Explanation:  
 Day 1: 30 pages  
 Day 2:  $\frac{1}{2} \times 30 = \frac{30}{2} = 15$   
 So, the book that has been typed is  $30 + 15 = 45$



(b)

Explanation:  
 The book that has been typed =  $\frac{1}{5} + \frac{1}{2} = \frac{7}{7}$   
 The book that has not been typed =  $1 - \frac{2}{7} = \frac{5}{7}$   
 $\frac{1}{5} = 30$  so  $\frac{5}{7} = \frac{5}{7} \times 30 = \frac{150}{7}$  pages



(c)

Explanation:  
 $\frac{1}{5} = 30$  pages  
 $\frac{1}{2} = \dots?$   
 $\frac{1}{5} = 30$  then  $1 = 30$   
 $5 = 5 \times 30 = 150$   
 $2 = 2 \times 30 = 60$   
 $\frac{1}{2} = 60$  pages  
 The book that has been typed is  $30 + 60 = 90$  pages

**Figure 2 Student work sheet for Problem 3**

Figure 2(a) showed that students cannot capture information that 1/5 is 30 pages. He thought that the second day Mia typed half the part of what he did on the first day. This assumption

makes students unable to solve this problem. In general, this student has not been able to understand the problems they face. Therefore, the error lies in misunderstanding the problem. Figure 2 (b) shows that this student is correct in stating the number of parts that Mia has typed up to day 2, which is  $1/5 + 1/2$  parts, but he does not understand the concept of adding fractions. Therefore, he did not arrive at the correct answer.

Figure 2(c) showing that a student's fatal error was that when he wrote  $1/5 = 30$  then  $1 = 30$ , he looked at the nominator as a determinant of the number of pages. Then, he writes  $2 = 2 \times 30 = 60$  so  $1/2 = 60$  pages, in this case he considers the denominator as a determinant of the number of pages. Overall, this student really does not understand the concept of fractional multiplication.

From 102 students whose follow the test, there are 80 students face the difficulties in making a picture or sketch a problem, or transferring the problem into their own perception. Most of the students make a picture for problem 5, a picture of a beam, but incorrect. Some of them do not think to make drawings, sketches or translate the problem into a form that could make them understand better. Although there are several students thought, but could not make it because they do not know how.

There are 86 students could not make connection between the problem at hand with mathematical prior knowledge. This problem is occurred do to the ability of math connection (Carpenter & Lehrer, 1999). For example, they have learned fractions, addition and subtraction of fractions, but they could not link it or apply it to problem number 3. In this case, they add up the nominators also add up the denominators.

For problem 2 and 5, as many as 89 students are wrong in creating mathematical model as a tool to solve the problem. The cause of this failure is due to mathematical concept they have. The concept of line and the concept of Pythagorean rule have not been stored firmly in their cognitive structure so it is hard to recall and apply in new situation. It means, they have not understand this concept yet (Marzano & Kendall, 2007). In order to understand, one should learning with understanding (Hiebert & Carpenter, 1992) so that knowledge/information embedded in the cognitive structure in a strong and durable manner and makes it easy to be recalled when needed in solving problems.

Some students still remember the concept of the line, planning to use the elimination technique to solve it but most of them are wrong in executing the technique because they do not understand elimination technique. For example,  $(a + 5b) - (3a + 9b) = -2a - 4b$ . Of course, there is no variable eliminated so that the result is still in equation form, not attain the value of each variable.

The problem of algebraic manipulation also complicates students. All students in this study could not solve Problem number 4, that is the problem related to algebraic manipulation. Most of the students were not able to make algebraic factorization. For this problem, students should factorize  $(cx^2 + cx - c - 1)$  to  $[(x - 1)(cx - (c - 1))]$  so that the factor  $(x - 1)$  in the nominator can be divided by factor  $(x - 1)$  in the denominator so that it produces a value of 1. So, the solution is  $[(cx - (c - 1))]$ .

Analysis towards all work sheets of the students reveal some types of mistakes they made as presented in Table 4.

**Tabel 4. Students Error in solving Math Problem**

No.	Type of Failure
1	Make a picture or sketch a problem.
2	Connecting the problem to prior knowledge.
3	Create a mathematical model for the problem at hand.
4	Adding up and subtracting variables
5	Algebraic manipulation.

**Cause of error**

The following is an example of an interview with one of the students whose answer problem 3 is incorrect as depicted at Figure 2.

Interviewer (I): Fikri, would you tell me how many sections of the manuscript did Mia make on the first day?

Fikri (F): 1/5

I: How many pages did Mia have on the first day?

F: 30

I: Can you calculate how many pages of the entire book text?

F: 15

I: How do you get 15?

F: I don't know. Just guessing.

I: Okay. How many parts have been done until the 2nd day?

F:  $\frac{1}{7}$  and  $\frac{1}{2}$

I: Tottaly?

F:  $\frac{1}{7}$

The following conversation also discusses Problem 3.

Interviewer (I) : Tia, tell me why your answer is 6 page for 1/5 part of the book?

Tia (T): Because  $\frac{1}{5} \times 30 = 6$

I: Why you say part of the book has been typed after two days is  $\frac{1}{7}$ ?

T: Because  $\frac{1}{2}$  and  $\frac{1}{5}$  is  $\frac{1}{7}$

I: If  $\frac{1}{5}$  parts of the book is 30 pages, then how many pages for 2/5 part of a book?

T:  $\frac{2}{5} \times 30 = \frac{60}{5} = 12$

I: How is  $\frac{5}{5}$  part of a book?

T:  $\frac{5}{5} \times 30 = 30$

I: Tia, look at the problem, isn't it say that 30 pages is typed at the first day and  $\frac{1}{5}$  is part of a book that is typed at

the first day. Isn't it means  $\frac{1}{5} = 30$ ?

T: hm.....

Based on the interview, it is revealed that the students had not understood the concept of part and the whole number. Then, students are not yet proficient in adding up fraction number where each fraction has a different denominator. Analysis of all interview revealed the following results:

1. Could not make a generalization as student' performance in Problem 1(a).
2. Could not recalling information/knowledge that store in their cognitive structure, for example, could not recalling formula of the  $n^{\text{th}}$  term and the number of  $n^{\text{th}}$  term of a sequence. They do not remember how to get the formula because the teacher told them the formula. Indeed, there are students who remember the formula for determining the  $n^{\text{th}}$  term and the sum of the 3<sup>rd</sup> and 7<sup>th</sup> term of the sequence, but they have difficulty in determining the values of variables searched through elimination or substitution techniques so that they do not reach a solution.
3. Related to question 2, the students view algebra as a complex material and does not have a definite pattern to conquer. They call it difficult and make dizzy. Of the 102 students who took this test, no one can solve this problem.
4. Related Problem number 3, students consider fractional problems is difficult to determine the connection between part and the whole part number. Difficulties are increasing as they do not creating picture to represent the problem or create other representations that make the problem become more obvious.
5. Could not create mathematical model for the problem at hand.

Overall, some mistakes students made in solving mathematical problems are due to lack of:

- (1) capability in algebraic operation
- (2) mathematical prior knowledge
- (3) mathematical connection and representation as component of mathematical understanding.

### **CONCLUSION**

The conclusions obtained from the results of this research data analysis are:

1. In general, mathematical problem solving ability of the students belongs to low category.
2. Failures students make in solving mathematical problems:
  - a. Make a picture or sketch a problem.
  - b. Connecting the problem to prior knowledge.
  - c. Create a mathematical model for the problem at hand.
  - d. Adding and subtracting variables
  - e. Algebraic manipulation.
3. Causes of failure the student made in solving MPS are due to:
  - a. inability in algebraic operation
  - b. insufficiency mathematical prior knowledge
  - c. lack of mathematical connection and representation as component of mathematical understanding.

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