Advances in Social Sciences Research Journal – Vol.5, No.7
Publication Date: July. 25, 2018
DoI:10.14738/assrj.57.4938.
Zhu, M. J. (2018). Risk Analysis of the Newsvendor Problem Based On Steady Risk Preference. *Advances in Social Sciences Research Journal, 5(7)* 428-433.

Risk Analysis of the Newsvendor Problem Based On Steady Risk Preference

Ming-Jun Zhu
1882 West Yan'an Rd., Glorious Sun School of Business and Management,
Donghua University, Shanghai, 200051,China

ABSTRACT
A new risk measurement method was used in this paper to analyze the risk of newsvendor problem. Through the study we found for the action of same order quantity decision, risk values of the action are different for decision makers who have different risk preference. For the same order quantity risk value of high profit product is always lower the low profit product. With increase of order quantity the increment of risk valuewill first decrease and then increase.

Keywords: newsvendor problem, risk measurement, steady risk preference

INTRODUCTION

As a fundamental problem in stochastic inventory control, the newsvendor problem has been studied for a long time and applied in a broad array of business settings. In the newsvendor problem a decision maker need to decide how many goods to order before a one period selling season with stochastic demand. If too much is ordered, stock is left over the end of the period, whereas if too little is ordered, sales are lost. In choosing an order quantity the newsvendor must balance the costs of ordering too little against the costs of ordering too much. Traditionally, risk-neutral decision makers are considered to either maximizing the expected profit or minimizing the expected cost. However, modern supply chains are become much more complex than ever. What concerns supply chain managers most is not the profit but the risk of their firms. So the assumption of risk-neutrality seems to be inadequate for contemporary supply chain management. And there are a number of papers devoted to the study of risk analysis in newsvendor models. We review some of them as follows.

Traditionally, the von Neumann-Morgenstern utility (UT) approach and the mean-variance (MV) approach are two well-known decision methodologies for studying problems with risk concerns.

To reflect the newsvendor's risk attitude, many researchers have adopted the utility function approach in their analysis. Representative papers in this area include [1][2][3]. Atkinson [1] studies the incentive issue in the newsvendor problem. He finds that a risk-averse manager will order a smaller quantity compared with the risk-neutral manager, and a delegation scheme which can improve the situation is proposed. In [2], the maximization of the von Neumann–Morgenstern's expected utility is discussed, and the optimal solution can only be solved via a numerical search. Eeckhoudt et al. [3] focus on the effects of risk and risk aversion in the newsvendor model when risk is measured by expected utility functions. In particular, they determine comparative-static (i.e., a qualitative sensitivity analysis) effects of changes in the various price and cost parameters in the risk aversion setting.

The MV formulation was proposed by Markowitz [4]. Under the MV formulation, the payoff of the investment is quantified by expected return and the risk is quantified by the variance of return. Instead of optimizing the expected cost/expected profit for the newsvendor problem, Lau[1] investigates the optimal stocking problem which maximizes an objective function of the expected profit and standard deviation of profit. Chen and Federgruen [5] first model a quadratic utility function for the risk-averse decision maker and then construct an MV efficient frontier. Choi *et al.* [6] carry out a mean-variance analysis of the newsvendor problem. They constructed analytical models and reveal the problem's structural properties, and proposed the solution schemes which help to identify the optimal solutions. Jun Wu *et al.* [7] studied the risk-averse newsvendor model with a MV objective function. They found that stockout cost has a significant impact on the newsvendor's optimal ordering decisions.

The UT approach is precise, but its application is limited, owing to the difficulty in assessing the utility function for decision maker in practice. MV theory measure the risk as it is purely objective event and have no relationship with actor's subjective behavior.

In this paper, we will use a new risk measurement method proposed by Jiang [8] to research the traditional newsboy problem. This method can build a relationship between subjective attitude of actor and the objective risk, and can incorporate different risk preference include risk-neutral, risk-aversion and risk-seeking. By this method we can measure the risk for different risk preference decision makers. And so try to find the reason of the experimental finding by Schweitzer and Cachon[8]. They describe two experiments that investigate newsvendor decisions across different profit conditions. Results from these studies demonstrate that choices systematically deviate from those that maximize expected profit. Subjects order too few for high-profit products and too many of low-profit products.

To be specific, we first discuss the characteristic of risk and risk preference, and the risk measurement formulation is given in Section 2. The review of the newsvendor problem and the risk express of newsvendor is given in Section 3. In Section 4, we study the newsvendor problem which described in [9] use the new formulation. Conclusions are given in section 5.

RISK PREFERENCE AND RISK MEASUREMENT

Characteristics of risk

Risk has three exceptional characteristics. One is subordinative, any risks have definite actor and happened in the action with explicit target of interest. Different actors may take different action for same event, so to get different targets. Second is harmful, the potential loss that lead by risk may threaten the actor. Third is associative, any risks that subordinate to some actions may threaten the actor and quite contrary to the target of actions. So risk has inherent relationship with subjective behavior of the actors.

When we describe risk there are 5 essentials should be involved. They are: action, actor, benefit, possible loss, harm. Action and actor are subjective elements. Benefit and possible loss are objective elements. Harm is the effect of subjective element on objective elements, it is also the subject of risk measurement. "Benefit" is the target of actions taken by actors, it can be indicated by currency gains, denoted by w. The probability of benefit is . "Possible loss" is the certain quantity of monetary loss, denoted by l. The probability of possible loss is . Benefit and possible loss are complement with each other. The risk of action p is defined as follows:

$$R(p) \sim [\ _1, l_1; \cdots;\ _n, l_n;\ _1, w_1; \cdots;\ _m, w_m] \quad (1)$$

where ${}_i$ is the probability of loss l_i, ${}_i$ is the probability of benefit w_i. For the complementary of possible loss and benefit, ${}^{n}_{i=1}\ {}_i + {}^{m}_{i=1}\ {}_i = 1$

l_i in formula (1) corresponding to loss, we call it "common risk". w_i in formula (1) corresponding to benefit, we call it "speculative risk". Avoid common risk and chase speculative risk is the most important psychological characteristic of actors.

Risk preference characteristic

According to standard gambling proposed by von-Neumann-Morgenstern [10], we call binomial distribution $[x, a, y]$ a gambling. *x and y* are two possible results. is the probability of *y*, so probability of *x* is 1 ,0≤ α ≤1. *x<y*, and *x, y* ∈*X*, *X* is subset of positive real numbers [0,+∞）. The preference of decision maker to uncertainty depend on the value assess of gambling $[x, \ , y]$, we call it equivalent of gambling, denoted by , the relationship of and $[x, \ , y]$is follows:

$$[x, \ , y] \sim \qquad (2)$$

We call formula (2) as no difference formula. It is apparently that $x < \ < y$. Decision makers must trade off repeatedly, and different risk preference will has different .

Definition 1: If decision maker's wealth level has changed, and his risk preference remains the same, that is for all $[0, + \ )$, we have$[x, \ , y]+ \ \sim \ +$, then the following relationship exist:

$$[x, \ , y] \sim \ \Leftrightarrow [x, \ , y] + \ \sim \ + \qquad (3)$$

We call such kind of decision maker had steady risk preference. This paper will focus on steady risk preference actor.

Risk preference can be identified by the value of . According to [11], there are 3 types of risk preference in the steady risk preference category. We define the risk preference of common risk and speculative risk separately.

Definition 2: Common risk preference

If *x, y* denote 2 different loss results, then the equivalent of gambling is loss equivalent. Decision maker assess the no difference formula. If $< E[x, \ , y]$, we call decision maker is risk aversion. If $= E[x, \ , y]$, we call decision maker is risk neutral. If $> E[x, \ , y]$, we call decision maker is risk seeking. Where $E[x, \ , y]$ is the mathematical expectation of$[x, \ , y]$, that is $(1 \quad)x + \ y$.

Definition 3: Speculative risk preference

If *x, y* denote 2 different income results, then the equivalent of gambling is speculative equivalent. Decision maker assess the no difference formula. If $< E[x, \ , y]$, we call decision maker is speculation prudent. If $= E[x, \ , y]$, we call decision maker is speculation neutral. If $> E[x, \ , y]$, we call decision maker is speculation seeking. Where $E[x, \ , y]$ is the mathematical expectation of $[x, \ , y]$, that is $(1 \quad)x + \ y$.

Make denotes the equivalent of loss distribution, and denotes the equivalent of income distribution. According to the above definitions, for different risk preference decision maker, for standard gambling [0, 1/2, 1], the characteristic of equivalents are as table 1.

Table 1 Equivalents of gambling[0， 1/2， 1]

	Risk neutral, speculation neutral	Risk aversion, speculation prudent	Risk aversion, speculation seeking	Risk seeking, speculation seeking	Risk seeking, speculation prudent
θ	=0.5	<0.5	<0.5	>0.5	>0.5
η	=0.5	<0.5	>0.5	>0.5	<0.5

Risk measurement

For a risk $p=[\ _i, l_i;\ _j, w_j], i=1,\cdots,n, j=1,\cdots,m$, common risk will bring loss, and speculate risk will bring benefit, and benefit equals to negative loss. So in the expression of risk, w_j is negative. If decision maker is steady risk preference, then the measurement formulation of risk *p* is as follows [8].

$$R(p)=\frac{(1\quad _2)(\ _1 \quad \overset{n}{\underset{i=1}{}} \ _i\ _1^{x_i})}{(1\quad _2)(\ _1 \quad \overset{n}{\underset{i=1}{}} \ _i\ _1^{x_i})+(1\quad _1)(\ _2 \quad \overset{m}{\underset{j=1}{}} \ _j\ _2^{y_j})} \tag{4}$$

Let $O_0=\min\limits_{i=1,\cdots n, j=1\cdots,m}(l_i,w_j)$, $O_*=\max\limits_{i=1,\cdots n, j=1\cdots,m}(l_i,w_j)$

the standard value of l_i and w_j is x_i, y_j separately then

$$x_i=\frac{l_i \quad O_0}{O_* \quad O_0}, i=1,\cdots,n \tag{5}$$

$$y_i=\frac{w_j \quad O_0}{O_* \quad O_0}, j=1,\cdots,m \tag{6}$$

$$_1=\overset{n}{\underset{i=1}{}} \quad _i \tag{7}$$

$$_2=\overset{m}{\underset{i=1}{}} \quad _j \tag{8}$$

The level of common risk preference and speculative risk preference of decision maker can be decided by two standard no difference formulas

$$[0,\quad ,1]\sim[1,\quad] \tag{9}$$

$$[0,\quad ,1]\sim[1,\quad] \tag{10}$$

Solve the equations

$$(1\qquad)=0 \tag{11}$$

$$(1\qquad)=0 \tag{12}$$

we get two non 1 roots. The root of (11) is $_1$, the root of (12) is $_2$.

If p is has continuous distribution, and the For the risk measurement of continuous distribution *p* and probability density function is *f(t),* then the measurement formulation of risk *p* is as follows:

$$R(p)=\frac{(1\quad {}_2)(\ {}_1\quad {}^{0}\ {}_{1}\ {}^{\frac{t}{O^*}} f(t)dt)}{(1\quad {}_2)(\ {}_1\quad {}^{0}\ {}_{1}\ {}^{\frac{t}{O^*}} f(t)dt)+(1\quad {}_1)(\ {}_2\quad {}_{0}^{+}\ {}_{1}\ {}^{\frac{t}{O^*}} f(t)dt)} \qquad (13)$$

where ${}_1 = {}^{0} f(t)dt$, ${}_2 = {}_{0}^{+} f(t)dt$, for other parameters are the same with discrete distribution.

NEWSVENDOR MODEL

Basic newsvendor model

In the classical single-period single-item newsvendor problem, a newsvendor orders a certain amount of newspaper from his supplier with a unit ordering cost *c*. The newspaper is sold with a unit revenue *r*, where *r>c*. The unsold newspaper has a unit value *v* which is defined as the unit salvage value minus the unit holding cost. To avoid trivial cases, we have *v<c*. The overage cost c_0, i.e. cost per unit for a remaining inventory at the end of period is *c-v*. The underage cost c_u ,i.e. the cost per unit for unsatisfied period demand is *r-c*.

The newspaper's demand *D* is uncertain and follows a certain distribution with a known probability density function *f(x)* and cumulative distribution function *F(x)*. Before the day starts, the newsvendor, as the decision maker in the newsvendor problem needs to determine the order quantity of the newspaper. The order quantity is represented by *q*, and it is the decision variable in the newsvendor problem. With all these details, we can derive the expressions for the profit *P(q)*, expected profit *EP(q)*, as follows:

$$P(q)=(r\quad v)\min(q,D)\quad (c\quad v)q \qquad (14)$$

$$EP(q)=(r\quad c)q\quad (r\quad v)\ {}_{0}^{q} F(x)dx \qquad (15)$$

In the classical newsvendor model the optimal order quantity *q** is derived by maximizing the expected profit *EP(q)*. The optimality condition is given by

$$F(q^*)=\frac{r\quad c}{r\quad v} \qquad (16)$$

Risk Expression of Newsvendor Problem

Newsvendor is the actor of the action of newspaper order decision. Order action will both bring loss and benefit with some probability. We should first express the risk of order action as formula (1).

Assume order number is *q*, and demand is *x*. If *q>x*, then there is purely benefit, without loss. If *q<x*, then the income is $c_u \cdot\ x$, loss is $c_o \cdot\ (q\quad x)$. If income>loss then when demand is *x*, the result of order *q* decision is benefit. If income<loss when demand is *x*, the result of order *q* decision is benefit. For each q, we can find a demand *b* to make income=loss, call demand *b* as balance point.

$$b=\frac{c_0\cdot\ q}{c_u+c_0} \qquad (17)$$

Risk of order decision can be expressed in 3 pieces. The first piece is about loss. The probability density function of this piece is $f(x), x\quad (\quad ,b)$, and loss is $C_0(q\quad x)\quad C_u x$. The second piece is about benefit. The probability density function of this piece is $f(x), x\quad [b,q)$, and benefit is $C_0(q\quad x)+C_u x$, i.e. negative loss. The third piece is also about benefit. The probability of this piece is ${}_{q}^{+}\ f(x)dx$, and benefit is $C_u q$, i.e. negative loss.

The risk of action of order *q* newspapers can be expressed as

$$R(q) \sim [\underset{(\ \ ,b)}{f(x)}, C_0(q \quad x) \quad C_u x; \underset{(b,q)}{f(x)}, \ C_u x + C_0(q \quad x); \ {}_{q}^{+}\ f(x)dx, \ C_u q] \qquad (18)$$

CONCLUSION

We use a new risk measurement method to study the risk of newsvendor problem in this paper. This method helps us to build a relationship between subjective attitude of actor and the objective risk. We can analyze all kind of risk attitude. We get some finding by our study. And the most interesting one is the third one, That is change of *Δ R(q)* coincide with the phenomena in [9]. But we can't give further explanation from such as behavior or psychology aspect.

One problem of use this method in practice is how to judge risk preference of decision maker. From the perspective of psychological measurement, the process of get equivalent of gambling is a heuristics process. We should chose suitable gambling and point estimate method to assess .

This paper focuses on steady risk preference, and in practice many decision makers may have variable risk preference. It may be a new research direction.

References

Anthony A. Atkinson, Incentives, uncertainty, and risk in the newsboy problem, *Decision Sciences*, 10(3), 1979, pp. 341-357.

H. S. Lau, The newsboy problem under alternative optimization objectives, *Journal of the Operational Research Society*, 31(6), 1980, pp. 525-535.

L. Eeckhoudt, C. Gollier, and H. Schlesinger, The risk-averse (and prudent) newsboy, Management Science, 41(5), 1995, pp. 786-794, May 1995.

H. M. Markowitz, Portfolio selection: efficient diversification of investment. *New York: Wiley,* 1959.

Chen F, Federgruen A. , Mean–variance analysis of basic inventory models, *Working Paper, Columbia Business School, New York, USA*, 2000.

Tsan-Ming Choi, Duan Li, Houmin Yan, Mean-variance analysis for the newsvendor problem, *IEEE transactions on systems, man, and cybernetics-Part a: systems and humans*, 38(5), 2008, pp.1169-1180.

Jun Wu, Jian Li, Shouyang Wang, T.C.E. Cheng, Mean-variance analysis of the newsvendor model with stockout cost, *Omega-The international journal of management science,* 37, 2009, pp.724-730.

Jiang Qingfang, Chen Fangzhen, Risk measurement principle, *Shanghai Tongji University Press*, 2000

Maurice E. Schweitzer, Gerard P. Cachon, Decision bias in the newsvendor problem with a known demand distribution: experimental evidence, *Management Science*, 46(3), 2000, pp.404-420.

von Neumann J. and O. Morgenstern, Theory of Games and Economic Behavior, *Princeton, Princeton University Press,* 1947

Jiang Shu-yuan, Jiang Qing-fang, The Functional Expressions of Utility Measured as Constant Risk Preference and Parameters Determination, *Chinese Journal of Management Science*,15(1),2007,16-20