ISSN 2055-0286
Advances in Social Sciences Research Journal; Vol. 13 No. 01 (2026) (136-144)
https://doi.org/10.14738/assrj.1301.19910

Randomly Correlated Data

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Abstract: The random correlation data algorithm allows the creation of two randomly correlated variables having a relatively strong positive or negative correlation between them. We have used this algorithm in project management and, more specifically, in project portfolio selection. This paper describes the most important concepts and portrays the relevant equations for the algorithm being considered. The equations can be applied to other domains of inquiry. Also, although the random distribution generator was uniformly distributed, other random generators can be used, such as normally distributed random generators.

Keywords: Random, correlation, data, positive, negative.

INTRODUCTION. WHAT IS A PROJECT?

A project consisting in the combination of two or more mutually inclusive tasks with pre-specified precedence relationships may as a matter of fact be considered a single project.

But, what is a project? A project is an organized set of activities with finite duration to be performed, having a given purpose or goal (well defined set of final results desired), with some unique elements and interested parties (customers, parent organization, project team, and the public). *A project is the combination of interrelated activities that must be executed in a pre-specified sequence to complete a full task* (Meredith & Mantel, 2008).

The Probabilistic Critical Path Method (PCPM) measures the way in which a project is managed by using three dimensions: time in the form of a project schedule, which can be appreciated in a Gantt chart for planning purposes (Meredith & Mantel, 2008), cost in the form of a budget that may be higher or lower depending on whether the activities are crashed or not (Elsayed & Boucher, 1994), and finally performance, in the form of the Internal Rate of Return (IRR), which is compared with the Minimally Attractive Rate of Return or MARR (Brealey & Myers, 1991).

SYSTEMIC RELATIONSHIPS BETWEEN THE PROJECT DIMENSIONS: TIME, COST, AND PERFORMANCE

Although the relations among the project management dimensions vary from time to time and from project to project, a systemic approach can be used to elucidate the nature of the underlying balances (Icmeli, 1996; Johnson & Schou, 1990; Sunde & Lichtenberg, 1995).

Figure 1a illustrates the systemic relationships between time and cost using influence diagrams. If the project is delayed (it takes longer) will cost more, so that there is a positive correlation between time and cost. But if to deliver the project on time, additional resources are used for critical activities, maintaining resources to a minimum for non-critical activities (which is called *crashing*) there is a negative correlation between cost and time (Winston, 1994). The existence of both a positive and a negative correlation between time and cost implies the existence of a balance point in which an optimal project completion can be achieved at a minimum cost. Figure 1b illustrates how the time/cost balancing is additionally influenced by performance. Improving the quality of the product requires investing more resources, which will increase cost and increase time if those resources are limited. But if more resources are invested and it takes longer to complete the project, it costs more, so that the Internal Rate of Return (IRR) of the project measuring its profitability is reduced. Therefore, there must also be an optimal balance between time/cost achieving an optimal performance as measured according to the project's IRR.

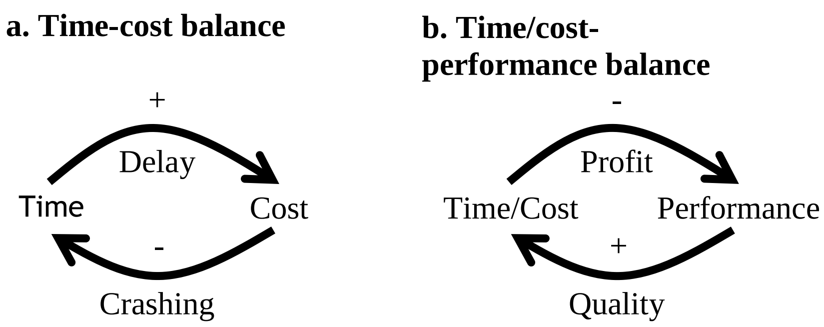

Figure 1: Balances among time, cost, and performance.

A PROJECT PORTFOLIO SELECTION MODEL

Considering a higher level of abstraction, we can have a set of projects to select or de-select. A project portfolio is a set of projects chosen to be carried out. Project selection is one of the first and most critical activities in project management. Deciding from a pool of available and competing projects which ones should be undertaken (thus assigning limited resources to them) and which ones should not be undertaken or terminated is a complex decision. Overall value maximization, balance among dimensions, and business strategy should be considered. The very essence of portfolio management portrayed by Cooper, Edgett and Kleinschmidt (2007) as a "dynamic decision process… constantly up-dated and revised… [where] new projects are evaluated, selected and prioritized; existing projects may be accelerated, killed or de-prioritized; and resources are allocated or re-allocated to the active projects" increases the difficulty. Furthermore, portfolio selection is a process characterized by uncertainty and changing information: new opportunities arise, multiple goals as well as strategic considerations are required, and interdependences among projects (either when competing for scarce resources or when synergies are achieved) exist, not to mention multiple decisionmakers and locations. Consequently, a mathematical model seems to be the best long-term approach to tackle such a complex decision-making process.

According to Meredith and Mantel (2008), project selection methods can be classified as nonnumeric (qualitative) or numeric (quantitative). The sacred cow, operating necessity, competitive necessity, product line extension and the comparative benefit model are among the qualitative methods. Profitability models* and scoring models† are among the quantitative methods.

A decision support system for project portfolio selection is presented by Archer and Ghasemzadeh (1999). There is no such thing as the optimal portfolio when we consider the tradeoffs among time, cost, and performance (not to mention risk preferences). Decisionmakers must weight multiple project dimensions and intuitively decide how adding or removing a specific project would have an impact on the portfolio. In other words, they face intuitive decisions on marginal contribution (gain or loss). Our conjecture is that *the best decision is achieved when overall cost and time are minimized while maximizing performance for a given risk profile*.

THE RELEVANT VARIABLES THROUGH AN ILLUSTRATIVE EXAMPLE

There are six basic (input) variables to consider for our portfolio selection model. The first such variable is the average (mean) completion time for each project. The Probabilistic Critical Path Method (PCPM) can be used to obtain the mean completion time of any given project as well as the related average cost and average rate of return (Copertari, 2020).

Also, we need the uncertainty associated to each of these three variables. Such uncertainty can be given as one or two times the standard deviation for each dimension (time, cost, and performance). Given the variance, the standard deviation is simply the square root of such variance. Thus, let k be any given project in a portfolio with a total of s projects. Then, t_k, c_k and i_k are the time, cost, and performance mean values (averages) for each project k, where k = 1, 2, …, s, respectively. Also, let Δt_k, Δc_k and Δi_k be the associated uncertainties for the time, cost, and performance dimensions, respectively, where k = 1, 2, …, s. For illustrative purposes, let us consider a portfolio of three alternative projects: Alpha, Beta and Gamma. Table 1 shows the relevant information.

Table 1: Small illustrative example.

k	Project	Time in weeks (t_k)	Time uncertainty (Δt_k)	Cost in dollars (c_k)	Cost uncertainty (Δc_k)	Performance in percentage (i_k)	Performance uncertainty (Δi_k)
1	Alpha	7	4	$2,000	$500	8%	2%
2	Beta	3	3	$1,500	$1,000	7%	3%
3	Gamma	10	4	$2,500	$500	5%	4%
	Average:	Irrelevant	Total:	$6,000	Average:	6.67%	

The portfolio's budget is $4,500. Since we are dealing with a portfolio, not a single project with a set of projects and a given precedence sequence for such projects, the

* Payback period, average rate of return, Net Present Value or NPV, Internal Rate of Return or IRR, profitability index, as well as others that subdivide the elements of the cash flow, include terms of risk or uncertainty, or consider the effect on other projects or the organization.
† Weighted and non-weighted factor models, with or without constraints, usually solved using integer programming as well as goal programming when multiple objectives are given.

average (or total) time dimension for the portfolio is irrelevant. However, the total (maximum) possible cost for the portfolio is important. Notice that such total is \$6,000, which is higher than the portfolio's budget. This means we cannot include all projects in the portfolio, but rather decide which ones should be undertaken.

THE RANDOM CORRELATION DATA EQUATIONS

It is possible to generate random data where the time dimension is positively correlated with the cost dimension to generate useful trial data. Let $Mint_k = 5$ and $Maxt_k = 50$ be the minimum and maximum possible time estimates. Also, let $Minc_k = 100$ and $Maxc_k = 1500$ be the minimum and maximum possible cost estimates. Finally, let $Mini_k = 5$ and $Maxi_k = 40$ be the minimum and maximum possible return rates.

How can we calculate the corresponding values for t_k, Δt_k, c_k, Δc_k, i_k and Δi_k, given these parameters? Let R be a 0-1 uniformly distributed random number such that $0 \leq R < 1$, R(a,b) be a uniformly distributed random number between a and b such that both a and b are integer values. Also, let the function f(x) indicate the rounding function given x such that if x = 5.3, f(5.3) = 5 and if x = 5.7, f(5.7) = 6, for example. Thus, R(a,b) = f(a+(b-a)R). Equation (1) shows how to estimate any given t_k, where k = 1, 2, …, s.

$$t_k = R(Mint_k, Maxt_k) \tag{1}$$

Equation (2) indicates how to calculate Δt_k given a previously calculated t_k.

$$\Delta t_k = f\Big(R\big(f(t_k/10), f(t_k/3)\big)\Big) \tag{2}$$

Calculating the value for c_k is the most complicated equation, because, in general, the values for t_k and c_k, although random, must have some degree of positive (or negative) correlation. Equation (3) indicates how to calculate c_k given a previously calculated t_k. Also, $Min\{t_k\}$ indicates the minimum value of all previously generated values for t_k, and $Max\{t_k\}$ indicates the maximum value of all previously generated values of t_k.

$$c_k = f\left(R(Minc_k, Maxc_k) \times \left(\frac{t_k - Min\{t_k\}}{Max\{t_k\} - Min\{t_k\}}\right) + Minc_k\right) \tag{3}$$

Unfortunately, equation (3) produces randomly correlated cost figures between $Minc_k$ and $Maxc_k + Minc_k$. To get randomly correlated data between $Minc_k$ and $Maxc_k$ we should modify equation (3) becoming equation (4).

$$c_k = f\left(R(0, Maxc_k - Minc_k) \times \left(\frac{t_k - Min\{t_k\}}{Max\{t_k\} - Min\{t_k\}}\right) + Minc_k\right), Maxc_k > Minc_k \tag{4}$$

Also, to get negatively random correlated data between t_k and c_k in the range $Minc_k$ and $Maxc_k$ for the cost figures, we should use equation (5).

$$c_k = f\left(R(0, Maxc_k - Minc_k) \times \left(\frac{Max\{t_k\} - t_k}{Max\{t_k\} - Min\{t_k\}}\right) + Minc_k\right), Maxc_k > Minc_k \tag{5}$$

Calculating Δc_k is like calculating Δt_k and the equation is shown as equation (6).

$$\Delta c_k = f\Big(R\big(f(c_k/10), f(c_k/3)\big)\Big) \tag{6}$$

Then comes how to calculate i_k. Remember that i_k is a percentage. Thus, a value given for $Mini_k$ = 5 means 5% and a value for $Maxi_k$ = 40 means 40%. Equation (7) shows how to calculate i_k.

$$i_k = f\big(R(Mini_i, Maxi_k)\big) \quad (7)$$

Finally, comes how to calculate Δi_k in equation (8).

$$\Delta i_k = f\Big(R\big(f(i_k/10), f(i_k/3)\big)\Big) \quad (8)$$

The projects are named Project k, where k = 1, 2, …, s, and as default, there are no pre-required projects or mutually exclusive projects.

There is a correlation between time and cost in the way the random data was generated. The Pearson correlation coefficient denoted as r (Walpole & Myers, 1989) was calculated between the time and the cost dimensions according to equation (9). Notice that $\bar{t}$ and $\bar{c}$ are the average duration time and the average cost as indicated in equations (10) and (11), respectively.

$$r = \frac{\sum_{k=1}^{s}(t_k-\bar{t})(c_k-\bar{c})}{\sqrt{\sum_{k=1}^{s}(t_k-\bar{t})^2 \sum_{k=1}^{s}(c_k-\bar{c})^2}} \quad (9)$$

$$\bar{t} = \frac{1}{s}\sum_{k=1}^{s} t_k \quad (10)$$

$$\bar{c} = \frac{1}{s}\sum_{k=1}^{s} c_k \quad (11)$$

SAMPLE OUTPUTS USING THE POSITIVE RANDOM CORRELATION AND THE NEGATIVE RANDOM CORRELATION ALGORITHMS

Following is output using the equations to generate a positive random correlation between t_k and c_k.

```
POSITIVE RANDOM CORRELATION
===========================
Give me the number of projects in the portfolio (2..254) ==> 20
Name         tk   Delta tk    ck     Delta ck   ik   Delta ik
Project 1    5    1           100    12         29   8
Project 2    6    2           107    30         15   2
Project 3    44   12          814    175        8    1
Project 4    14   2           128    39         27   3
Project 5    17   3           355    78         15   2
Project 6    35   7           183    43         36   10
Project 7    19   4           478    154        11   3
Project 8    12   2           262    68         15   3
Project 9    22   6           557    100        24   6
Project 10   24   4           573    58         39   8
Project 11   9    2           180    48         11   3
Project 12   26   4           288    95         10   2
Project 13   8    3           172    48         14   2
Project 14   43   7           907    249        11   3
Project 15   8    3           198    29         25   7
Project 16   18   6           397    53         22   3
Project 17   46   10          1498   469        15   2
Project 18   22   6           273    77         10   1
Project 19   40   11          936    222        26   3
Project 20   20   2           277    83         31   4
r = 0.8315
Press intro to continue...
```

Notice that the Pearson correlation coefficient for this set of data values between t_k and c_k equals 0.8315, which indicates a strong positive correlation. Also, notice that we used equation (4) instead of equation (3) to generate the cost figures, because the maximum cost occurs for Project 17 and it is \$1,498, which is less than the maximum allowed cost of \$1,500.

Next comes using equation (5) to generate negatively random correlated data between t_k and c_k. The relevant outcome is shown following. Notice that in this case the Pearson correlation coefficient (r) equaled -0.6630, which is a negative correlation, although not as strong. The reason such correlation is not as strong is because the data is randomly generated. The user could try other outcomes if a stronger correlation is wanted.

```
NEGATIVE RANDOM CORRELATION
===========================
Give me the number of projects in the portfolio (2..254) ==> 20
Name         tk   Delta tk   ck     Delta ck   ik   Delta ik
Project 1    5    1          383    46         29   8
Project 2    6    2          380    107        15   2
Project 3    44   12         137    30         8    1
Project 4    14   2          200    61         27   3
Project 5    17   3          716    157        15   2
Project 6    35   7          130    30         36   10
Project 7    19   4          829    267        11   3
Project 8    12   2          884    230        15   3
Project 9    22   6          745    133        24   6
Project 10   24   4          648    66         39   8
Project 11   9    2          845    224        11   3
Project 12   26   4          280    92         10   2
Project 13   8    3          1013   284        14   2
Project 14   43   7          164    45         11   3
Project 15   8    3          1344   196        25   7
Project 16   18   6          740    99         22   3
Project 17   46   10         100    31         15   2
Project 18   22   6          344    98         10   1
Project 19   40   11         243    58         26   3
Project 20   20   2          407    123        31   4
r = -0.6630
Press intro to continue...
```

DISCUSSION AND CONCLUSION

Why do equations (4) and (5) work? Let us consider our small illustrative example from Table 1. The values given for t_k are $t_1 = 7$, $t_2 = 3$ and $t_3 = 10$. Let us assume we want to create new cost figures using equations (4) and (5). The minimum value we want to assign is $Minc_k$ = \$100 and the maximum value we want to assign is $Maxc_k$ = \$1,500.

We begin analyzing equation (4). This could happen if the time and cost figures are positively correlated due to delay (refer to Figure 1a). The function R(0,1500-100) = R(0,1400) generates uniformly distributed random numbers between 0 and 1400. The minimum such value could be 0 and the maximum such value could be 1400. We multiply that randomly generated number by the factor $(t_k\text{-Min}\{t_k\})/(\text{Max}\{t_k\}\text{-Min}\{t_k\})$ and finally we add to the result the value 100. We will see that the factor just described is a number between 0 and 1. The factor equals 0 if the time it corresponds to is the minimum time and the factor equals 1 if the time it is related to is the maximum time. We have that $\text{Min}\{t_k\}$ = Min{7,3,10} = 3 and $\text{Max}\{t_k\}$ = Max{7,3,10} = 10. Thus, for t_2 we have the following: (3-3)/(10-3) = 0/7 = 0, which makes sense since t_2 is the lowest time estimate and for t_3 we have (10-3)/(10-3) = 7/7 = 1, which also makes sense since t_3 is the highest value for the given time figures. Regardless of the outcome given by the uniformly generated random function, the lowest time estimate will always yield the lowest cost: R(0,1400)×((3-3)/(10-3))+100 =

R(0,1400)×0+100 = \$100. The highest time estimate is a little different. It depends on the value given by the uniformly distributed random function. Thus, we have: R(0,1400)×((10-3)/(10-3))+100 = R(0,1400)×1+100. We do not know a priori the value the uniformly distributed random function will give. Only that its minimum possible value is 0 and its maximum possible value is 1400. If it gives the minimum possible value the cost associated to the largest time figure would be 0x1+100 = \$100. But if it gives the largest possible value the cost associated would be 1400×1+100 = 1500, which is the highest cost we would want. Consequently, the highest time figure would tend to have a large cost figure associated to it. To illustrate further, let us assume the outcome given by the uniformly distributed random function is given by its mean. The mean for the function R(0,1400) is the value between the extreme points. In this case, we would have (1400+0)/2 = 1400/2 = \$700. Thus, for t_3 = 10 we would have \$700×1+\$100 = \$800. What happens with the intermediate value given as t_1 = 7? We would have R(0,1400)×(7-3)(10-3)+100 = \$700×(4/7)+\$100 = \$500. We can see that the lowest time estimate (t_2 = 3) has the lowest cost (c_2 = \$100), the highest time estimate (t_3 = 10) is associated (on average) with the highest cost estimate (c_3 = \$800) and the intermediate cost estimate (t_1 = 7) is associated (also on average) with an intermediate cost estimate (c_1 = \$500). In this case, the Pearson correlation coefficient (r) would be equal to 1. In practice, the actual correlation coefficient would be less than 1 because the uniformly distributed random function would yield randomly distributed numbers, not a single number (\$700). The reason we have a perfect correlation coefficient of 1 when using the average instead of the uniform random distribution is because equation (4) becomes equation (12), where α = \$100, β = \$700 and x are all the factors multiplying β. Notice equation (12) corresponds to the equation of a straight line with a positive slope.

$$y = \alpha + \beta x \tag{12}$$

What about equation (5) for negatively random correlated data? (refer to Figure 1a for crashing, when there is a negative correlation between time and cost). In that case, the highest time estimate (t_3 = 10) would have associated to it the lowest cost, that is: R(0,1400)×((10-10)/(10-3))+100 = R(0,1400)×0+100 = \$100. The lowest time estimate (t_2 = 3) would have associated to it (on average) the largest cost figure, that is: R(0,1400)×((10-3)/(10-3))+100 = \$700×1+\$100 = \$800. The intermediate time estimate (t_1 = 7) would have (on average) associated to it an intermediate cost estimate, that is: R(0,1400)×((10-7)/(10-3))+100 = \$700×(3/7)+\$100 = \$400. Thus, the Pearson correlation coefficient would be r = -1. In practice, that correlation coefficient would be less than perfect negative correlation because the uniformly distributed random function would give uniformly distributed random numbers. The reason in this other case there is a perfect negative correlation coefficient when we use the average value for the uniformly distributed random function is because equation (5) becomes equation (13), which is also a straight line with a negative slope, where α = \$100, β = \$700, and (1-x) are all the factors multiplying β. If we define x to be equal to (t_k-Min{t_k})/(Max{t_k}-Min{t_k}), then we can see (1-x) = 1-(t_k-Min{t_k})/(Max{t_k}-Min{t_k}) = (Max{t_k}-Min{t_k})/(Max{t_k}-Min{t_k}) - (t_k-Min{t_k})/(Max{t_k}-Min{t_k}) = (Max{t_k}-~~Min{t_k}~~-t_k+~~Min{t_k}~~)/(Max{t_k}-Min{t_k}) = (Max{t_k}-t_k)/(Max{t_k}-Min{t_k}), which is precisely how the factor in equation (5) is defined.

$$y = \alpha + \beta(1 - x) = (\alpha + \beta) - \beta x \tag{13}$$

The data randomly generated in section 6 shows first the case of positive correlation (r = 0.8315) and second a case of negative correlation (r = -0.6630). These correlations are not perfect because the uniformly distributed random function generates values between $0 and $1,400, but still, the trend is such that there is a positive and a negative correlation in each case associated to the use of equations (4) and (5), respectively.

In conclusion, we can see that both equations (4) and (5) work as long as it remains true that $Minc_k < Maxc_k$, that is, when the minimum possible cost is less than the maximum possible cost, because otherwise, the uniformly distributed random function would have a negative high limit. We can apply the ideas behind equations (4) and (5) to other domains of inquiry when we require a pair of positively or negatively random correlated variables. Also, instead of using a uniformly distributed random function R(a,b), other randomly distributed functions, such as a normally distributed random function $G(\mu,\sigma)$, where μ is the mean and σ the standard deviation, could be used, yielding, if desired, more perfectly correlated functions if the standard deviation is relatively small compared to the mean.

A standard normally distributed random function, Z, can be generated using equation (14), where R is a 0-1 uniformly random distribution such that $0 \le R < 1$ (Coss Bu, 1991).

$$Z = R + R + R + R + R + R + R + R + R + R + R + R - 6 = \sum_{i=1}^{12} R - 6 \tag{14}$$

A Gaussian or normally distributed random function, $G(\mu,\sigma)$, can be generated using the value (Z) generated in equation (14) and applying equation (15), where μ is the mean and σ the standard deviation (Kvanli, Guynes, & Pavur, 1989).

$$G(\mu, \sigma) = \mu + \Sigma z \tag{15}$$

Notice that the standard normally distributed random values Z obtained using equation (14) have (on average) a mean of 0 and a standard deviation equal to the variance of 1. Also, notice that all the values R in equation (14) are different 0-1 uniformly distributed random variables. Other random distributions are also possible (Copertari, 2025).

However, applying equations (4) and (5) for positively and negatively randomly correlated Gaussian functions, respectively, is not as easy as applying it for uniformly distributed random variables. That is because the Gaussian function varies between $-\infty$ and $+\infty$ and not between $Minc_k$ and $Maxc_k$. However, it is well known that between $\mu+3\sigma$ and $\mu-3\sigma$ there are 99.73% of all data values. Thus, we could set the lower interval to $a=Minc_k$ and the upper interval to $b=Maxc_k$. Since we are multiplying these values by 0 or 1 for the lowest and highest values in a positive correlation (or alternatively by 1 and 0 for a negative correlation) the actual upper interval is b-a, whereas the lower interval remains being a, because at the end of equations (4) and (5) we are adding $a=Minc_k$. Consequently, we have $a = \mu-3\sigma$ and $b-a = \mu+3\sigma$. As a result, $(b-a)-a = b-2a = (\mu+3\sigma)-(\mu-3\sigma) = \cancel{\mu}+3\sigma-\cancel{\mu}+3\sigma = 6\sigma$. The standard deviation for the randomly correlated Gaussian function is given by equation (16).

$$\sigma = \frac{b-2a}{6}, b > 2a, a = Minc_k, \text{and } b = Maxc_k \tag{16}$$

Also, the mean is given by the average of these two extreme values: a and b-a, which is indicated in equation (17).

$$\mu = \frac{a+(b-a)}{2} = \frac{b}{2}, b = Maxc_k \tag{17}$$

Equations (4) and (5) then become equations (18) and (19) for Gaussian positively and negatively random correlated data, respectively.

$$c_k = f\left(G(\mu, \sigma) \times \left(\frac{t_k - Min\{t_k\}}{Max\{t_k\} - Min\{t_k\}}\right) + Minc_k\right) \quad (18)$$

$$c_k = f\left(G(\mu, \sigma) \times \left(\frac{Max\{t_k\} - t_k}{Max\{t_k\} - Min\{t_k\}}\right) + Minc_k\right) \quad (19)$$

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