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Competition between Multiproduct Firms with Heterogeneous Costs

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Abstract

This paper draws upon Feenstra and Ma (2007, 2008), to develop a model of asymmetric competition between multiproduct firms. The model is used to analyze how cost asymmetry affects the equilibrium, with determination of quantity/price as well as product scope per firm. By treating the number of firms as a continuous variable, the model is extended to account for the endogenous determination of the number of firms in a long-run, monopolistically competitive equilibrium, with free entry by heterogeneous firms.

Keywords: Multiproduct firm, monopolistic competition, product scope, cost asymmetry.

INTRODUCTION

A recent literature on international economics has focused on the effects of trade openness when firms are heterogeneous and multiproduct, since the seminal work by Melitz (2003). A number of oligopolistic and monopolistically competitive models have been proposed, and used to assess the theoretical implications of operating in a larger market and/or exposing domestic firms to international competition (Neary, 2009). The same models could be fruitfully be employed to explore other interesting issues, which may be of less interest for international economics but still very relevant for other fields, like general industrial economics.

Oligopolistic models for multiproduct firms, for example those introduced by Feenstra and Ma (2007, 2008), Eckel and Neary (2010), Luong (2010), allow for the endogenous determination of the number of product varieties offered by the same firm (product scope). The optimal product scope is then found where marginal profits of expanding scope are zero. Decreasing returns to scope are obtained in Eckel and Neary (2010) by assuming that firms possess a "core competence" in the production of a particular variety, becoming less efficient as more varieties are produced. Luong10 takes a different but somehow equivalent approach, by assuming that managing multiple brands requires organizational and managerial skills, which are scarce resources subject to decreasing marginal productivity.

By contrast, Feenstra and Ma (2007, 2008) obtain decreasing returns to scope by just relaxing the simplifying assumption, usually adopted in most models based on the Dixit-Stiglitz framework, of ignoring own-price effects on the aggregate price index. Departing from this assumption may be important when firms cover a non negligible share of the market, and this is likely to be the case when firms produce several products, rather than just one. When new products are added, demand for all existing varieties, including those produced by the same firm, decreases. This effect, which is sometimes referred to as "cannibalization", reduces the marginal benefit of expanding the product scope and, since the cannibalization effect is stronger when more varieties are in place, this generates decreasing returns to scope.

The Feenstra and Ma model is sufficiently general and analytically tractable. In this work, I present the basic setting of this model, with only minor modifications, with the aim of exploring equilibrium in the market when firms have different production costs and make different choices about the product scope. I then extend the basic model to account for free entry and monopolistic competition

Understanding the strategic role played by variations in the range of offered products may be especially important for some markets, like those of media services. Many differentiated services are provided on the Internet, where large multiproduct platforms, like Google, may coexist with smaller providers, typically focusing on one type of service. Another example is advertisement-based television broadcasting. In Italy (as well as in most European countries), this industry is concentrated, with three players covering much of the market. After the transition from analog to digital broadcasting, which allows for the existence of many more channels into the same frequency spectrum, the former State-owned monopoly RAI expanded its supply from 3 to 11 channels. Private companies Mediaset and Telecom Italia increased their number of channels from 3 to 7 and from 1 to 2, respectively.

Theoretical models like the one described in this paper can provide a conceptual framework to better understand the strategic response obtained through variations in the product scope. As a key characteristic of many markets in which multiproduct firms compete is firms' heterogeneity, it is also important to explicitly address the issue of asymmetric equilibria.

This is the primary aim of the paper, which is organized as follows. The next section introduces the model and illustrates its structure. Section three is devoted to a qualitative analysis of an asymmetric duopolistic equilibrium, which is done through a numerical example. The model is then extended in section four, to allow for free entry by heterogeneous firms in a monopolistic competition setting. A final section concludes.

THE MODEL

Basic Setting

We present here the basic assumptions and some preliminary results of the model, derived from Feenstra and Ma (2007, 2008).

There is a market, where a continuum of N differentiated goods or services, indexed i, is supplied. Total aggregate expenditure in the market, R, is given. The sub-utility function of a representative consumer in the market is expressed as a CES function:

$$U = \left[\int_0^N {q_i}^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \quad (1)$$

The goods or services are produced by *M* firms, indexed *j*, each one supplying N_j varieties. Therefore:

$$\sum_1^M N_j = N \quad (2)$$

or, if even the number of firms is treated as a continuous variable:

$$\int_0^M N_j dj = N \quad (3)$$

Maximization of (1) under budget constraint (over R) gives raise to the standard expression for CES demand functions:

$$q_i = \frac{R}{P^{1-\epsilon}} p_i{}^{-\epsilon} \qquad (4)$$

where P is the CES price index:

$$P = \left[\int_0^N p_i^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}} \qquad (5)$$

With a discrete and finite number M of firms, assuming that all varieties produced by each firm j are priced the same at p_j, (5) can be also expressed as:

$$P = \left[\sum_1^M N_j p_j{}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \qquad (6)$$

It is easy to show that the perceived price elasticity of demand for each good produced by the j-th firm is linked to the market share s_j of that firm, that is:

$$\frac{\frac{\partial q_j}{q_j}}{\frac{\partial p_j}{p_j}} = -[\epsilon(1 - s_j) + s_j] \qquad (7)$$

Where:

$$s_j = \frac{N_j q_j p_j}{\sum_1^M N_j q_j p_j} = \frac{N_j p_j^{1-\epsilon}}{\sum_1^M N_j p_j^{1-\epsilon}} \qquad (8)$$

Production takes place on the basis of a technology, involving constant marginal costs c_j per variety (possibly differing by firm, not by variety), fixed costs per variety (possibly scope-dependent, therefore marginal costs in terms of scope)[1] $F_j(N_j)$, and total "headquarters" fixed costs h_j. Therefore, profits of a representative firm are given by:

$$\Pi_j = N_j[q_j(p_j - c_j) - F_j(N_j)] - h_j \qquad (9)$$

Profit maximization brings about the standard mark-up rule, where the elasticity (7) is taken into account:

$$\Pi_j = N_j[q_j(p_j - c_j) - F_j(N_j)] - h_j \qquad (10)$$

This result deserves some comment. The higher the market share of a firm, the lower the perceived elasticity (7), the higher the price is set (10). This is because a variation in the price of a specific variety changes its demand but also changes, in opposite way, the demand for all

[1] In the original formulation by Feenstra and Ma this cost is constant.

other varieties produced by the same firm. The latter effect is negligible only if the market share is very low, so that the firm is "small" in the market. Also, notice that profit-maximizing prices depend on market shares (10), but shares themselves depend on prices (8). Taking the number of firms M and varieties N_j as given, the market equilibrium is found by simultaneously solving (8) and (10) (determining p_j and s_j) for all firms in the market.

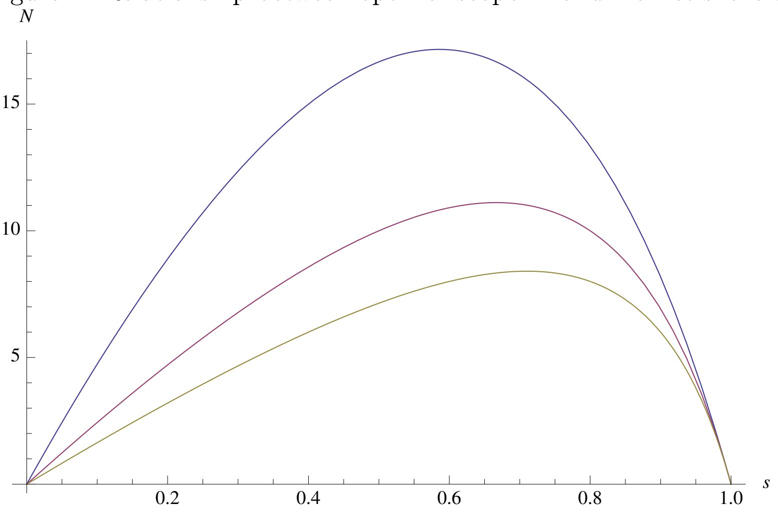
Feenstra and Ma (2007, 2008) derive an optimality condition for the profit maximizing choice of N_j. Interestingly, the optimal number of varieties (or "scope") is also a function of a firm market share:

$$N_j = \left[\frac{s_j(1-s_j)}{\epsilon(1-s_j)+s_j}\right]\frac{R}{f_j} \qquad (11)$$

Where $f_j = {}^{dF_j}/_{dN_j}$ is the cost of adding one more variety.

The optimal scope N_j is strictly increasing in R, decreasing in F_j and ε. These relationships are all easy to interpret. The relationship between N_j and s_j, on the other hand, is not a trivial one. Figure 1 plots the optimal N_j as a function of the market share $0 \leq s_j \leq 1$, for arbitrary values of R and f_j, and various values of ε.

Figure 1: Relationship between optimal scope N and market share s



The optimal N_j is first increasing, then decreasing, reaching a maximum whose position depends on ε. This is because there are two forces at work. A higher market share reduces the perceived elasticity in (10), thereby determining higher mark-ups per variety. This induces to expand the scope N_j . On the other hand, adding one more variety reduces demand for all existing varieties. This effect is stronger when the market share is quite significant and prevails over the previous one for sufficiently large values of s_j.

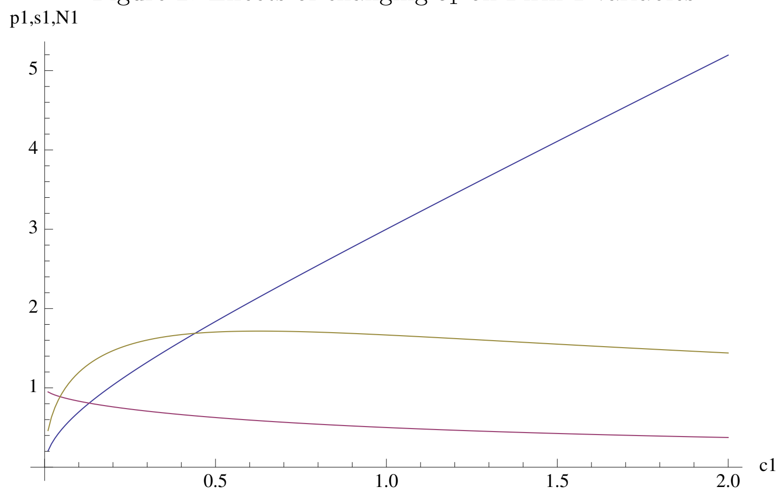
With the addition of (11) it is possible to identify a (short-run) equilibrium in a market of "size" R, where M firms are active.[2] Each firm is characterized by its cost parameters $(c_j, F_j(N_j), h_j)$, and it is associated with equilibrium conditions (8), (10) and (11). In other words, finding the market equilibrium entails solving a system of $3M$ equations, for the determination of the endogenous variables p_j, s_j, N_j.Quantities q_j and profits Π_j immediately follow on the basis of (4) and (9).

An Asymmetric Duopoly

To illustrate the properties of the oligopolistic equilibrium as described in the previous section, and in particular the effect of cost differentials on market asymmetry, let us consider a duopoly with parameters specified as follows: $\varepsilon = 2, P = 10, c_2 = 1, f_2 = 1$. If we also set $c_1 = 1$ and $f_1 = 1$, we get the symmetric equilibrium with $p_1 = p_2 = 3$, $N_1 = N_2 = 1.67$ and, of course, $s_1 = s_2 = 0.5$.

Now, keep $f_1 = 1$ and analyze how the equilibrium variables change for different values of the marginal cost c_1 of the first firm. Figure 3 shows how the price p_1, the share s_1 and the scope N_1 would vary.

Figure 2: Effects of changing c_1 on Firm 1 variables



Figures 3, 4 and 5 display the effects of varying c_1 on: prices set by the two firms (Figure 3), market shares (Figure 4) and number of varieties provided by the two firms (Figure 5).

Figure 3: Effects of changing c_1 on prices

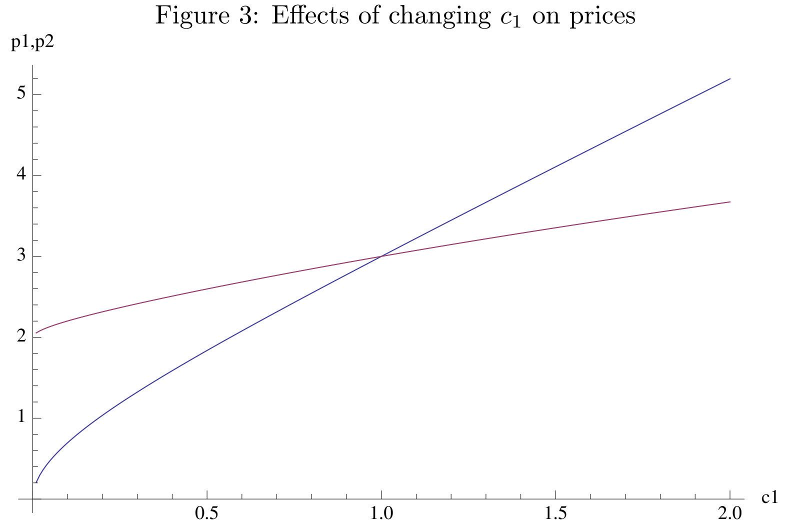


Figure 4: Effects of changing c_1 on market shares

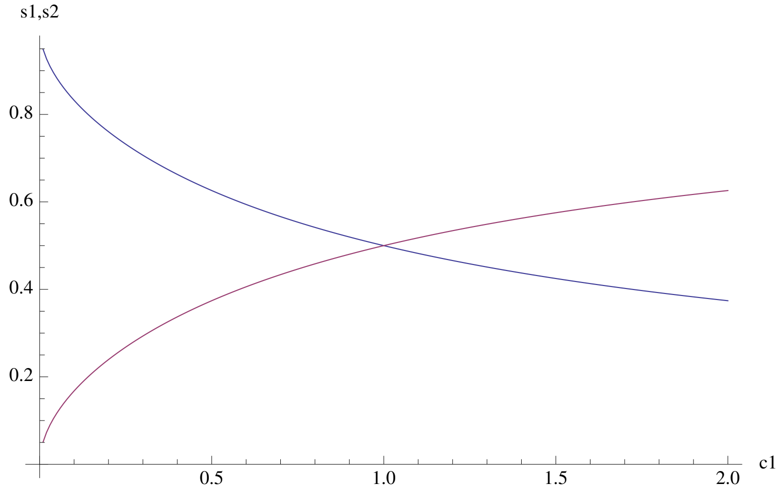
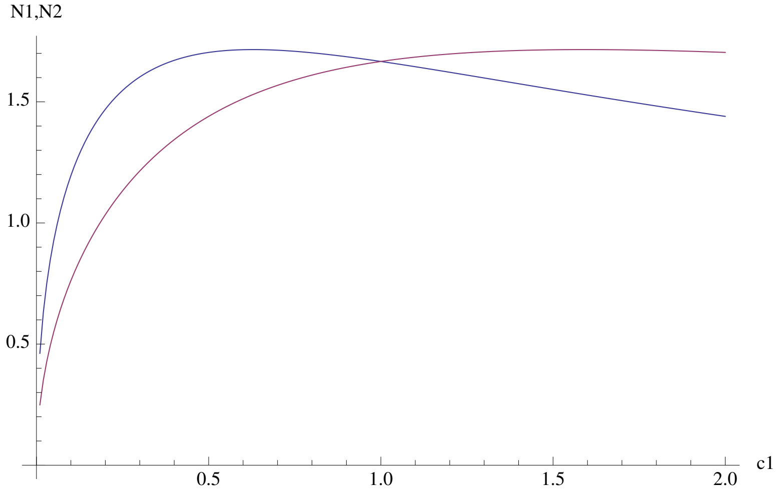


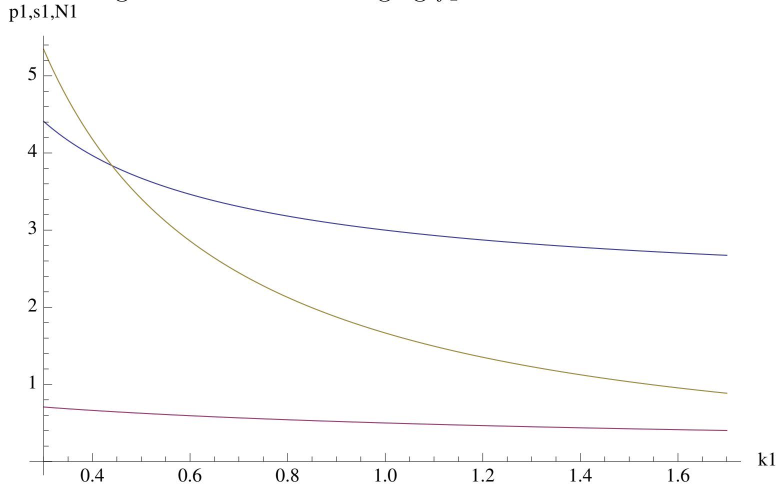
Figure 5: Effects of changing c_1 on product scopes



When c_1 increases, prices set by both firms increases because of the direct effect of variable costs (firm 1) and because prices are strategic complements (firm 2). The market shares move symmetrically, and when the marginal cost of the first firm approaches zero, its market share approaches one. The evolution of the variables N_j is more complicated. Both N_1 and N_2 are concave functions of c_1, reaching maxima at $N_1^{MAX} < N_2^{MAX}$. Consequently, when c_1 is sufficiently smaller or sufficiently larger than c_2, the number of products put on the market, N_1 and N_2, move to the same direction, that is, an increase in N_1 is associated with an increase in N_2, and vice versa. However, for intermediate values of c_1, N_1 and N_2 move in an opposite way.

A similar comparative exercise can be undertaken by fixing c_1=1, and observing how the variables of interest change for various values of the marginal scope cost f_1. Figure 6 is analogous of Figure 3 and shows how the price p_1, the share s_1 and the scope N_1 would vary with f_1.

Figure 6: Effects of changing f_1 on Firm 1 variables



The main effect of a higher set-up cost f_1 is reducing the number of product varieties offered by the first firm. With a lower N_1, the market share s_1 declines, increasing the perceived demand elasticity, which reduces the price p_1. This implies an increase in the quantity volumes q_1 which partly compensates for the fall in market share due to the reduction in product varieties and price. For this reason, the market share s_1 appears to be less sensitive to f_1 than to c_1.

Figures 7, 8 and 9 show the effects of varying f_1 on the corresponding variables (p_i, s_j, N_j) of the two firms.

Figure 7: Effects of changing f_1 on prices

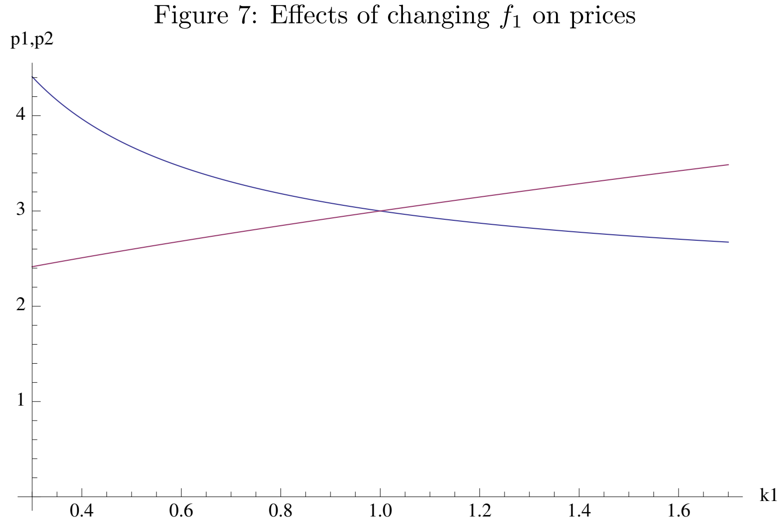


Figure 8: Effects of changing f_1 on market shares

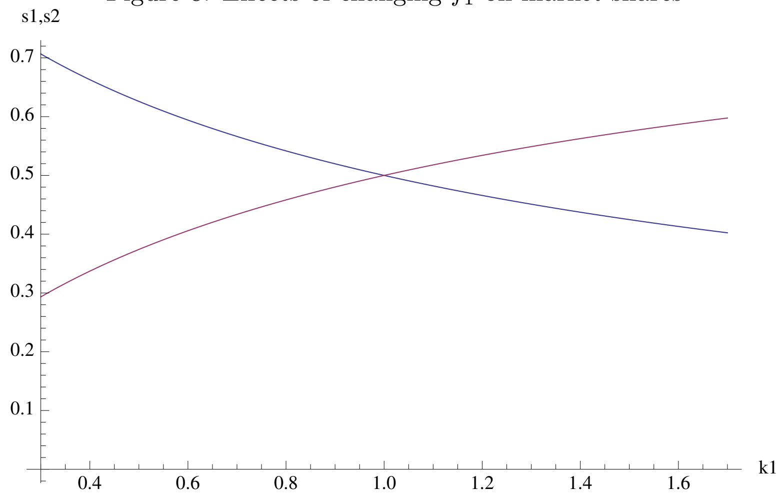
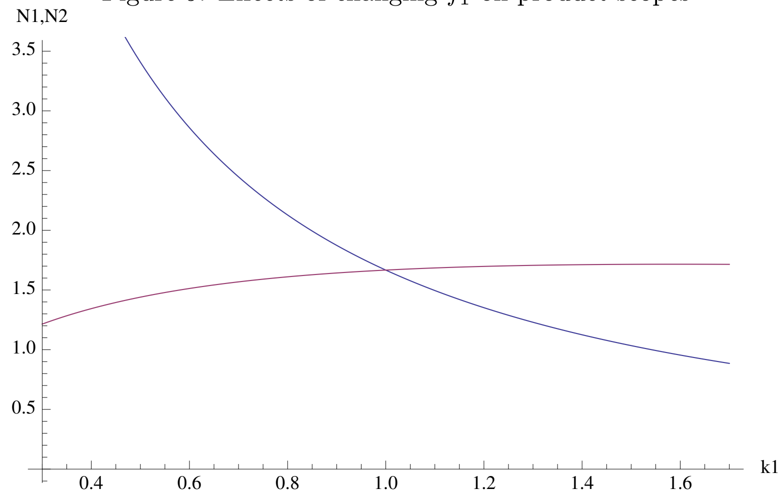


Figure 9: Effects of changing f_1 on product scopes



The main difference here, with respect to the case of changing c_1, can be seen in the variation of prices. Whereas with a higher c_1 we noticed that both prices increase (Figure 3), now a higher f_1 induces a *reduction* in the price p_1, but still an increase in the price p_2. Indeed, in both cases higher costs bring about a reduction in market share s_1 and an increase in s_2 (therefore, in p_2). However, whereas c_1 directly affects p_1, the main impact of f_1 is on N_1, which is partly compensated through adjustments in the price p_1.

Monopolistic Competition

We consider now a monopolistic competition setting, with free entry driven by expected profits. Firms are assumed to have different marginal production costs c_j but, to simplify, have the same cost sub-function $F_j(N_j)$ and no headquarters costs h_j. Unlike Feenstra and Ma (2007, 2008)[3] but similarly to Montagna (1995) we assume that firms are continuously distributed over a cost range, so that $G(c)$ expresses the density of firms having marginal production cost c.

When the optimal product scope N_j is chosen (see (11)) the following condition holds, for positive N_j:

$$q_j(p_j - c_j)(1 - s_j) = f_j = \frac{dF_j}{dN_j} \qquad (12)$$

By combining the latter equation with (4) and (10) it is possible to derive a relationship, linking the marginal cost c_j of a firm to its market share s_j:

$$c_j = P\left[\frac{1}{(\epsilon-1)(1-s_j)} + 1\right]^{\frac{\epsilon}{1-\epsilon}} \left[\frac{R}{(\epsilon-1)f_j}\right]^{\frac{1}{1-\epsilon}} \qquad (13)$$

Entry will occur for all firms having positive profits, which applies to those firms having production costs c_j lower than a cutoff level c_0. The latter is easily identified, by noting that the least productive firm will have a zero market share, so that (13) can be applied with $s_o = 0$. Furthermore, the market share of a firm can be expressed in terms of its relative cost ratio:

$$\frac{c_j}{c_0} = \left[\frac{1 + \frac{1}{(\epsilon-1)(1-s_j)}}{1 + \frac{1}{(\epsilon-1)}}\right]^{\frac{\epsilon}{1-\epsilon}} \qquad (14)$$

Which implies (with $c_j \leq c_0$):

$$s_j(c_j) = \left[1 - \frac{1}{1 + \epsilon\left(\left(\frac{c_0}{c_j}\right)^{\frac{\epsilon-1}{\epsilon}} - 1\right)}\right] \qquad (15)$$

Observe that $s_j'(c_j) < 0$ and $s_j(c_0) = s_o = 0$.

In a monopolistic competition model, the number M of active firms is endogenously determined through the free entry condition, which in this case amounts to selecting the cutoff cost c_o in such a way that the sum (integral) of market shares sums up to one. This condition is:

$$\int_{\underline{c}}^{c_0} s_j(\gamma)G(\gamma)d\gamma = 1 \qquad (16)$$

where $\underline{c}>0$ is the minimum marginal cost. Equation (16) can be solved to find c_o. As a consequence:

$$M = \int_{\underline{c}}^{c_0} G(\gamma)d\gamma \qquad (17)$$

For example, suppose that $\underline{c}=0$, $\varepsilon=2$, and firms are uniformously distributed with density G, that is $G(c)=G$. Then (15) becomes:

$$s_j(c_j) = \frac{2\left(\sqrt{c_0} - \sqrt{c_j}\right)}{2\sqrt{c_0} - \sqrt{c_j}} \qquad (18)$$

and (16) becomes:

$$6c_0G - 8\ln 2c_0G = 1 \qquad (19)$$

bringing about:

$$c_0 = \frac{1}{(6 - 8\ln 2)G} \qquad (20)$$

$$M = c_0 G = \frac{1}{6 - 8\ln 2} \quad (21)$$

In this setting, each of the M active firms is associated with a marginal cost $\underline{c} \geq c_j \geq c_0$. Its market share is determined on the basis of (15). The number of product varieties is set through (11), and the price of each product is given by (10).

CONCLUSIONS

Multiproduct firms are not just large scale clones of single product firms. When market conditions change, firms revise their policy in terms of price, production volume, quality, etc., but also in terms of the number of offered products. The latter effect is especially important in many service industries (telecommunications and media, in particular). Unfortunately, the industrial organization literature on multiproduct firms and endogenous scope choice is quite thin. Strategic scope choice has been studied mostly in relation with entry deterrence (since Judd (1985)) or as a response to entry (Johnson and Myatt, 2003). The determination of market structure when firms are heterogeneous and multiproduct has not been directly addressed, to the best of my knowledge.

Fortunately, some new models proposed in the field of international economics have the potential to fill the gap. In this work, I have discussed a recent model by Feenstra and Ma (2007, 2008), which I have extended and used to address the issue of asymmetry in equilibrium. Cost differentials are at the basis of differences in strategic responses by the firms. However, costs in multiproduct firms are multidimensional, as they refer to the production of a specific good, or to the addition of a new product line, or to fixed headquarters costs. Firms may differ in all these dimensions. Depending on the nature of these differences, a different asymmetric market structure may emerge.

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