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A Hybrid NSGA-II - AHP Approach for Multi-Objective Optimization of Two-Stage Helical Gearboxes: Cross-Sectional Area-Efficiency Trade-Off

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Abstract: In modern mechanical systems, the design of two-stage helical gearboxes requires a careful balance between compactness and performance. This study presents a hybrid multi-objective optimization approach integrating the Non-dominated Sorting Genetic Algorithm II (NSGA-II) and the Analytic Hierarchy Process (AHP) to simultaneously minimize the cross-sectional area and maximize the efficiency of two-stage helical gearboxes. A comprehensive mathematical model is developed to describe the relationships between design variables, objective functions, and engineering constraints. The NSGA-II algorithm is employed to generate a Pareto-optimal set of feasible solutions, revealing the trade-off characteristics between structural compactness and transmission efficiency. Subsequently, AHP is applied as a decision-making tool to identify the most suitable solution from the Pareto front based on relative importance weights of the objectives. The results demonstrate that the proposed hybrid framework effectively provides a diverse set of optimal solutions and supports rational decision-making in gearbox design. The study offers a practical and systematic methodology for achieving an optimal balance between size reduction and efficiency improvement, contributing to the development of high-performance and compact transmission systems.

Keywords: Multi-objective optimization, NSGA-II, Analytic Hierarchy Process (AHP), Two-stage helical gearbox, Design optimization, Cross-sectional area, Efficiency, Trade-off analysis.

INTRODUCTION

Gear transmission systems, particularly two-stage helical gearboxes, are widely used in industrial machinery due to their high load-carrying capacity, smooth operation, and transmission efficiency. However, modern engineering applications increasingly demand not only high efficiency but also compact structural design to reduce material consumption, installation space, and overall system cost. Consequently, the design of gearboxes has evolved into a complex multi-objective optimization problem, where conflicting objectives such as minimizing size and maximizing efficiency must be addressed simultaneously [1], [2].

Traditional gearbox design methods often rely on deterministic or single-objective approaches, which are insufficient to capture the inherent trade-offs among multiple performance criteria. Early studies focused on optimizing individual aspects such as gear geometry or weight using classical techniques [3], while later works incorporated heuristic and evolutionary approaches to automate gear design processes [4], [5]. The development of metaheuristic algorithms has significantly enhanced the capability to solve complex engineering optimization problems, especially in the context of multi-objective design [6], [7].

Among these methods, the NSGA-II has become one of the most widely used algorithms due to its fast convergence, elitism mechanism, and ability to maintain diversity in Pareto-optimal solutions [8]. NSGA-II has been successfully applied to various gearbox optimization problems, including spur and helical gear systems, to address multiple objectives such as weight reduction, efficiency improvement, and vibration minimization [9], [10], [11]. More recent studies have demonstrated the effectiveness of evolutionary algorithms in generating a diverse set of optimal solutions for gearbox design, enabling engineers to explore trade-offs between competing objectives [12], [13].

In parallel, several studies have focused specifically on the multi-objective optimization of two-stage helical gearboxes. For instance, Le and Vu [14] applied the Taguchi method combined with grey relational analysis to optimize gearbox performance, while Dinh et al. [15], [16] employed multi-criteria decision-making (MCDM) techniques such as TOPSIS and MARCOS to select optimal solutions from a set of alternatives. Similarly, Tran et al. [17] and Maputi and Arora [18] integrated optimization algorithms with decision-making methods to enhance the practical applicability of optimization results. These studies highlight the importance of combining optimization algorithms with decision-support tools to obtain engineering-feasible and practically meaningful solutions.

Despite these advances, several limitations remain. First, many existing studies either rely on traditional optimization techniques or focus on limited objective functions, without fully exploring the trade-off between structural compactness (e.g., cross-sectional area) and efficiency in a unified framework. Second, although NSGA-II has been widely used, the selection of the most suitable solution from the Pareto front is often overlooked or handled subjectively. Third, while various MCDM methods have been applied, the integration of NSGA-II with the Analytic Hierarchy Process (AHP)-a well-established method for structured decision-making-remains limited in gearbox design problems [19].

To address these gaps, this study proposes a hybrid NSGA-II-AHP approach for the multi-objective optimization of two-stage helical gearboxes. The objective is to simultaneously minimize the cross-sectional area and maximize the transmission efficiency while satisfying engineering constraints. NSGA-II is employed to generate a Pareto-optimal set of feasibility, capturing the trade-off characteristics between the objectives. Subsequently, AHP is applied to systematically evaluate and rank the solutions based on their relative importance, enabling the selection of the most appropriate design.

The main contributions of this study can be summarized as follows:

(i) development of a comprehensive mathematical model for the multi-objective design of two-stage helical gearboxes;

(ii) application of NSGA-II to obtain a diverse and well-distributed Pareto front;

(iii) integration of AHP as a decision-support tool to select the optimal solution; and

(iv) provision of engineering insights into the trade-off between gearbox compactness and efficiency. The proposed framework offers a practical and effective methodology for advanced gearbox design and can be extended to other mechanical systems with conflicting design objectives.

OPTIMIZATION PROBLEM

Cross-Sectional Area Calculation

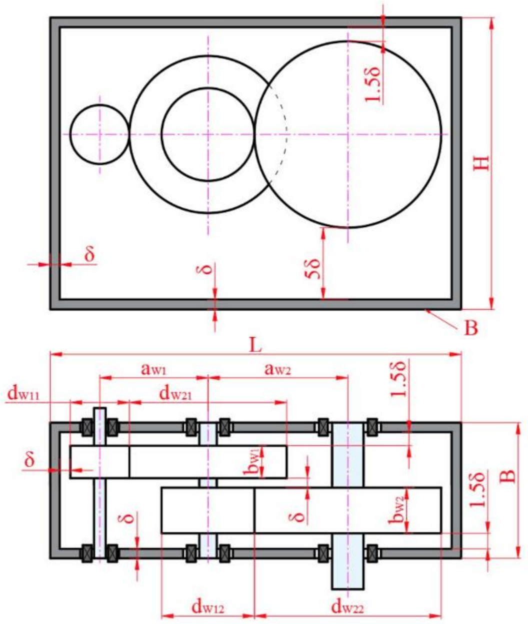

Fig. 1: Schema for finding gearbox cross-sectional area.

The cross-section area A_b of the gearbox can be found by (Figure 1):

$$A_c = (L \cdot H) \tag{1}$$

Where L and B are found by (Figure 1):

$$L = d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 4 \cdot \delta \tag{2}$$

$$H = \max(d_{w21}, d_{w22}) + 8.5 \cdot \delta \tag{3}$$

where δ=7÷10 (mm) [17]; b_{wi}, d_{w1i}, d_{w2i} are the face width, the pitch diameter of the pinion and the gear of ith stage (i=1÷2), respectively. These parameters are computed by:

$$b_{wi} = \mathrm{X}_{bai} \cdot a_{wi} \tag{4}$$

$$d_{w1i} = 2 \cdot a_{wi}/(u_i + 1) \tag{5}$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i \cdot/(u_i + 1) \tag{6}$$

In which, X_{bai} and a_{wi} (i=1÷2) denote the face width coefficient and the center distance of the ith stage. The center distance a_{wi} is found by [20]:

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta}/([AS_i]^2 \cdot u_i \cdot X_{bai})} \tag{7}$$

In which, T_{1i} (i=1÷2) is the torque on the pinion of ith stage which can be computed by:

$$T_{11} = T_{out}/\left(u_{gb} \cdot \eta_{hg}^2 \cdot \eta_b^3\right) \tag{8}$$

$$T_{12} = T_{out}/\left(u_2 \cdot \eta_{hg} \cdot \eta_{be}^2\right) \tag{9}$$

Efficiency Calculation

The gearbox efficiency (%) can be calculated by:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \tag{10}$$

In which, Pl is the total power loss in the gearbox which can be computed by [21]:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{Z0} \tag{11}$$

In (11), P_{lg}, P_{lb}, P_{ls}, and P_{Z0} are the power losses due to gear meshing, bearing friction, seal drag, and idle motion, respectively. These components are determined as in [22].

Objective Functions and Constraints

This study formulates a multi-objective optimization problem with two primary objectives: (i) minimizing the cross-sectional area $A_c$and (ii) maximizing the mechanical efficiency η_{gb}. For implementation in the NSGA-II framework, the second objective is transformed into a minimization problem by considering the negative value of efficiency.

$$min f_1 = A_c \tag{12}$$

$$min f_2 = -\eta_{\mathrm{gb}} \tag{13}$$

Design Variables

The decision variables include the transmission ratio of the first stage (u_1) and the gear face width coefficients for both the first and second stages.

Constraints:

- Gear ratio bounds [20]:

$$1 \le u_i \le 9 \tag{14}$$

- Face width coefficient bounds [20]:

$$0.25 \le X_{\mathrm{bai}} \le 0.4 \tag{15}$$

The proposed framework utilizes NSGA-II to generate the Pareto front of optimal trade-off solutions, followed by the application of AHP to identify the most appropriate alternative based on the designer’s preferences.

METHODOLOGY

NSGA-II Method

The Non-dominated Sorting Genetic Algorithm II (NSGA-II) is a widely used evolutionary approach for solving multi-objective optimization problems, owing to its high computational efficiency and strong capability to preserve solution diversity [8]. In this study, NSGA-II is applied to simultaneously address two conflicting objectives: minimizing the gearbox cross-sectional area and maximizing its mechanical efficiency.

The working principle of NSGA-II involves the following key steps:

- **Initial population generation:** A set of candidate solutions is randomly created within the predefined bounds of the design variables u_1, X_{ba1}, and X_{ba2}.
- **Non-dominated sorting:** The population is classified into different fronts based on Pareto dominance, where the first front contains the best non-dominated solutions.
- **Crowding distance assignment:** A diversity-preserving mechanism is applied by assigning a crowding distance to each solution, ensuring a well-distributed Pareto front.
  Selection, crossover, and mutation: Binary tournament selection is performed based on rank and crowding distance. Simulated binary crossover (SBX) and polynomial mutation are then used to explore the design space.
- **Elitism and population update:** The parent and offspring populations are merged, and the best individuals are selected for the next generation according to non-domination and diversity criteria.

This iterative procedure continues until a termination condition is satisfied, such as reaching the maximum number of generations. The final output is a set of Pareto-optimal solutions representing the trade-off between cross-sectional area and efficiency.

AHP Method

The AHP is a MCDM method designed to support rational selection among alternatives in the presence of conflicting criteria. In this study, AHP is applied to rank and identify the most suitable solution from the Pareto front obtained using NSGA-II.

The AHP procedure is carried out through the following steps:

- **Hierarchy construction:** A hierarchical structure is defined, with the overall goal (selection of the best design) at the top, followed by the evaluation criteria (efficiency and cross-sectional area), and the set of alternatives (Pareto solutions) at the lowest level.
- **Pairwise comparison matrix:** The decision-maker expresses the relative importance of criteria using a numerical scale (typically from 1 to 9). In this study, efficiency is assumed to be three times more important than cross-sectional area, resulting in the following comparison matrix:

$$\begin{bmatrix} 1 & 1/3 \\ 3 & 1 \end{bmatrix} \tag{16}$$

- **Priority vector calculation:** The geometric mean method is employed to determine the normalized weights of each criterion.
- **Alternative scoring:** Each Pareto solution is normalized with respect to the two criteria and evaluated using a weighted sum approach.

- **Best solution selection:** The alternative with the highest AHP score is identified as the most balanced design, reflecting both technical performance and designer preferences.

By integrating NSGA-II with AHP, the proposed hybrid approach combines powerful global search capability with a structured decision-making process, providing a robust framework for optimal gearbox design.

RESULTS AND DISCUSSIONS

This section presents and discusses the results of the multi-objective optimization of a two-stage spur gearbox using the integrated NSGA-II and AHP approach. The analysis focuses on: (1) the relationship between the overall transmission ratio u_h and the optimal first-stage ratio u_1; (2) the effectiveness of AHP-based selection compared with mean-value solutions; and (3) the trade-off characteristics between efficiency and cross-sectional area across different values of u_h.

Relationship Between u_h and Optimal Gear Ratio u_1 Respect u_h

Figure 1 illustrates the linear regression relationship between the total transmission ratio u_h and the mean value of the first-stage ratio $u_1$derived from the NSGA-II Pareto solutions. The resulting model exhibits a coefficient of determination of $R^2 = 0.9417$.

$$u_1 = 0.1607 \cdot u_h + 1.9746 \tag{17}$$

This result indicates a strong positive linear correlation. As the total transmission ratio increases, the optimizer tends to assign a larger portion of the ratio to the first stage, implying that the first stage carries a greater load under higher reduction requirements.

In contrast, Figure 2 presents the linear relationship based on the optimal solutions selected using AHP. The corresponding regression model (with $R^2 = 0.9908$) is expressed as:

$$u_1 = 0.1071 \cdot u_h + 0.9490 \tag{18}$$

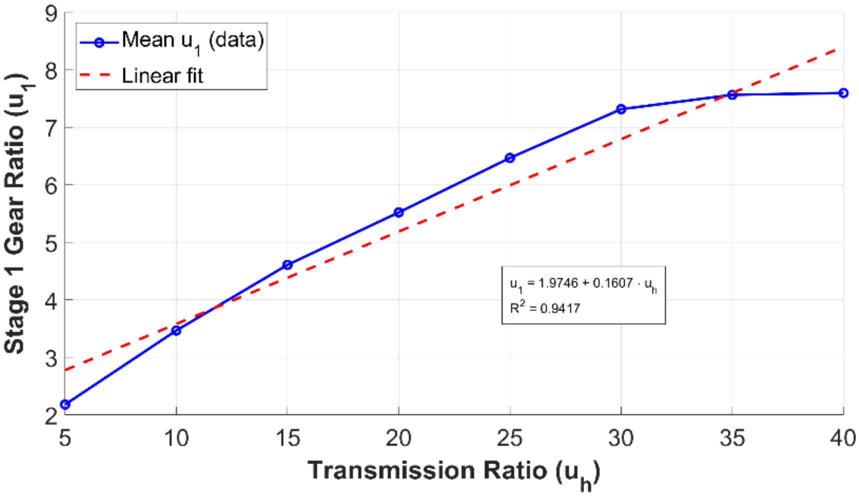

Fig. 2: Linear regression between u_h and mean first-stage gear ratio u_1 from NSGA-II Pareto solutions.

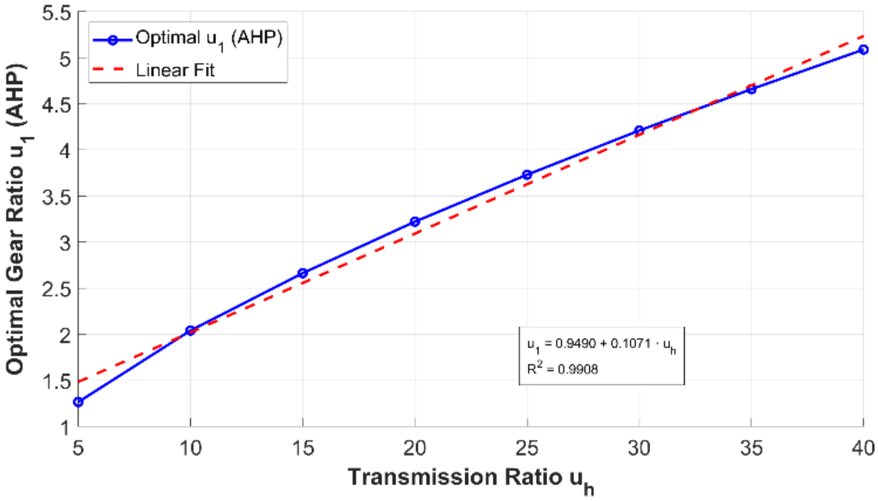

Fig. 3: Linear regression between u_h and AHP-selected optimal first-stage gear ratio u_1.

Trade-Off Between Efficiency and Cross-Sectional Area

Figure 4 presents the Pareto fronts corresponding to different values of u_h in the objective space, where the cross-sectional area is plotted on the horizontal axis and efficiency on the vertical axis. Each colored cluster represents the set of Pareto-optimal solutions for a fixed transmission ratio u_h ranging from 5 to 40. The Pareto fronts exhibit a distinct non-linear, concave shape, clearly illustrating the inherent trade-off between maximizing efficiency and minimizing cross-sectional area. For lower values of u_h, the fronts extend toward regions of higher efficiency and smaller structural area, reflecting more favorable gear configurations under modest transmission requirements. As u_h increases, however, the feasible region shifts toward larger cross-sectional areas and reduced efficiency, primarily due to mechanical constraints such as increased gear diameters and less effective meshing conditions. These observations highlight the necessity of a multi-objective optimization approach. A single-objective formulation would either excessively favor compactness-potentially compromising performance-or prioritize efficiency at the expense of increased structural size.

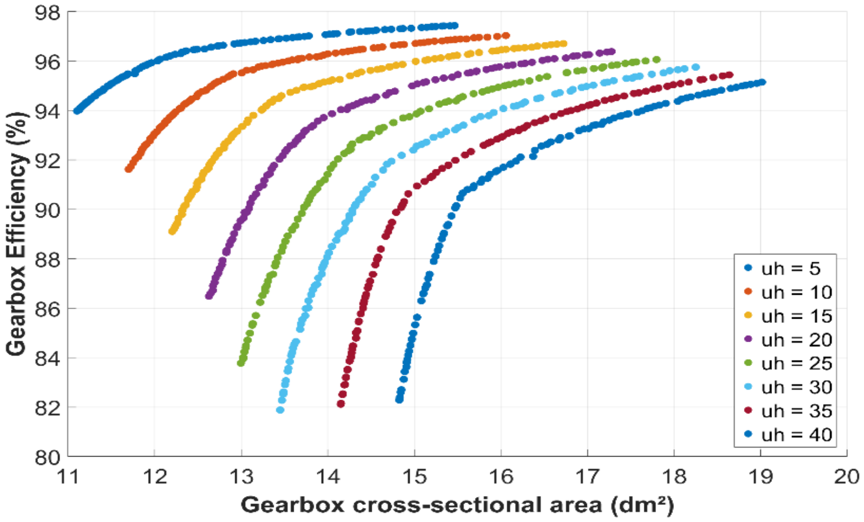

Fig. 4: Pareto fronts for various u_h.

Evaluation of AHP-Based Selection

To facilitate practical decision-making, the AHP method was employed to select a representative solution from each Pareto front corresponding to a given overall transmission ratio u_h . The selection reflects the decision-maker's preference, in which gearbox efficiency is considered three times more important than compactness (cross-sectional area). Table 1 summarizes the optimal design parameters obtained via AHP for each value of u_h.

The results reveal a smooth and consistent trend as $u_h$increases. Specifically, higher transmission ratios lead to a gradual increase in the optimal first-stage ratio $u_1$and a slight enlargement of the cross-sectional area, which is expected due to geometric scaling requirements. At the same time, efficiency shows a marginal decline with increasing u_h, reflecting the inherent trade-off between compactness and performance.

Notably, all selected solutions maintain constant values of $X_{ba1} = 0.25$ and $X_{ba2} = 0.40$, indicating a consistent preference for wider gears in the second stage, where higher torque is typically transmitted. The stability of these parameters across all cases suggests that the AHP-based selection effectively balances competing objectives by prioritizing improved meshing conditions in critical stages while controlling structural size.

Overall, the application of AHP proves to be an effective strategy for identifying well-balanced gearbox designs that are consistent with engineering intuition and manufacturing feasibility, while also aligning with the designer's preferences.

Table 1: Optimal gear design factors selected using AHP for different u_h.

u_h	u_1	X_{ba1}	X_{ba2}	Ac (dm²)	Efficiency (%)
5	1.27	0.25	0.40	15.46	97.43
10	2.04	0.25	0.40	16.09	97.03
15	2.66	0.25	0.40	16.72	96.70
20	3.22	0.25	0.40	17.30	96.38
25	3.73	0.25	0.40	17.79	96.06
30	4.21	0.25	0.40	18.25	95.75
35	4.66	0.25	0.40	18.64	95.44
40	5.09	0.25	0.40	19.01	95.14

CONCLUSIONS

This study presents a hybrid optimization framework that integrates the Non-dominated Sorting Genetic Algorithm II (NSGA-II) with the Analytic Hierarchy Process (AHP) to address conflicting objectives in the design of a two-stage spur gearbox. Two primary objectives-minimizing cross-sectional area and maximizing mechanical efficiency-were formulated and evaluated using a comprehensive parametric model that accounts for gear geometry, housing dimensions, and power loss mechanisms.

NSGA-II was effectively applied to generate a diverse set of Pareto-optimal solutions over a wide range of transmission ratios u_h. The resulting Pareto fronts clearly illustrate the trade-off between compactness and efficiency, particularly at higher transmission ratios. Subsequently, AHP was employed to identify the most balanced solution from each Pareto set, based on a preference structure in which efficiency is weighted three times higher than cross-sectional area.

The results indicate a consistent trend, with the optimal first-stage ratio $u_1$increasing alongside u_h, while maintaining high efficiency and only moderate growth in cross-sectional area. Linear regression analysis further confirms the strong stability and predictability of the AHP-based selections. Notably, all optimal solutions share the same gear face width coefficients ($X_{ba1} = 0.25$ and $X_{ba2} = 0.40$), suggesting a preferred structural configuration for achieving efficient performance.

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