



The Effect of Optical Element Calibration Error on the Performance of the High-precision Light Rangefinder

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Abstract: Observations were carried out to study the influence of calibration and alignment of individual optical elements in the electro-optical channel on the performance of an ultra-precise photometer. The amount of light in the minimum signal of the demodulation curve was determined depending on the calibration of the crystal and the analyzer in the transmitting and receiving channels. Tolerances for calibration errors of the analyzer and the crystal are presented. Acceptable alignment errors of optical elements were identified and mathematically substantiated, showing that in the modulator and demodulator these must be achieved with an accuracy of 0.5° . The relationship between crystal orientation and residual light under increased voltage applied to the crystals was also established. It was further demonstrated that when using two separate electro-optical crystals in the transmitting and receiving channels, the error magnitude is determined by the orientation errors of the crystals and the analyzer, which must not exceed 0.25° .

Keywords: analyzer, mirrors, light modulator-demodulator, optical path, orientation, phase plate.

INTRODUCTION

The basis for the construction of high-precision phase light range finders is electro-optical modulation-demodulation of light, which provides high-precision phase detection, as well as high stability of the constant correction.

The well-known high-precision light range finders based on the modulation method ME-5000 [1], Geomancer GR-204 [2] and CD-1200 [3,4], in real conditions provided an error of linear measurements in the range of $0.5 \dots 0.25$ mm. However, the fundamental solutions underlying the development of these light range finders allow the construction of the "0"-th category light range finders providing an error in determining the phase $m_\phi = 0.01 - 0.02$ mm [5]. Light range finders with an accuracy of measuring distances of the order of 10^{-7} open up new prospects for the use of high-precision phase light range finders for linear measurements in special engineering and geodetic works. For example, in comparators as an intermediate link for transmitting a unit of length to working means of linear measurements [6], when adjusting optical telescopes, when monitoring the movements of the Earth's crust in order to predict earthquakes, in hydraulic structures, etc., it is necessary to limit the error in determining the phase to a value of $m_\phi = 0.01 \dots 0.02$ mm.

To this end, it is necessary to apply more precise principles for determining the fractional part of the phase cycle, for example, the formation of two receiving signals offset relative to each other by 180° , the functional diagram of which is shown in Fig. 1 [7].

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The accuracy of measurements largely depends on the quality of the alignment of the optical path of the light range finder. The tolerances for the orientation of optical elements can be set based on the specified accuracy of the light range finder based on the analysis of the interaction of the light flux with optical elements.

The optical path of the light range finder consists of the following optical elements: a source of polarized radiation, electro-optical crystals of a light modem, a reflector and an analyzer.

Each element, included in the optical path, can introduce an additional deviation of the phase of the modulated beam, the magnitude of which depends on the quality of these elements and the accuracy of the orientation. The additional phase shift reduces the modulation efficiency and creates additional noise signals at the output of the photo detector. The residual light does not shift the position of the minimum value, but increases its width. For that reason, it is necessary to evaluate the effects of optical element calibration errors and to determine at least the magnitude of the residual light current, which is caused by the additional phase deviation, in order to build an optical path with as few losses as possible.

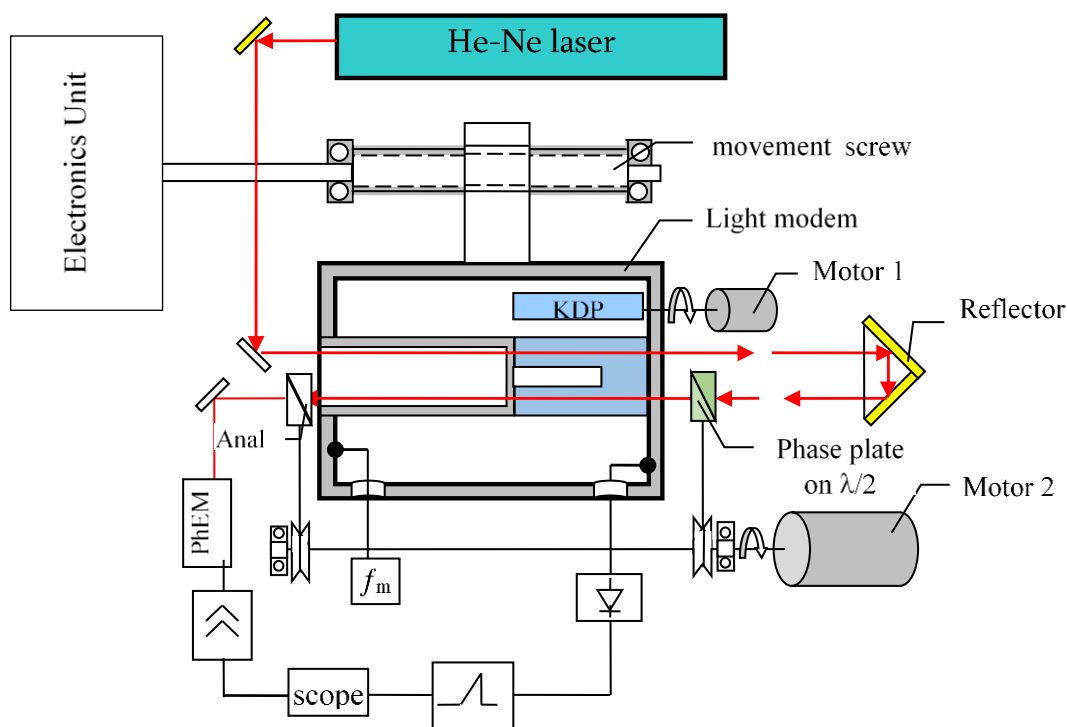


Fig.1: Functional scheme of high-precision light range finder

MATERIALS AND METHODS

It is possible to estimate the errors of the optical elements on the basis of the Jones matrix.

In the matrix of the crystal, instead of θ' , you should put the size of the orientation of the crystal θ'_1 in fig. For the scheme described in fig.1, and fig. 2: instead of θ' , the magnitude of $\theta' = \frac{\pi}{2} + \theta'_2$.

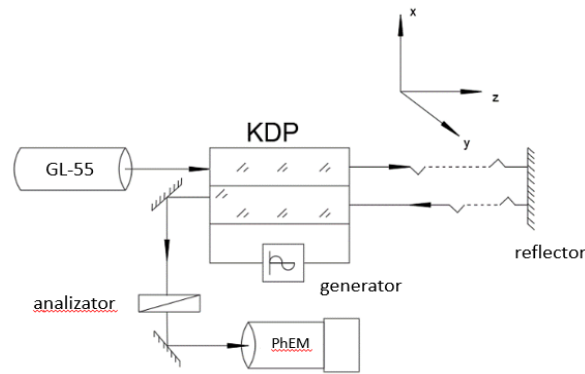


Fig. 1: Passage of direct and reverse light rays through the same crystal

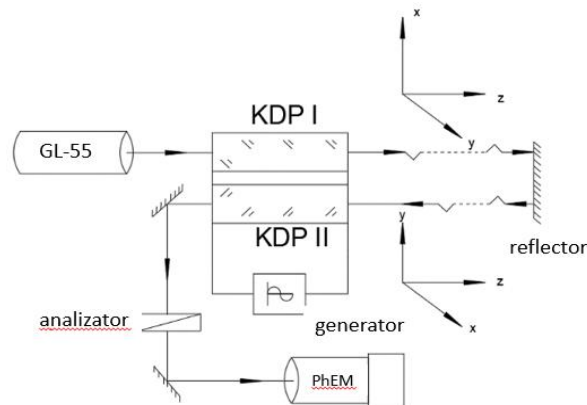


Fig. 2: Passage of direct and reverse light beams through KDP crystals, one of which is rotated 90° with respect to the Z axis.

It is necessary to take into account that the values of θ'_1 and θ'_2 of crystal orientation are small and in this case K_1 and K_2 matrices will be presented in the following form.

$$K_{1,2}(\theta)_{1,2} = K_{1,2}(0) + \theta_{1,2}K'_{1,2}(0), \quad (1)$$

where $K'_{1,2}$ is the derivative of the $K_{1,2}$ matrix in case $\theta'_1 = \theta'_2 = 0$, θ is the orientation of the crystal. If the same crystal is used in the transmitting and receiving channels, then $\theta'_1 = \theta'_2$. In this case, the crystal matrix of the transmission path will be written in the following form.

$$K_1(\theta_1) = \begin{bmatrix} e^{i\Gamma_1/2} & 0 \\ 0 & e^{-i\Gamma_1/2} \end{bmatrix} + \theta'_1 \begin{bmatrix} 0 & 2i \sin \Gamma_1/2 \\ 2i \sin \Gamma_1/2 & 0 \end{bmatrix}; \quad (2)$$

For the receiving path crystal, the matrix has the same form as expression (2), only instead of Γ_1 it will be written as Γ_2 where Γ_1 and Γ_2 are the phase deviations in the receiving and transmitting channels. In the case when the crystals in the transmitting and receiving channels are separate, such as in "Mekometre" and ДBCД-1200, $\theta'_1 \neq \theta'_2$. In this case, the matrix of the crystal of the transmitting path will be written in the form of expression (2), and for the crystal of the receiving path it will look like this:

$$K_2(\theta_2) = \begin{bmatrix} e^{i\Gamma_2/2} & 0 \\ 0 & e^{-i\Gamma_2/2} \end{bmatrix} + \theta'_2 \begin{bmatrix} 0 & 2i \sin \Gamma_2/2 \\ 2i \sin \Gamma_2/2 & 0 \end{bmatrix} \quad (3)$$

When the second crystal is rotated by $(\pi/2 + \theta_2)$, the crystal matrix is written as:

$$K_3\left(\frac{\pi}{2} + \theta_2\right) = \begin{bmatrix} e^{-i\Gamma_2/2} & 0 \\ 0 & e^{i\Gamma_2/2} \end{bmatrix} + \theta_2' \begin{bmatrix} 0 & -2i \sin \Gamma_2/2 \\ -2i \sin \Gamma_2/2 & 0 \end{bmatrix} \quad (4)$$

To create the matrix of the general system, it is also necessary to write the matrix of the analyzer, in case of small errors of orientation with respect to the coordinate axes:

$$A\left(\frac{\pi}{4} + \theta_3\right) = A\left(\frac{\pi}{4}\right) + \theta_3 A'\left(\frac{\pi}{4}\right) = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \theta_3 \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad (5)$$

To estimate the magnitude of the residual light intensity at the minimum value after beam demodulation, it is sufficient to determine the matrix of the entire system with the accuracy of the second order of smallness:

$$M_I = \left[A\left(\frac{\pi}{4}\right) + \theta_3 A'\left(\frac{\pi}{4}\right) \right] [K_2(0) + \theta_2' K_2'(0)] [K_1(0) + \theta_1' K_1'(0)] \sqrt{\frac{I_0}{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad (6)$$

Inserting the matrix values of the elements into this expression and multiplying them, the number of field components E_x and E_y will be given:

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \sqrt{\frac{I_0}{2}} \left[i \sin \frac{\Gamma_1 + \Gamma_2}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2 \sqrt{\frac{I_0}{2}} (\theta_1' - \theta_2') \sin \Gamma_1/2 \sin \Gamma_2/2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \theta_3 \begin{bmatrix} e^{i(\frac{\Gamma_1 + \Gamma_2}{2})} \\ e^{-i(\frac{\Gamma_1 + \Gamma_2}{2})} \end{bmatrix} \right] \quad (7)$$

The light intensity at the output of the analyzer is determined as the square of the modulus of the vector whose components E_x and E_y are equal to:

$$I = E_x^2 + E_y^2 = \frac{I_0}{2} \left\{ \left(i \sin \frac{\Gamma_1 + \Gamma_2}{2} - \theta_3 e^{i\frac{\Gamma_1 + \Gamma_2}{2}} + 2(\theta_1' + \theta_2') \sin \frac{\Gamma_1}{2} \sin \frac{\Gamma_2}{2} \right)^2 + \left[i \sin \frac{\Gamma_1 + \Gamma_2}{2} - \theta_3 e^{-i\frac{\Gamma_1 + \Gamma_2}{2}} + 2(\theta_1' - \theta_2') \sin \frac{\Gamma_1}{2} \sin \frac{\Gamma_2}{2} \right]^2 \right\} = I_0 \left[\sin^2 \frac{\Gamma_1 + \Gamma_2}{2} + \theta_3^2 - 4\theta_3(\theta_1' - \theta_2') \cos \frac{\Gamma_1 + \Gamma_2}{2} \sin \Gamma_1/2 \sin \Gamma_2/2 + 4(\theta_1' - \theta_2')^2 \sin^2 \Gamma_1/2 \sin^2 \Gamma_2/2 \right] \quad (8)$$

The first term of this expression is the light intensity at the output of the analyzer in case of correct orientation of the optical elements, the second term is caused by the error of the analyzer calibration, the last term of the expression (8) characterizes the residual light, which is caused by the calibration error of the crystals and the analyzer.

At the position of the minimum value $\Gamma_1 = -\Gamma_2 = \Gamma$, then equation (8) will yield:

$$I = I_0 [\theta_3^2 + 4\theta_3(\theta_1' - \theta_2') \sin^2 \Gamma + 4(\theta_1' - \theta_2')^2 \sin^4 \Gamma] \quad (9)$$

The expression (9) shows that if the crystals in the transmitting and receiving channels are similarly calibrated: $\theta_1' = \theta_2'$, then the residual light flux at the minimum value depends only on the analyzer calibration error θ_3 .

When $\theta_1' \neq \theta_2'$, at the point of the minimum value there is a residual luminous flux due to the error of crystal alignment. In order to estimate the average value of the residual intensity during the oscillation period, it is necessary to determine the integral, assuming that

$$\Gamma = \pi \frac{U_0}{U_\pi} \sin 2\pi \frac{t}{T}, \quad (10)$$

$$\frac{I}{I_0} = \frac{1}{T} \int I(t) dt = \theta_3^2 + 2\theta_3(\theta_1' - \theta_2') \left[1 - I_0 \left(2\pi \frac{U_0}{U_\pi} \right) \right] + 2(\theta_1' - \theta_2')^2 \left[\frac{3}{4} + \frac{1}{4} I_0 \left(4\pi \frac{U_0}{U_\pi} \right) - I_0 \left(2\pi \frac{U_0}{U_\pi} \right) \right] \quad (11)$$

The obtained expressions show that the residual intensity in the minimum value depends on the effectiveness of the beam modulation in the case of wrong orientation of

the crystals. In the case when the second crystal is rotated by $\frac{\pi}{2} + \theta'_2$, the expression that determines the residual intensity coincides with the expression (11). The dependence of the residual light intensity at the minimum value on the modulator-demodulator operation mode, when $\theta_3 = (\theta'_1 - \theta'_2) = 1^\circ$, is shown in fig. 3 in [8].

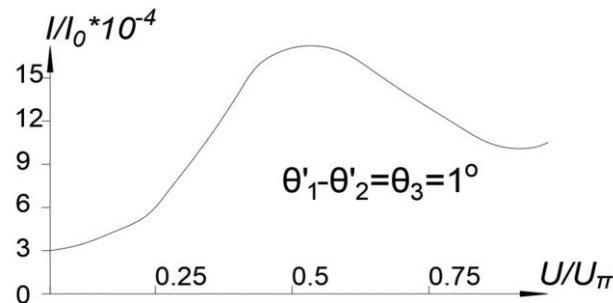


Fig. 3: The dependence of residual light intensity at the minimum value on the mode of operation of the modulator-demodulator when $\theta_3 = (\theta'_1 - \theta'_2) = 1^\circ$

It can be seen from the graph in Figure 3 that when the voltage on the crystal is increased by the value $U/U_\pi \approx 0,5$, the residual light at the minimum value increases, and when U/U_π is increased further, it decreases, but remains always more than U/U_π in the case of 0 value. To describe the flow of the graph in Figure 3, let's assume that the crystal is correctly oriented in the transmitting channel and incorrectly in the receiving channel. In this case, line 1 (Fig. 4), which shows the dependence of the phase difference Γ on the voltage applied to the receiving crystal, makes a smaller angle with the U/U_π path than the same line 2 in the case of the correct orientation of the crystal.

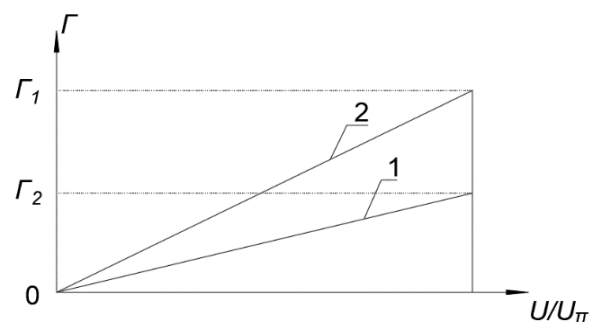


Fig. 4: Dependence of the phase difference Γ on the voltage applied to the receiving crystal

It can be seen from 4 that the greater the voltage applied to the crystal, the greater the phase difference ($\Gamma_1 - \Gamma_2$), which leads to the generation of residual light intensity at the output of the analyzer. A certain reduction of the residual intensity at a voltage value of $\frac{U}{U_\pi} > 0,5$ depends on the re-modulation of the light beam. In order to estimate the magnitude of the error in the determination of optical elements, it is necessary to build the dependence of the residual light intensity at the minimum value on the analyzer orientation error when $U/U_\pi = 0$, as well as on the same error of the analyzer and crystal orientation when $\theta = (\theta'_1 - \theta'_2) = \theta$: In this case, the expression (11) will have the following form:

$$\frac{I}{I_0} = 4,5\theta^2 + \frac{1}{2}\theta^2 I_0 \left(4\pi \frac{U}{U_\pi}\right) - 4\theta^2 I_0 \left(2\pi \frac{U}{U_\pi}\right) \quad (12)$$

RESULTS AND DISCUSSION

The dependence of I/I_0 on θ in different operating modes of the modulator is shown in fig. 5, from which it follows:

1. if one crystal is used as a light modulator-demodulator: $(\theta'_1 - \theta'_2) = 0$, then the residual light is caused by the orientation error of the analyzer, which according to fig. 5 can be within $\pm 0,5^\circ$.
2. in the case when different crystals are used for the transmitting and receiving channels, the orientation error of both the crystals and the analyzer is possible. The limits of orientation error are $\pm 0,25^\circ$ [9,10]
3. when the analyzer is correctly oriented: $\theta_3 = 0$, then the crystal orientation error at the value of voltage $\frac{U}{U_\pi} = 0,5$ is allowed within $\pm 0,5^\circ$,
4. in the case of increasing the voltage on the crystals above the value $\frac{U}{U_\pi} \approx 0,4$, the orientation of the crystals has a greater influence on the residual light than the similar error of the analyzer.

It should be noted that in the case of light modulation and demodulation, the residual light in the crystal cells leads to a reduction in the modulation efficiency, as a result of which the working distance and the measurement accuracy of the photometer decrease. The residual light does not move the position of the minimum value, but increases its width ΔL .

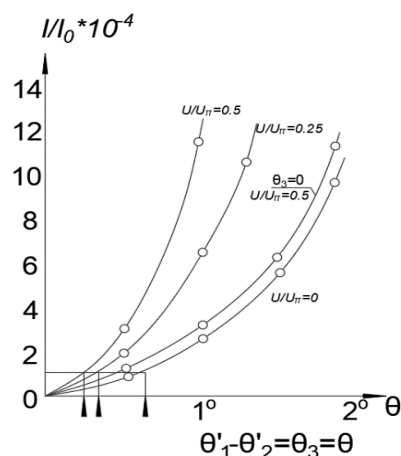


Fig. 5: Dependence of I/I_0 on θ in different operating modes of the modulator

The magnitude of the residual light in the case of using a phase plate, when the phase in the optical path deviates by $\lambda/2$, then the Jones matrix, when the light passes through the quarter-wave plate twice, will take the following form:

$$T^2\left(\frac{\pi}{4} + \theta_4\right) = \begin{bmatrix} i \cos\left(\frac{\pi}{2} + 2\theta_4\right) & i \sin\left(\frac{\pi}{2} + 2\theta_4\right) \\ i \sin\left(\frac{\pi}{2} + 2\theta_4\right) & -i \cos\left(\frac{\pi}{2} + 2\theta_4\right) \end{bmatrix}.$$

Since the value of θ_4 is small, then

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + i\theta_4 \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix} \quad (13)$$

The matrix of an optical system with a $\lambda/4$ phase plate will be expressed as:

$$M_3 = A \left(\frac{\pi}{4} + \theta_3 \right) \cdot K_2(\theta_2') \cdot T^2 \left(\frac{\pi}{4} + \theta_4 \right) \cdot K_1(\theta_1') \cdot \sqrt{\frac{I_0}{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad (14)$$

Analyzing this expression, we get:

$$I = I_0 \left(\sin^2 \frac{\Gamma_2 - \Gamma_1}{2} + \theta_3^2 + 4(\theta_1' + \theta_2')^2 \sin^2 \frac{\Gamma_1}{2} \sin^2 \frac{\Gamma_2}{2} + 4\theta_4^2 \cos^2 \frac{\Gamma_1 + \Gamma_2}{2} + 8(\theta_1' + \theta_2')\theta_4 \sin \frac{\Gamma_1}{2} \sin \frac{\Gamma_2}{2} \cos \frac{\Gamma_1 + \Gamma_2}{2} - 4(\theta_1' + \theta_2')\theta_3 \sin \frac{\Gamma_1}{2} \sin \frac{\Gamma_2}{2} \cos \frac{\Gamma_2 - \Gamma_1}{2} - 4\theta_4\theta_3 \cos \frac{\Gamma_1 + \Gamma_2}{2} \cos \frac{\Gamma_1 - \Gamma_2}{2} \right) \quad (15)$$

At the minimum value, $\Gamma_1 = \Gamma_2 = \Gamma$ and the residual light will be equal to:

$$I = I_0 \left[\theta_3^2 + 4(\theta_1 + \theta_2)^2 \sin^4 \frac{\Gamma}{2} + 4\theta_4^2 \cos^2 \Gamma + 8(\theta_1 - \theta_2)\theta_4 \sin^2 \frac{\Gamma}{2} \cos \Gamma - 4(\theta_1 + \theta_2)\theta_3 \sin^2 \frac{\Gamma}{2} - 4\theta_3\theta_4 \cos \Gamma \right] \quad (16)$$

The average value of residual intensity during period T can be determined as:

$$\frac{I}{I_0} = \frac{1}{T} \int_0^T I(t) dt = \theta_3^2 + 2(\theta_1' + \theta_2')^2 \left[\frac{3}{4} + \frac{1}{4} I_0 \left(4\pi \frac{U}{U_\pi} \right) - I_0 \left(2\pi \frac{U}{U_\pi} \right) \right] + 2\theta_4^2 \left[1 + I_0 \left(4\pi \frac{U}{U_\pi} \right) \right] + 4(\theta_1' + \theta_2')\theta_4 \left[I_0 \left(2\pi \frac{U}{U_\pi} \right) - \frac{1}{2} - \frac{1}{2} I_0 \left(4\pi \frac{U}{U_\pi} \right) \right] - 2(\theta_1' + \theta_2')\theta_3 \left[1 - I_0 \left(2\pi \frac{U}{U_\pi} \right) \right] - 4\theta_3\theta_4 I_0 \left(2\pi \frac{U}{U_\pi} \right) \quad (17)$$

Comparing these two expressions (11) and (17), the following can be noted:

1. when a quarter-wave phase plate is introduced into the optical channel, the residual light at a minimum value depends on the crystal orientation error,
2. if all the elements in the optical path, except for the phase plate, are correctly oriented, then the phase plate orientation error at a minimum value leads to the generation of residual intensity, which depends on the power mode of the light modulator-demodulator. The misorientation of the plate leads to twice as much residual intensity as the same orientation of the crystals.

CONCLUSION

The error in the orientation of the optical elements leads to a decrease in the efficiency of light modulation-demodulation, that is, to a decrease in the accuracy of the phase determination and the radius of the device's operation. In order for the residual light intensity not to leave significant errors on the measurement results, the orientation of the optical elements of the light modulator and demodulator should be carried out with an error of no more than $\pm 0.5^\circ$. In the case when one crystal is used as a modulator demodulator, the residual light depends on the orientation of the analyzer to the extent of $\pm 0.5^\circ$, in the case of correct orientation of the analyzer and $U/U_\pi = 0.5$, the crystal orientation error should be $\pm 0.5^\circ$ within limits. In the case of using two separate crystals in the receiving and transmitting channels, the amount of error is determined by the orientation error of both the crystals and the analyzer, which should be within $\pm 0.25^\circ$.

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