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Maxwell Equations derived from (Coulomb' Law + velocity), Maxwell-type Gravity derived from (Newton's Law + velocity), Spin-Electromagnetics derived from (Coulomb' Law + spin), Spin-Gravity derived from (Newton's Law + spin) --- Duality, Symmetry, Unification, and New Spin-related Forces

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ABSTRACT

The Universal Mathematical Field Theory (UMFT) is established, which states that the combination of inverse-square laws and either a vector field or an axial vector field create the curl of an axial vector field. Utilizing UMFT, we mathematically: (1) derive the Maxwell electromagnetics (Maxwell-EM) and the Lorentz force from Coulomb's law and the velocity of an electric source; (2) establish Maxwell-type gravity (Gravito-EM) and Lorentz-type gravitational force from Newton's law and the velocity of a gravitational source; (3) establish Spin-Electromagnetics (Spin-EM) from the Coulomb's law and the spin of the electric source; (4) establish Spin-gravity (Spin-gravity) from the Newton's law and the spin of the mass (such as neutron); (5) predicate the spin related Lorentz-type force. The so-derived Maxwell-EM justifies UMFT and shows that the experiments-based Maxwell equations have mathematical origin. UMFT shows that mathematical identities lead to physical dualities, such as the duality between Maxwell-EM, Gravito-EM, Spin-EM and Spin-gravity. Therefore, the concepts, effects and phenomena of Maxwell-EM may be directly converted to that of gravity and Spin related forces. The Gravito-EM can be quantized, along the line of quantizing Maxwell-EM, and unified with Maxwell-EM force. Spin-EM predicts several new effects, the Spin-Lorentz-type force, and Spin-Lagrangian-Lorentz-type force. If experimentally confirmed, the Spin-related forces may be the New Forces. Spin-gravity can be utilized to describe the new spin related interaction between neutrons. The Maxwell-EM, Gravito-EM, Spin-EM, and Spin-gravity are all derived from UMFT and thus, they are dual to each other, they have same symmetry, such as U (1) symmetry of Maxwell-EM, and they can be unified in the frame of UMFT.

Keywords: Maxwell-electromagnetics, Coulomb law, Gravito-electromagnetics, Newton law, spin-electromagnetics, spin-gravity, spin, duality, symmetry, unification.

INTRODUCTION

Historically, Maxwell equations were established based on series experiments, except the introduction of the displacement current. Consider the situation of a stationary electric charge (e-charge). The Coulomb's law states that a stationary e-charge Q_e induces a static vector electric field **E.** The created electric field is determined by only one quantity, e-charge. A heuristic phenomenon is that an observer uniformly moving relative to the same e-charge (or the e-charge moves relative to the observer) will observe not only the electric field but a magnetic field. The magnetic field is determined by three quantities, e-charge, electric field and velocity (Table 1.1).

Table 1.1: Static e-charge vs. moving e-charge vs spin e-charge

	Static e-charge: Electric field	moving e-charge: Static Magnetic field	Spin e-charge: Spin-EM field
Source of field	Q_e	$Q_e\boldsymbol{v}$	$Q_e\boldsymbol{S}$
How sources produce fields	$\mathrm{Q_e} \to \mathbf{E}$ $\mathrm{Q_e} \to \nabla\cdot\mathbf{E}$	$(\mathrm{Q_e}, \mathbf{v}, \mathbf{E}) \to \mathbf{B}$ $\mathrm{Q_e}\mathbf{v} \to \nabla\times\mathbf{B}$	$(\mathrm{Q_e}, \mathbf{S}, \mathbf{E}) \to \mathbf{B_s}$
Nature of Field	vector field	First order axial vector field	Second order axial vector field
Force due to field	$\mathrm{q_e}\mathbf{E}$	$\mathrm{q_e}\mathbf{v} \times \mathbf{B}$	

Let's first review the fundamental differences between a static electric field **E** and a static magnetic field **B.** A physics student may ask questions: Why the motion of e-charge creates magnetic fields? Does the generation of magnetic fields relate with the Coulomb's law? The standard answer is that the magnetism is the combination of electric field with Special Relativity (SR) and does not relate with the Coulomb's law. We argue that the answer is not proper to explain the above four fundamental differences.

Now we ask a further question: Is the generation of magnetic fields by a moving e-charge inevitable? The answer is yes. Since, physically, magnetic phenomena are related with the motion of e-charge and inevitable, we suggest that:

1. The Coulomb's law can be established only by experiments and thus, is a primary law;
2. Maxwell equations related with magnetic field, including the experimental Ampere's law, experimental Faraday's law and experimental Gauss's law of magnetism, which are all related with the motion of e-charges, should be derivable mathematically from the Coulomb' law and the velocity of source, and thus, are secondary laws.

Universal Mathematical Field Theory (UMFT): To mathematically derive Maxwell equations from the Coulomb's law and velocity of an e-charge, we establish Universal Mathematical Field Theory first (Section 2).

Extended-Maxwell equations (EM): then the Extended-Maxwell-Electromagnetism (EM) is derived from UMFT, Coulomb's law and the velocity of the source, which demonstrates the validity of UMFT (Section 3).

Maxwell-type Gravity (Gravito-EM): Based on the similarity between the Coulomb's law and Newton's law, naturally, the same UMFT is employed to derive Maxwell-type gravitational equations from Newton's law and the linear motion of a source, denoted as Gravito-electromagnetics or Gravito-EM, which provides a clear physical picture of gravity. We show that both theories, Maxwell theory and Gravito-EM are convertible to each other by converting the e-charge to gravitational mass (g-charge), and vice versa, which we denote as "Ultra-symmetry" or the "Duality between EM and Gravito-EM" (Section 4).

Historically, Maxwell-type vector field theories of gravity were studied, which described gravity as a physical field. However, physicists gave up this direction before Einstein's General Relativity (GR), because they realized that such vector theories faced the issue: the potential energy of Newton field is negative. After SR was established, second issue raised that those vector theories of gravity did not comply with SR. We address those two issues in this article.

The benefits of the Gravito-EM are: (1) Address the long-standing issues of negative potential energy; (2) comply with SR; (3) explain the accelerating expansion of the universe and to predict a jerking expansion of the universe. Therefore, we argue that the Gravito-EM provides, at least, a "bridge" between the Newton's theory and an ultimate theory of gravity. To understand and describe all of forces in one framework, a possible and simple way is to treat gravity as a physical field. Recently, a gauge theory of gravity has been proposed, quantized and unified with other forces [1]. The Gravito-EM is consistent with this gauge theory of gravity.

Spin-Electromagnetics (Spin-EM): an e-charge has a spin that is an axial vector. Utilizing UMFT, Coulomb's law, e-charge and spin of the e-charge, Spin-EM is derived. Spin-EM provide classical foundations of variety phenomena of spintronics (Section 5).

Spin-gravity: for example, a neutron has mass and spin. Utilizing UMFT, Newton's Law and Spin of mass, Spin-gravity is derived (Section 6). The Spin-gravity is the dual of the Spin-EM.

UNIVERSAL MATHEMATICAL FIELD THEORY (UMFT)

Motivation

Motivation: To address above differences between a static electric field **E** and a magnetic field **B** inspires us to establish UMFT [2]. The UMFT is equally applicable to derive Maxwell-EM and Gravito-EM. Thus, there is duality between Maxwell-EM and Gravito-EM, so that many concepts and effects of the well-established EM can be transferred directly to gravity. Most important, gravity and EM are linked together clearly and closely. Gravito-EM is powerful and fruitful.

The significances of UMFT are the following.

1. combining UMFT, the Coulomb's law and velocity of e-source re-derives *mathematically* Maxwell's equations, which addresses the above-mentioned differences related with EM. The derivation shows that the experiments-based Maxwell equations have their mathematical origin, and justifies UMFT.
2. combining UMFT, the Newton's law and velocity of mass derives *mathematically* Gravito-EM.
3. combining UMFT, the Coulomb's law and the spin of e-charge derives *mathematically* spin-EM.
4. combining UMFT, the Newton's law and the spin of mass derives *mathematically* spin-gravity.
5. UMFT provides the mathematic origin of dualities between different physic fields derived from it, such as duality between electricity and magnetism, duality between Maxwell-EM, Gravito-EM, spin-EM, and spin-gravity. Duality is a powerful tool to find intrinsic similarities between apparently different phenomena, and predict new effects.

"It turns out that most of the important concepts and theories of physics can be unified and understood by their common attribute of duality" (Damian P Hampshire).

General UMFT

To establish UMFT, we need to find a mathematical identity that connecting divergences of either a vector or an axial vector with curls of induced axial vectors. For this aim, the following vector analysis identity is the most significant foundation of UMFT,

$$\nabla \times (\mathbf{S} \times \mathbf{T}) = \mathbf{S}(\nabla \cdot \mathbf{T}) - \mathbf{T}(\nabla \cdot \mathbf{S}) + (\mathbf{T} \cdot \nabla)\mathbf{S} - (\mathbf{S} \cdot \nabla)\mathbf{T}, \tag{2.1}$$

which indicates that the combination of gradient and divergence of two arbitrary vectors including axial vector, $\mathbf{S}$ and $\mathbf{T}$, induces inevitably an axial vector ($\mathbf{S} \times \mathbf{T}$). One of two terms, either ($\nabla \cdot \mathbf{T}$) or ($\nabla \cdot \mathbf{S}$), represents fundamental inverse-square laws, and introduce a "charge". Now there are three quantities, $\mathbf{S}$, $\mathbf{T}$ and "charge" in Eq. (2.1).

It is useful to write Eq. (2.1) in a different but equivalent form. By using another mathematical identity,

$$(\mathbf{T} \cdot \boldsymbol{\nabla})\mathbf{S} = \boldsymbol{\nabla}(\mathbf{S} \cdot \mathbf{T}) - (\mathbf{S} \cdot \boldsymbol{\nabla})\mathbf{T} - \mathbf{S} \times (\boldsymbol{\nabla} \times \mathbf{T}) - \mathbf{T} \times (\boldsymbol{\nabla} \times \mathbf{S}),$$

Eq. (2.1) can be rewritten as an identity,

$$\nabla \times (\mathbf{S} \times \mathbf{T}) = \mathbf{S}(\nabla \cdot \mathbf{T}) - \mathbf{T}(\nabla \cdot \mathbf{S}) + \nabla(\mathbf{S} \cdot \mathbf{T}) - 2(\mathbf{S} \cdot \nabla)\mathbf{T} - \mathbf{S} \times (\nabla \times \mathbf{T}) - \mathbf{T} \times (\nabla \times \mathbf{S}). \tag{2.2}$$

Eq. (2.1) and Eq. (2.2) are mathematical equivalent. When apply UMFT to describe physical fields, the "$\mathbf{S}$" and "$\mathbf{T}$" in Eq. (2.1) and Eq. (2.2) represent different physical quantities respectively.

Based on mathematical vector identities, we establish self-consistent UMFT that universally describes classical physical fields induced respectively by velocity and classical spin of a source.

UMFT Related with Velocity of Charges

General UMFT Related with Velocity/Spin of Charges:

In physics, there are two the inverse square laws, Coulomb law and Newton law: $\nabla \cdot \mathbf{E} = Q_e$ and $\nabla \cdot \mathbf{g} = Q_m$. The basic concept is that the combination of the inverse-square laws and the motion of charges must induce axial vector fields. To show, we set "$\mathbf{S}$" being motion parameters, either vector velocity $\mathbf{v}$ or the axial vector spin $\mathbf{S}_c$,

$$\mathbf{S} = \mathbf{v} \text{ (vector velocity) or } \mathbf{S} = \mathbf{S}_c \text{ (axial vector spin)} \tag{2.3}$$

Note: the physical quantities $\mathbf{S}$ representing are not limited to velocity and spin. Substituting Eq. (2.3) into Eq. (2.1) and Eq. (2.2) respectively, we obtain,

$$\nabla \times (\mathbf{v} \times \mathbf{T}) = \mathbf{v}(\nabla \cdot \mathbf{T}) - \mathbf{T}(\nabla \cdot \mathbf{v}) + (\mathbf{T} \cdot \nabla)\mathbf{v} - (\mathbf{v} \cdot \nabla)\mathbf{T}, \tag{2.4a}$$

$$\nabla \times (\mathbf{S}_c \times \mathbf{T}) = \mathbf{S}_c(\nabla \cdot \mathbf{T}) - \mathbf{T}(\nabla \cdot \mathbf{S}_c) + (\mathbf{T} \cdot \nabla)\mathbf{S}_c - (\mathbf{S}_c \cdot \nabla)\mathbf{T}, \tag{2.4b}$$

$$\nabla\times(\mathbf{v}\times\mathbf{T})=\mathbf{v}(\nabla\cdot\mathbf{T})-\mathbf{T}(\nabla\cdot\mathbf{v})+\nabla(\mathbf{v}\cdot\mathbf{T})-2(\mathbf{v}\cdot\nabla)\mathbf{v}-\mathbf{v}\times(\nabla\times\mathbf{T})-\mathbf{T}\times(\nabla\times\mathbf{v}).\quad(2.5a)$$

$$\nabla\times(\mathbf{S}_c\times\mathbf{T})=\mathbf{S}_c(\nabla\cdot\mathbf{T})-\mathbf{T}(\nabla\cdot\mathbf{S}_c)+\nabla(\mathbf{S}_c\cdot\mathbf{T})-2(\mathbf{T}\cdot\nabla)\mathbf{S}_c-\mathbf{S}_c\times(\nabla\times\mathbf{T})-\mathbf{T}\times(\nabla\times\mathbf{S}_c)\quad(2.5b)$$

Fortunately, the right-hand side of Eq. (2.4a) contains both the motion of source and the inverse-square law, while the left-hand side contains the curl of an axial vector field. Eq. (2.4a) connects both.

In Section 2, we start with Eq. (2.4a) to derive the Maxwell-type equations for the fields induced by the velocity of sources. Note: in Eq. (2.4a), the velocity is spatially varying, e.g., $\mathbf{T}(\nabla\cdot\mathbf{v})\neq 0$, $(\mathbf{T}\cdot\nabla)\mathbf{v}\neq 0$, and is instantaneous velocity at a given space point. Eq. (2.4a) implies that a velocity and its spatial variations induce the axial vector $(\mathbf{v}\times\mathbf{T})$ field equally.

In Section 3, we start with Eq. (2.4b) to derive the Maxwell equations for the fields induced by the velocity of e-charges, referred it as Maxwell-EM.

In Section 4, we start with Eq. (2.4b) to derive the Maxwell-type equations for the fields induced by the velocity of mass, referred it as Maxwell-type-gravity.

In Section 5, we start with Eq. (2.4b) to derive the Maxwell-type equations for the fields induced by the spin of e-charges, referred it as Spin-EM.

In Section 6, we start with Eq. (2.4b) to derive the Maxwell-type equations for the fields induced by the spin of g-charge, mass, referred it as Spin-gravity.

Ampere-Maxwell-type UMFT:
Firstly, we derive the Ampere-Maxwell-type UMFT. Let's assume the "**T**" is an arbitrary vector field **G** and satisfies the inverse square law,

$$\nabla\cdot\mathbf{G}\neq 0.\quad(2.6)$$

Since

$$-(\mathbf{v}\cdot\nabla)\mathbf{G}=-\left[\mathrm{v_x}\frac{\partial}{\partial \mathrm{x}}+\mathrm{v_y}\frac{\partial}{\partial \mathrm{y}}+\mathrm{v_z}\frac{\partial}{\partial \mathrm{z}}\right]\mathbf{G}=\frac{\partial\mathbf{G}}{\partial \mathrm{t}}-\frac{\mathrm{d}\mathbf{G}}{\mathrm{dt}},$$

substituting it into Eq. (2.4a), we obtain the Ampere-Maxwell-type UMFT,

$$\nabla\times(\mathbf{v}\times\mathbf{T})=\mathbf{v}(\nabla\cdot\mathbf{T})-\mathbf{T}(\nabla\cdot\mathbf{v})+(\mathbf{T}\cdot\nabla)\mathbf{v}-(\mathbf{v}\cdot\nabla)\mathbf{T},$$

$$\nabla\times(\mathbf{v}\times\mathbf{G})+\frac{\mathrm{d}\mathbf{G}}{\mathrm{dt}}=\mathbf{v}(\nabla\cdot\mathbf{G})+\frac{\partial\mathbf{G}}{\partial \mathrm{t}}-\mathbf{G}(\nabla\cdot\mathbf{v})+(\mathbf{G}\cdot\nabla)\mathbf{v}.\quad(2.7)$$

Defining a First level axial vector field **M**, i.e., both **v** and **G** are vector fields,

$$\mathbf{M}\equiv\mathbf{v}\times\mathbf{G}.\quad(2.8)$$

For the axial vector field **M**, by mathematical definition, we have,

$$\nabla \cdot \mathbf{M} \equiv 0. \tag{2.9}$$

Substituting Eq. (2.8) into Eq. (2.4a) and Eq. (2.7) respectively, we obtain

$$\nabla \times \mathbf{M} = \mathbf{v}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{v}) + (\mathbf{G} \cdot \nabla)\mathbf{v} - (\mathbf{v} \cdot \nabla)\mathbf{G}. \tag{2.10}$$

$$\nabla \times \mathbf{M} + \frac{\mathrm{d}\mathbf{G}}{\mathrm{dt}} = \mathbf{v}(\nabla \cdot \mathbf{G}) + \frac{\partial \mathbf{G}}{\partial \mathrm{t}} - \mathbf{G}(\nabla \cdot \mathbf{v}) + (\mathbf{G} \cdot \nabla)\mathbf{v}. \tag{2.11}$$

All terms on the right-hand side of Eq. (2.11) induce equally the axial vector field M. The interpretations for those terms are,

1. The term, $\mathbf{v}(\nabla \cdot \mathbf{G})$, plays the role of the "current";
2. The term, $\frac{\partial \mathbf{G}}{\partial \mathrm{t}}$, plays the role of the "displacement current";
3. The term, $\mathbf{G}(\nabla \cdot \mathbf{v})$, describes stretching of the G field due to source velocity compressibility;
4. The term, $(\mathbf{G} \cdot \nabla)\mathbf{v}$, describes the stretching or tilting of the G field due to the velocity gradients;
5. The terms, $\frac{\partial \mathbf{G}}{\partial \mathrm{t}}$ and $\mathbf{G}(\nabla \cdot \mathbf{v})$, have the same direction; while the terms, $\mathbf{v}(\nabla \cdot \mathbf{G})$ and $(\mathbf{G} \cdot \nabla)\mathbf{v}$, have the same direction.

The Ampere-Maxwell-type UMFT, Eq. (2.7), Eq. (2.10), Eq. (2.11), can be written respectively in the integral form as,

$$\oint(\mathbf{v} \times \mathbf{G}) \cdot \mathrm{d}\mathbf{l} + \frac{\mathrm{d}}{\mathrm{dt}}\iint \mathbf{G} \cdot \mathrm{ds} = \iint \left[\mathbf{v}(\nabla \cdot \mathbf{G}) + \frac{\partial \mathbf{G}}{\partial \mathrm{t}} - \mathbf{G}(\nabla \cdot \mathbf{v}) + (\mathbf{G} \cdot \nabla)\mathbf{v}\right] \cdot \mathrm{ds}, \tag{2.12}$$

$$\oint \mathbf{M} \cdot \mathrm{d}\mathbf{l} = \int [\mathbf{v}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{v}) + (\mathbf{G} \cdot \nabla)\mathbf{v} - (\mathbf{v} \cdot \nabla)\mathbf{G}] \cdot \mathrm{ds}, \tag{2.13}$$

$$\oint \mathbf{M} \cdot \mathrm{d}\mathbf{l} + \frac{\mathrm{d}}{\mathrm{dt}}\iint \mathbf{G} \cdot \mathrm{ds} = \int \left[\mathbf{v}(\nabla \cdot \mathbf{G}) + \frac{\partial \mathbf{G}}{\partial \mathrm{t}} - \mathbf{G}(\nabla \cdot \mathbf{v}) + (\mathbf{G} \cdot \nabla)\mathbf{v}\right] \cdot \mathrm{ds}. \tag{2.14}$$

Faraday-type UMFT:

For deriving the Faraday-type UMFT, taking the **T** field as the First level axial vector field **M** defined by Eq. (2.8), $\mathbf{T} = \mathbf{M}$. For the **M** field, we have,

$$-(\mathbf{v} \cdot \nabla)\mathbf{M} = -\left[\mathrm{v_x}\frac{\partial}{\partial \mathrm{x}} + \mathrm{v_y}\frac{\partial}{\partial \mathrm{y}} + \mathrm{v_z}\frac{\partial}{\partial \mathrm{z}}\right]\mathbf{M} = \frac{\partial \mathbf{M}}{\partial \mathrm{t}} - \frac{\mathrm{d}\mathbf{M}}{\mathrm{dt}}. \tag{2.15}$$

Substituting Eq. (2.15) into Eq. (2.4a), we obtain Faraday-type UMFT,

$$\nabla \times (\mathbf{v} \times \mathbf{M}) + \frac{\mathbf{dM}}{\mathbf{dt}} = \mathbf{v}(\nabla \cdot \mathbf{M}) + \frac{\partial \mathbf{M}}{\partial \mathbf{t}} - \mathbf{M}(\nabla \cdot \mathbf{v}) + (\mathbf{M} \cdot \nabla)\mathbf{v}. \tag{2.16}$$

Let's define a Second level axial vector field **N**, i.e., **v** is a vector and **M** is a first level axial vector,

$$\mathbf{N} \equiv -\mathbf{v} \times \mathbf{M}, \tag{2.17}$$

which satisfies mathematically,

$$\nabla \cdot \mathbf{N} = 0. \tag{2.18}$$

Substituting Eq. (2.17) into Eq. (2.16), we obtain Faraday-type UMFT,

$$\nabla \times \mathbf{N} - \frac{\mathrm{d}\mathbf{M}}{\mathrm{dt}} = -\mathbf{v}(\nabla \cdot \mathbf{M}) - \frac{\partial \mathbf{M}}{\partial \mathrm{t}} + \mathbf{M}(\nabla \cdot \mathbf{v}) - (\mathbf{M} \cdot \nabla)\mathbf{v}. \tag{2.19}$$

The Faraday-type UMFT, Eq. (2.16) and Eq. (2.19), can be written respectively in the integral form as,

$$\oint(\mathbf{v} \times \mathbf{M}) \cdot \mathrm{d}\mathbf{l} + \frac{\mathrm{d}}{\mathrm{dt}}\iint \mathbf{M} \cdot \mathrm{ds} = \iint \left[\mathbf{v}(\nabla \cdot \mathbf{M}) + \frac{\partial \mathbf{M}}{\partial \mathbf{t}} - \mathbf{M}(\nabla \cdot \mathbf{v}) + (\mathbf{M} \cdot \nabla)\mathbf{v}\right] \cdot \mathrm{ds}. \tag{2.20}$$

$$\oint \mathbf{N} \cdot \mathrm{d}\mathbf{l} - \frac{\mathrm{d}}{\mathrm{dt}}\iint \mathbf{M} \cdot \mathrm{ds} = -\iint \left[\mathbf{v}(\nabla \cdot \mathbf{M}) + \frac{\partial \mathbf{M}}{\partial \mathbf{t}} - \mathbf{M}(\nabla \cdot \mathbf{v}) + (\mathbf{M} \cdot \nabla)\mathbf{v}\right] \cdot \mathrm{ds}. \tag{2.21}$$

The term $\mathbf{v}(\nabla \cdot \mathbf{M})$ is the source term. We still keep this source term in Eq. (2.16), (2.19), (2.20) and (2.21), because this source term leaves a door open for a possible existence of a monopole physically, although, mathematically, the monopole of the **M** field does not exist.

The interpretations of those right-hand side terms of Eq. (2.19) are,

1. The term, $\mathbf{v}(\nabla \cdot \mathbf{M})$, plays the role of the "current", which is mathematically zero;
2. The term, $\frac{\partial \mathbf{M}}{\partial \mathrm{t}}$, describes the time change of the **M** fields as the source;
3. The term, $\mathbf{M}(\nabla \cdot \mathbf{v})$, describes stretching of the **M** field due to source velocity compressibility;
4. The term, $(\mathbf{M} \cdot \nabla)\mathbf{v}$, describes the stretching or tilting of the M field due to the velocity gradients;
5. The terms, $\frac{\partial \mathbf{M}}{\partial \mathrm{t}}$ and $\mathbf{M}(\nabla \cdot \mathbf{v})$, have the same direction; while the terms, $\mathbf{v}(\nabla \cdot \mathbf{M})$ and $(\mathbf{M} \cdot \nabla)\mathbf{v}$, have the same direction.

Next, substituting Eq. (2.9), $\nabla \cdot \mathbf{M} = 0$, into Eq. (2.16), Eq. (2.19), Eq. (2.20) and Eq. (2.21) respectively, we obtain source-free Faraday-type UMFT,

$$\nabla \times (\mathbf{v} \times \mathbf{M}) + \frac{\mathbf{dM}}{\mathbf{dt}} = \frac{\partial \mathbf{M}}{\partial \mathbf{t}} - \mathbf{M}(\nabla \cdot \mathbf{v}) + (\mathbf{M} \cdot \nabla)\mathbf{v}, \tag{2.22}$$

$$\nabla \times \mathbf{N} - \frac{\mathbf{dM}}{\mathbf{dt}} = -\frac{\partial \mathbf{M}}{\partial \mathbf{t}} + \mathbf{M}(\nabla \cdot \mathbf{v}) - (\mathbf{M} \cdot \nabla)\mathbf{v}. \tag{2.23}$$

$$\oint(\mathbf{v} \times \mathbf{M}) \cdot \mathrm{d}\mathbf{l} + \frac{\mathrm{d}}{\mathrm{dt}}\iint \mathbf{M} \cdot \mathrm{ds} = \iint \left[\frac{\partial \mathbf{M}}{\partial \mathbf{t}} - \mathbf{M}(\nabla \cdot \mathbf{v}) + (\mathbf{M} \cdot \nabla)\mathbf{v}\right] \cdot \mathrm{ds}. \tag{2.24}$$

$$\oint \mathbf{N} \cdot \mathrm{d}\mathbf{l} - \frac{\mathrm{d}}{\mathrm{dt}}\iint \mathbf{M} \cdot \mathrm{ds} = -\iint \left[\frac{\partial \mathbf{M}}{\partial \mathbf{t}} - \mathbf{M}(\nabla \cdot \mathbf{v}) + (\mathbf{M} \cdot \nabla)\mathbf{v}\right] \cdot \mathrm{ds}. \tag{2.25}$$

Type-2 Duality:

The **M** field is a First level axial vector field, while the **N** field is a Second level axial vector field. The **M** field and the **N** field are determined respectively by Eq. (2.11) and Eq. (2.19), which have the same form. Thus, there is type-2 duality between the First level axial vector field **M** and the Second level axial vector field **N**, which is pre-determined mathematically. For different type of duality and different level of axial vector (see Appendix A). We have derived the basic UMFT for the fields induced by velocity of sources.

UMFT Related with Spatially Uniform Velocity:

For spatially uniform velocity, we have

$$\mathbf{M}(\nabla\cdot\mathbf{v}) = (\mathbf{M}\cdot\nabla)\mathbf{v} = \boldsymbol{\nabla}\times\mathbf{v} = 0, \tag{2.26}$$

Substituting Eq. (2.26) into Eq. (2.11), we obtain Ampere-Maxwell-type UMFT for the M field,

$$\nabla\times\mathbf{M} + \frac{\mathrm{d}\mathbf{G}}{\mathrm{dt}} = \mathbf{v}(\nabla\cdot\mathbf{G}) + \frac{\partial\mathbf{G}}{\partial\mathrm{t}}. \tag{2.27}$$

Or in the integral forms,

$$\oint\mathbf{M}\cdot\mathrm{d}\mathbf{l} + \frac{\mathrm{d}}{\mathrm{dt}}\iint\mathbf{G}\cdot\mathrm{ds} = \iint\left[\mathbf{v}(\nabla\cdot\mathbf{G}) + \frac{\boldsymbol{\partial}\mathbf{G}}{\boldsymbol{\partial}\mathbf{t}}\right]\cdot\mathrm{ds}. \tag{2.28}$$

Substituting Eq. (2.26) into Eq. (2.23), we obtain Faraday-type UMFT for the N field,

$$\nabla\times\mathbf{N} - \frac{\mathbf{dM}}{\mathbf{dt}} = -\frac{\boldsymbol{\partial}\mathbf{M}}{\boldsymbol{\partial}\mathbf{t}}. \tag{2.29}$$

Or in the integral forms,

$$\oint\mathbf{N}\cdot\mathrm{d}\mathbf{l} - \frac{\mathrm{d}}{\mathrm{dt}}\iint\mathbf{M}\cdot\mathrm{ds} = -\iint\frac{\partial\mathbf{M}}{\partial\mathrm{t}}\cdot\mathrm{ds}. \tag{2.30}$$

MAXWELL EQUATIONS DERIVED FROM UMFT, COULOMB'S LAW AND VELOCITY OF E-SOURCE

In this Section, the Extended-Maxwell equations are derived mathematically from the combination of UMFT, the Coulomb's law and velocity of the electric source, e-source. Under condition of spatially uniform moving electric charge (e-charge), Extended-Maxwell equations reduce to regular Maxwell equations, which demonstrates that UMFT is valid for studying physic fields. Thus, leads us to apply UMFT to study other physical fields, such as gravity and spin related phenomena.

Extended-Faraday's Law

Starting from the Faraday-type UMFT, Eq. (2.20). Let the **M** field is a magnetic field **B**, $\mathbf{M}\equiv\mathbf{B}$, Eq. (2.20) gives,

$$\oint -(\mathbf{v}\times\mathbf{B})\cdot\mathrm{d}\mathbf{l} - \frac{\mathrm{d}}{\mathrm{dt}}\iint\mathbf{B}\cdot\mathrm{ds} = -\iint\left[\mathbf{v}(\nabla\cdot\mathbf{B}) + \frac{\boldsymbol{\partial}\mathbf{B}}{\boldsymbol{\partial}\mathbf{t}} - \mathbf{B}(\nabla\cdot\mathbf{v}) + (\mathbf{B}\cdot\nabla)\mathbf{v}\right]\cdot\mathrm{ds}. \tag{3.1}$$

And the definition of **M**, Eq. (2.9), gives mathematically Gauss' s law of magnetism,

$$\nabla \cdot \mathbf{B} \equiv 0. \tag{3.2}$$

Let's show that Eq. (3.1) is consistent with Faraday's law. The Faraday's law gives

$$\frac{\mathrm{d}}{\mathrm{dt}} \iint \mathbf{B} \cdot \mathrm{ds} = -\oint \mathbf{E}' \cdot \mathrm{dl} \tag{3.3}$$

where $\mathbf{E}'$ is the electric field at the circuit d**l** in a reference frame in which d**l** is at rest. The **B** is a magnetic field at the neighborhood of the circuit.

Applying Eq. (3.3), Eq. (3.1) becomes the Extended-Faraday law,

$$\oint (\mathbf{E}' - \mathbf{v} \times \mathbf{B}) \cdot \mathrm{d}\mathbf{l} = -\iint \left[\mathbf{v}(\nabla \cdot \mathbf{B}) + \frac{\partial \mathbf{B}}{\partial \mathbf{t}} - \mathbf{B}(\nabla \cdot \mathbf{v}) + (\mathbf{B} \cdot \nabla)\mathbf{v}\right] \cdot \mathrm{ds}, \tag{3.4}$$

where the **v** is the velocity of the circuit d**l** relative to a laboratory frame.

Let's define an electric field **E** in the laboratory frame,

$$\mathbf{E} \equiv \mathbf{E}' - \mathbf{v} \times \mathbf{B}. \tag{3.5}$$

Applying Eq. (3.5), Eq. (3.4) gives the integral and differential forms of Extended-Faraday's law,

$$\oint \mathbf{E} \cdot dl = -\iint \left[\mathbf{v}(\nabla \cdot \mathbf{B}) + \frac{\partial \mathbf{B}}{\partial \mathbf{t}} - \mathbf{B}(\nabla \cdot \mathbf{v}) + (\mathbf{B} \cdot \nabla)\mathbf{v}\right] \cdot \mathrm{ds}, \tag{3.6}$$

$$\nabla \times \mathbf{E} = -\mathbf{v}(\nabla \cdot \mathbf{B}) - \frac{\partial \mathbf{B}}{\partial t} + \mathbf{B}(\nabla \cdot \mathbf{v}) - (\mathbf{B} \cdot \nabla)\mathbf{v}. \tag{3.7}$$

In order to show the duality between the Extended-Faraday's and Extended-Ampere-Maxwell's law, we still keep the term $\nabla \cdot \mathbf{B}$ in Eq. (3.7), although it is equal to zero (Eq. 3.2).

The interpretations of terms of the right-hand side of Eq. (3.7) are,

1. The term, $\mathbf{B}(\nabla \cdot \mathbf{v})$, describes stretching of the B field due to source velocity compressibility;
2. The term, $(\mathbf{B} \cdot \nabla)\mathbf{v}$, describes the stretching or tilting of the B field due to the velocity gradients.

Let's define a "current" generating induced electric field,

$$\mathbf{j}_{\mathbf{v-E}} \equiv \mathbf{v}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{v}) + (\mathbf{B} \cdot \nabla)\mathbf{v}. \tag{3.8}$$

Where the subscripts "v-E" represent the quantities related with velocity and electric field respectively.

Then Extended-Faraday's law, Eq. (3.7), may be rewritten as,

$$\nabla \times \mathbf{E} = -\mathbf{j}_{\mathbf{v-E}} - \frac{\partial \mathbf{B}}{\partial t}. \tag{3.9}$$

Eq. (3.9) shows the equation of continuity as

$$\nabla \cdot \left(\mathbf{j}_{\mathbf{v-E}} + \frac{\partial \mathbf{B}}{\partial t}\right) = 0. \tag{3.10}$$

For the situation in which, (1) $\nabla \cdot \mathbf{B} = 0$; and (2) the velocity is spatially-uniform, i.e., $\mathbf{B}(\nabla \cdot \mathbf{v}) = (\mathbf{B} \cdot \nabla)\mathbf{v} = 0$, Extended-Faraday's law, Eq. (3.7), gives the Faraday's law,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{3.11}$$

where the $\mathbf{E}$ field is an axial vector field and $\mathbf{j}_{\mathbf{v-E}} = 0$.

Therefore, Extended-Faraday's law, Eq. (3.1), is consistent with Faraday's law. Extended-Faraday's law predicts that the spatially-varying velocity, via terms $\mathbf{B}(\nabla \cdot \mathbf{v})$ and $(\mathbf{B} \cdot \nabla)\mathbf{v}$**,** create axial vector $\mathbf{E}$ fields.

To detect the effects of the terms, $\mathbf{B}(\nabla \cdot \mathbf{v})$ and $(\mathbf{B} \cdot \nabla)\mathbf{v}$, will confirm whether Extended-Faraday's law and thus, UMFT are correct.

Indeed, by the definition of the axial vector field $\mathbf{B}$, its divergence is zero mathematically, namely, from the perspective of the UMFT and Coulomb's law, there is no "magnetic monopole charge".

Extended-Ampere-Maxwell's Law (1)

The combination of UMFT and Coulomb's law leads us to let $\mathbf{G} \equiv \mathbf{E}$. Then Eq. (2.12) of UMFT provides,

$$\oint(\mathbf{v} \times \mathbf{E}) \cdot d\mathbf{l} + \frac{d}{dt}\iint \mathbf{E} \cdot ds = \iint \left[\mathbf{v}(\nabla \cdot \mathbf{E}) + \frac{\partial \mathbf{E}}{\partial t} - \mathbf{E}(\nabla \cdot \mathbf{v}) + (\mathbf{E} \cdot \nabla)\mathbf{v}\right] \cdot ds. \tag{3.12a}$$

Substituting the Coulomb's law, Eq. 3.12 becomes,

$$\oint(\mathbf{v} \times \mathbf{E}) \cdot d\mathbf{l} + \frac{d}{dt}\iint \mathbf{E} \cdot ds = \iint \left[Q_e\mathbf{v} + \frac{\partial \mathbf{E}}{\partial t} - \mathbf{E}(\nabla \cdot \mathbf{v}) + (\mathbf{E} \cdot \nabla)\mathbf{v}\right] \cdot ds. \tag{3.12b}$$

Eq. (3.12b) shows that the three quantities, Q_e, $\mathbf{E}$, $\mathbf{v}$, create magnetic field as mentioned in Table 1.1.

Type-2 Duality between Electric and Magnetic Fields

The First level axial vector field $(\mathbf{v} \times \mathbf{E})$ and Eq. (3.12) are the type-2 dual to the Second level axial vector field $(\mathbf{v} \times \mathbf{B})$ and Eq. (3.1) respectively. Moreover, Eq. (3.6) is equivalent to Eq. (3.1). Based on the Transfer Rules between dualities of Appendix A, there is a dual of Eq. (3.6), i.e., under transformation,

$$\mathbf{E} \leftrightarrow \mathbf{B} \text{ and } \mathbf{B} \leftrightarrow -\mathbf{E},$$

we have a type-2 dual of Eq. (3.5) and Eq. (3.6), which are,

$$\mathbf{E} \equiv \mathbf{E}' - \mathbf{v} \times \mathbf{B} \quad \leftrightarrow \quad \mathbf{B} \equiv \mathbf{B}' + \mathbf{v} \times \mathbf{E},$$

$$\oint \mathbf{B} \cdot dl = \iint \left[\mathbf{v}(\nabla \cdot \mathbf{E}) + \frac{\partial \mathbf{E}}{\partial \mathbf{t}} - \mathbf{E}(\nabla \cdot \mathbf{v}) + (\mathbf{E} \cdot \nabla)\mathbf{v}\right] \cdot \mathrm{ds}, \tag{3.13}$$

which are equivalent to Eq. (3.6) and thus, Eq. (3.1), and finally, Eq. (3.12).

With distinguishable feature of "type-1 duality" and "type-2 duality", the duality between induced electric field determined by the Faraday's law and magnetic field determined by Ampere-Maxwell's equation is actually a type-2 duality. UMFT provides the mathematical originations of the type-2 duality between axial electric field and magnetic field.

Extended-Ampere-Maxwell's Law (2)
Eq. (3.13) gives Extended-Ampere-Maxwell law,

$$\nabla \times \mathbf{B} = \mathbf{v}(\nabla \cdot \mathbf{E}) + \frac{\partial \mathbf{E}}{\partial \mathbf{t}} - \mathbf{E}(\nabla \cdot \mathbf{v}) + (\mathbf{E} \cdot \nabla)\mathbf{v}. \tag{3.14}$$

The magnetic field **B** is, in the laboratory frame,

$$\mathbf{B} = \mathbf{B}' + \mathbf{v} \times \mathbf{E}. \tag{3.15}$$

Where $\mathbf{B}'$ is the magnetic field at the circuit d**l** in a reference frame in which d**l** is at rest. The **E** is an electric field at the neighborhood of the circuit. The **v** is the velocity of the circuit relative to a laboratory frame.

Note there is no negative sign in front of the $\frac{\partial \mathbf{E}}{\partial \mathbf{t}}$, because that the time change of the **E** field through the circuit d**l** purely induces a magnetic field $\mathbf{B}'$ that does not accumulate e-particles to against the time change of the **E** field.

Equation of Continuity
Utilizing Coulomb's law, $\nabla \cdot \mathbf{E} = \rho_e$, let's define a "current" generating magnetic field, denote it as

$$\mathbf{j}_{\mathbf{v-B}} = \rho_e \mathbf{v} - \mathbf{E}(\nabla \cdot \mathbf{v}) + (\mathbf{E} \cdot \nabla)\mathbf{v}. \tag{3.16}$$

Where the subscripts "v-B" represent the quantity related with velocity and magnetic field respectively. Then Eq. (3.14) becomes,

$$\nabla \times \mathbf{B} = \mathbf{j}_{\mathbf{v-B}} + \frac{\partial \mathbf{E}}{\partial \mathbf{t}}. \tag{3.17}$$

The current $\mathbf{j}_{\mathbf{v-B}}$ satisfies the equation of continuity,

$$\nabla \cdot \mathbf{j_{v-B}} + \frac{\partial \rho_e}{\partial \mathbf{t}} = \mathbf{0}. \tag{3.18}$$

For the situation of the spatially uniform velocity, i.e.,

$$\mathbf{E}(\nabla \cdot \mathbf{v}) = (\mathbf{E} \cdot \nabla)\mathbf{v} = 0,$$

we have $\mathbf{j_{v-B}} = \rho_e \mathbf{v}$, Extended-Ampere-Maxwell equation, Eq. (3.14), becomes the Ampere-Maxwell law,

$$\nabla \times \mathbf{B} = \mathbf{v}(\nabla \cdot \mathbf{E}) + \frac{\partial \mathbf{E}}{\partial t}. \tag{3.19}$$

Extended-Ampere-Maxwell's law Eq. (3.14): (1) predicts that the products, $\mathbf{E}(\nabla \cdot \mathbf{v})$ and $(\mathbf{E} \cdot \nabla)\mathbf{v}$, create respectively axial vector $\mathbf{B}$ field; and (2) provides a mathematical origin of why and how e-current, $\rho_e \mathbf{v}$, and displacement-current, $\frac{\partial \mathbf{E}}{\partial \mathbf{t}}$, create inevitably magnetic fields.

Extended-Maxwell Equations

Now, from UMFT, Coulomb's law and the velocity of the source, we derived Extended-Maxwell equations,

$$\nabla \cdot \mathbf{E} = \rho_e, \nabla \times \mathbf{B} = \mathbf{v}(\nabla \cdot \mathbf{E}) + \frac{\partial \mathbf{E}}{\partial \mathbf{t}} - \mathbf{E}(\nabla \cdot \mathbf{v}) + (\mathbf{E} \cdot \nabla)\mathbf{v}, \tag{3.14}$$

$$\nabla \cdot \mathbf{B} \equiv 0,\ \nabla \times \mathbf{E} = -\mathbf{v}(\nabla \cdot \mathbf{B}) - \frac{\partial \mathbf{B}}{\partial t} + \mathbf{B}(\nabla \cdot \mathbf{v}) - (\mathbf{B} \cdot \nabla)\mathbf{v}, \tag{3.7}$$

Where the $\mathbf{E}$ field is a combination of vector field (Coulomb's field) and axial vector field,

$$\mathbf{E} = \mathbf{E}_{\text{vector}} + \mathbf{E}_{\text{axial-vector}}, \nabla \cdot \mathbf{E}_{\text{axial-vector}} \equiv 0, \nabla \cdot \mathbf{E} = \nabla \cdot \mathbf{E}_{\text{vector}},$$

$$\nabla \times \mathbf{E}_{\text{vector}} \equiv 0, \nabla \times \mathbf{E} = \nabla \times \mathbf{E}_{\text{axial-vector}}.$$

According to above derivation of Extended-Maxwell equations, we have

$$\mathbf{B} = \mathbf{B}_{\text{vector}} + \mathbf{B}_{\text{axial-vector}} = \mathbf{B}_{\text{axial-vector}},\ \nabla \cdot \mathbf{B} = \nabla \cdot \mathbf{B}_{\text{axial-vector}} \equiv 0.$$

Therefore, it is $\mathbf{E}_{\text{axial-vector}}$ that is type-2 dual to $\mathbf{B}_{\text{axial-vector}}$. There is no magnetic dual of $\mathbf{E}_{\text{vector}}$.

Extended-Maxwell Equations Reducing to Maxwell Equations for Uniform Motion of e-Charge

Note the Extended-Maxwell equations are not compatible with SR, the main reason is that SR was derived from and applied to inertial frames. So, when we ignore the non-inertial term, $(\nabla \cdot \mathbf{v})$, $(\mathbf{E} \cdot \nabla)\mathbf{v}$, $(\mathbf{B} \cdot \nabla)\mathbf{v}$, the extended-Maxwell equations reduce to standard Maxwell equation,

$$\nabla \cdot \mathbf{E} = \rho_e, \nabla \cdot \mathbf{B} = 0,$$

$$\nabla \times \mathbf{B} = \mathbf{v}(\nabla \cdot \mathbf{E}) + \frac{\partial \mathbf{E}}{\partial \mathbf{t}},\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial \mathrm{t}}. \tag{3.20}$$

Writing Eq. (3.16), Eq. (3.20), Eq. (3.11) and Eq. (3.2) in tensor form,

$$\frac{\partial F_e^{\alpha\beta}}{\partial x^\beta} = J_e^\alpha,\ \frac{\partial F_e^{\alpha\beta}}{\partial x^\gamma} + \frac{\partial F_e^{\beta\gamma}}{\partial x^\alpha} + \frac{\partial F_e^{\gamma\alpha}}{\partial x^\beta} = 0,\ J_e^\alpha = J_{e+}^\alpha + J_{e-}^\alpha = (\rho_e,\ \mathbf{J}_e). \tag{3.21}$$

Where $F_e^{\alpha\beta} = \partial^\alpha A_e^\beta - \partial^\beta A_e^\alpha$ is the field strength tensor, A_e^α is four-vector potential, J_e^α is four-current. Eq. (3.20) and Eq. (3.21) satisfy Lorentz transformation and comply with Special Relativity.

Lorentz Force Derived from Coulomb Force

In a reference frame S' in which a test e-particle q_e is at rest, an electric field is denote as E'. The force acting on the test e-particle is the Coulomb force,

$$\mathbf{F} = q_e \mathbf{E}'. \tag{3.22}$$

Transferring to a laboratory frame in which the test e-particle is moving with velocity **v**, Eq. (3.5) gives,

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \tag{3.23}$$

where the electric field **E** is measured in the laboratory frame. The magnetic field **B** is measure in the Laboratory frame at the neighborhood of the test e-particle. Substituting Eq. (3.23) into Eq. (3.22), the Coulomb force extended to the Lorentz force measured in the laboratory frame,

$$\mathbf{F} = q_e \mathbf{E} + q_e \mathbf{v} \times \mathbf{B}. \tag{3.24}$$

Positive and Negative Energy of Electromagnetic Field

To form an assemble of e-charges with the same signs, an external force is needed to bring those same sign e-charges together, namely the external force done the work $W_{External}$ and transfer the energy to the electric field of the assemble. The work $W_{External}$ done by an external force to assemble a volume configuration with a e-charge density ρ_e is

$$W_{External} = \frac{1}{2} \int \rho_e V_e d\tau. \tag{3.25}$$

Thus, the change of the energy of an electric field is defined as positive,

$$\Delta U_e = W_{External} > 0, \tag{3.26}$$

Which represents the energy stored in the configuration, which is also considered conventionally as the potential energy of the electric field.

On the contrary, for bring e-charges with opposite signs together, no external force is needed, and the electric field of e-charges done the work, i.e., the electric field losses energy. The change of the potential energy is defined as the negative of the work done by the electric field, i.e., the electric field loss energy,

$$\Delta U_e = -W_{field} < 0. \tag{3.27}$$

Where the potentials of positive and negative e-charges are respectively,

$$V_{e+} = \frac{Q_{e+}}{r} > 0, V_{e-} = \frac{Q_{e-}}{r} < 0. \tag{3.28}$$

Therefore, the definition of the positive or negative potential energy of an electric field is a matter of bookkeeping [3].

Using Coulomb's law and vector formula, Eq. (3.25) gives

$$\Delta U_e = W_{External} = \frac{1}{2}\int E^2\, d\tau. \tag{3.29}$$

Similarly, the work done to retain the e-current going is, setting $\epsilon_0 = \mu_0 = 1$,

$$W_B = \frac{1}{2}\int B^2\, d\tau, \tag{3.30}$$

Which is also considered as the energy stored in a magnetic field. The total energy, W_{EM}, stored in electromagnetic field is the sum of energies of both electric field and magnetic field,

$$W_{EM} = \frac{1}{2}\int (E^2 + B^2) d\tau. \tag{3.31}$$

Extended-Poynting Theorem

Now let's calculate the rate at which work "W" done to move e-changes under the influence of electric and magnetic fields, $dW = F \cdot dl$. Substituting Lorentz-force, Eq. (3.24), then

$$\frac{dW}{dt} = \int (E \cdot J) d\tau. \tag{3.32}$$

Substituting Eq. (3.14), $\nabla \times B = J + \frac{\partial E}{\partial t} - E(\nabla \cdot v) + (E \cdot \nabla)v$, into Eq. (3.32), we obtain

$$E \cdot J = E \cdot (\nabla \times B) - E \cdot \frac{\partial E}{\partial t} + E^2(\nabla \cdot v) - E \cdot [(E \cdot \nabla)v]. \tag{3.33}$$

The formula of vector analysis gives $E \cdot (\nabla \times B) \equiv B \cdot (\nabla \times E) - \nabla \cdot (E \times B)$. Substituting Eq. (3.7), $\nabla \times E = -\frac{\partial B}{\partial t} + B(\nabla \cdot v) - (B \cdot \nabla)v$, into it, then we have $E \cdot (\nabla \times B) = B \cdot \left[-\frac{\partial B}{\partial t} + B(\nabla \cdot v) - (B \cdot \nabla)v\right] - \nabla \cdot (E \times B)$. Or

$$E \cdot (\nabla \times B) = -\frac{1}{2}\frac{\partial B^2}{\partial t} + B^2(\nabla \cdot v) - B \cdot [(B \cdot \nabla)v] - \nabla \cdot (E \times B). \tag{3.34}$$

Substituting Eq. (3.34) into Eq. (3.33), we obtain

$$\mathbf{E}\cdot\mathbf{J} = -\frac{1}{2}\frac{\partial}{\partial t}\left(B^2 + E^2\right) - \nabla\cdot(\mathbf{E}\times\mathbf{B}) + \left(B^2 + E^2\right)(\nabla\cdot\mathbf{v}) - \mathbf{B}\cdot[(\mathbf{B}\cdot\nabla)\mathbf{v}] - \mathbf{E}\cdot[(\mathbf{E}\cdot\nabla)\mathbf{v}]. \quad (3.35)$$

Substituting Eq. (3.31) and Eq. (3.35) into Eq. (3.32), we derived the *Extended-Poynting's theorem*,

$$\frac{dW}{dt} = -\frac{dW_{EM}}{dt} - \oiint \mathbf{S}\cdot d\mathbf{a} + \int\{(B^2 + E^2)(\nabla\cdot\mathbf{v}) - \mathbf{B}\cdot[(\mathbf{B}\cdot\nabla)\mathbf{v}] - \mathbf{E}\cdot[(\mathbf{E}\cdot\nabla)\mathbf{v}]\}\, d\tau, \quad (3.36)$$

Where the Poynting's vector S is defined as,

$$\mathbf{S} \equiv \mathbf{E}\times\mathbf{B}. \quad (3.37)$$

For the special situation of spatially uniform velocity, $\nabla\cdot\mathbf{v} = (\mathbf{B}\cdot\nabla)\mathbf{v} = (\mathbf{E}\cdot\nabla)\mathbf{v} = 0$, the Extended-Poynting theorem deduces to the Poynting's theorem.

Origin of Differences between Static Electric Field and Magnetic Field

In the Section of **Motivation**, we have mentioned several fundamental differences: (1) "e-charge" vs. "e-current"; (2) "$\nabla\cdot\mathbf{E}$" vs. "$\nabla\times\mathbf{B}$"; (3) "vector field **E**" vs. "axial vector field **B**"; (4) "$q_e\mathbf{E}$" vs. "$q_e\mathbf{v}\times\mathbf{B}$".

In this Section we explain those differences below:

Comparison of Eq. (3.7) and Eq. (3.14) explains the first, second and third differences as the following: e-charge ρ_e induces **E** via $\nabla\cdot\mathbf{E} = 4\pi\rho_e$; it is UMFT combining with the Coulomb law that makes the term, $\mathbf{v}(\nabla\cdot\mathbf{E}) = 4\pi\rho_e\mathbf{v}$, induces **B** field via $\nabla\times\mathbf{B}$; Coulomb e-field **E** is a vector field, and $\mathbf{B}\sim\mathbf{v}\times\mathbf{E}$ is a first level axial vector field. Eq. (3.22) to Eq. (3.23) show that due to the fact that motion of e-charge creates magnetic field, thus, when transferring from an e-charge-rest frame to an e-charge-moving frame, the Coulomb's force in the e-charge-rest frame transfers to the Lorentz force in the e-charge-moving frame.

All of above-mentioned differences between static electric field and magnetic field have a mathematical origin, Eq. (2.1), which shows how a divergence field creates a curl field.

No Magnetic Monopole Charge Mathematically

In the derivation of Extended-Ampere-Maxwell equation, magnetic field is defined in Eq. (3.1) as an axial vector field, $\mathbf{B} \equiv \mathbf{v}\times\mathbf{E}.$ By the mathematical definition, the divergence of an axial vector is identically to zero, which is represented as Eq. (3.2), $\nabla\cdot\mathbf{B} \equiv 0$.

Indeed, mathematically, there is no magnetic monopole charge in vector electromagnetic theory, as long as the magnetic field is an axial vector field.

Discussion and Summary

Combining UMFT, Coulomb's law and velocity, we derived Extended EM, which: (1) justifies UMFT; (2) shows that the experiments-based EM have their mathematical origin; and (3) the uniform motion of an e-particle inevitably induce an axial vector magnetic field, which is predetermined mathematically. Moreover Extended EM predicts new effects that the spatially varying velocity of e-particle induces axial magnetic field and axial induced electric field.

We, in Section 3, have shown that, mathematically, the movement of e-charge produces the magnetic field. Namely the existence of the magnetic field is inevitable. Historically, the effects of magnetic field were discovered experimentally started in 1860s. Meanwhile, the vector analysis was established in the same era, 1860s [4]. It is reasonable to assume that physicists were not familiar with vector analysis at those years. We argue that this might be the reason why UMFT was not developed when Maxwell equation were established. Moreover, then Maxwell equations were well applied to variety of areas, so no mathematical derivation of Maxwell equations was focused.

MAXWELL-TYPE GRAVITY DERIVED FROM UMFT, NEWTON'S LAW AND VELOCITY OF SOURCE

We apply the UMFT to gravity in this Section to derive a Maxwell-type vector theory, Gravito-EM, which is one step further than Newton's theory to understand gravity, although may not be the ultimate theory of gravity. Indeed, since the Gravito-EM is derived mathematically, thus, any further theory of gravity should have not only Newton's law but Gravito-EM as approximations under certain condition. Moreover, we have shown that Gravito-EM has other fundamental benefits that it can be quantized and unified with other interaction without difficulty [5].

Positive and Negative Gravitational Charge (g-Charge)

Introduction of g-charge and Negative g-charge:

Two paths of studying gravity are: First path is to treat gravity, as other three forces, as physical fields; Second path is to treat gravity and other forces as geometric phenomena. Along the first path, other three forces, except gravity, were physically understood, quantized and unified successfully. Along the second path, to geometrize gravity was quite successful, but not other three forces. Tremendous efforts have been devoted on to quantize geometric theory of gravity and unify it with theories of other three forces, so far, without commonly accepted approach. Facing the tremendous difficulties in quantizing geometric theory of gravity and unification, Extended-Maxwell equations have been derived mathematically in Section 3 from the combination of UMFT and the Coulomb's law.

The fact that the Newton's law is similar to the Coulomb's law strongly suggests to study whether and under what condition, one can mathematically derived a counterpart of Extended-Maxwell equation from the UMFT and the Newton's law. For this aim, let's compare the Newton's law and Coulomb's law at the level of the classical field theory.

Table 4.1: Comparison between Newton's law and Coulomb's law

	Coulomb's law	Newton's law
Charge	Two kinds e-charge: Positive/negative	One kind g-charge: Positive/?
Coulomb's law vs Newton's law	$\nabla \cdot \mathbf{E} = \rho_e$	$\nabla \cdot \mathbf{g} = -\rho_g$

Force	Long-range inverse-square-law $\mathbf{F} = q_e\mathbf{E}$ Attractive/repulsive	Long-range inverse-square-law $\mathbf{F} = m\mathbf{g}$ Attractive/?

Table 4.1 shows the comparison and suggests that it is the fact, two kinds of e-charge vs. one kind of gravitational charge (denoted as g-charge), that leads to that attractive/repulsive electric force vs. attractive gravitation force. The significant differences are in three aspects.

Firstly, the difference in forces: The Coulomb's forces are either attractive or repulsive, while conventional gravity force is only attractive. However, this understanding of gravitational force was changed. In 1998, scientists reported that the universe is expended with acceleration [6-8], even the acceleration is accelerating, which indicates the existence of repulsive gravitational force. Astronomers invoke dark energy for this repulsive force that creates a constant acceleration.

An effective dynamic model for explaining the accelerated expansion of the universe was proposed [9]. The model is based on a hypothesis of the existence of the negative g-charges. The results of this model contain those of the dark energy models and consistent with the observations. Moreover, an alternative physical interpretation of the cosmological constant is that it is equivalent to the negative g-charges as a source of gravitational fields. This interpretation avoids the fine-tuning problem of the cosmological constant.

We suggest that the accelerated expansion of the normal universe is observational evidence of the existence of the negative g-charges. On the other hand, this model predicts that a sub-universe filled with negative g-charges is accelerated collapse and thus, that the equation of state and the acceleration of the expansion of the universe are time dependent, namely the acceleration of the expansion of the universe is accelerating, called jerking.

Although GR is now a hundred-year-old theory, it remains a powerful and controversial idea in cosmology. It is one of the basic assumptions behind the current cosmological model: a model that is both very successful in matching observations but implies the existence of both dark matter and dark energy. These indicates that our understanding of gravity is incomplete. We likely need a new profound idea to explain these mysteries and require more powerful observations and experiments to light the path toward our new insights.

Secondly, the difference in charges: There are positive and negative e-charges in EM, while in gravity only one positive gravitational mass. To establish a perfect duality between the Coulomb's law and Newton's law, we need the concept of g-charge. The positive g-charge, $Q_{g+} \equiv +\sqrt{G}m$, was introduced [10]. After the discovery of the accelerating universe, the repel force emerged. We proposed that the origin of the repulsive force is gravity, and introduced the concept of negative g-charge [11] to explain the universe expansion.

Before introducing negative g-charge into the theories of gravity, let's postulate two hypotheses:

Hypothesis 1: An object having rest mass, $m_0 > 0$ ($m_0 = \int \rho_{m0} d^3x$), may carry either positive or negative g-charge defined as,

$$Q_{g+} \equiv +\sqrt{G} m_0, \qquad \rho_{g+} = +\sqrt{G}\rho_{m0}, \tag{4.1}$$

$$Q_{g-} \equiv -\sqrt{G} m_0, \qquad \rho_{g-} = -\sqrt{G}\rho_{m0}. \tag{4.2}$$

Eq. (4.1) and Eq. (4.2) can be combined as,

$$Q_{g\pm} \equiv \pm\sqrt{G} m_0, \quad \rho_{g\pm} = \pm\sqrt{G}\rho_{m0}. \tag{4.3}$$

$$Q_{gNet} = Q_{g+} + Q_{g-}, \quad \rho_{gNet} = \rho_{g+} + \rho_{g-}. \tag{4.4}$$

Where "G" is the Newton constant; the quantities with subscript "+", "−", "±" and "Net" are related with positive, negative, either positive or negative, and Net g-charges, respectively.

When one kind of g-charges is defined as positive, then g-charges that repels it is defined as negative. For convenience, let's define the g-charges carried by the ordinary matters in the universe as the positive g-charges.

The existence of negative g-charge has been justified at quantum level [3].

Thirdly, Duality: When one converts e-charge Q_e to negative g-charge ($-Q_g$), and the electric field strength **E** to the Newton gravitational field strength **g**, then the Coulomb's field equation converts to Newton's field equation, and vice versa. However, duality between force laws is different, one needs to converts e-charge Q_e to g-charge, Q_g, and the electric field strength **E** converted to the Newton gravitational field strength **g**, then the Coulomb's force law converts to Newton's force law, and vice versa.

We propose a positive/negative g-charge conjugation, which leads to:

Hypothesis 2: the laws governing gravitational interaction generated respectively by positive and negative g-charges have the same form.

By analogy to e-charge conjugation, the g-charge conjugation is that under transformation of g-charge conjugation, gravitation laws are unchanged. Repulsive gravity then exists, since that would imply the existence of g-charges with opposite sign.

Interaction between Positive and Negative g-Charges:
According to Hypothesis 2, the Newton's law,

$$\nabla \cdot \mathbf{g}_+ = -Q_{g+} = -\sqrt{G} m_{0+}, \tag{4.5}$$

$$\mathbf{F}_+ = q_{g+}\mathbf{g}_+, \tag{4.6}$$

is extended for the negative g-charge as,

$$\nabla\cdot\mathbf{g}_- = -Q_{g-} = \sqrt{G}m_{0-}, \tag{4.7}$$

$$\mathbf{F}_- = q_{g-}\mathbf{g}_-. \tag{4.8}$$

Combining Eq. (4.5) and Eq. (4.7) leads to the extended-Newton's law,

$$\nabla\cdot\mathbf{g} = -Q_g, \tag{4.9}$$

$$\mathbf{F}_\pm = q_{g\pm}\mathbf{g}, \tag{4.10}$$

where,

$$\mathbf{g} \equiv \mathbf{g}_+ + \mathbf{g}_-, \tag{4.11}$$

$$Q_g = Q_{g+} + Q_{g-}. \tag{4.12}$$

Q_{g+}/Q_{g-} and q_{g+}/q_{g-} are g-charges of source and test-object respectively. Accordingly, two "negative g-charges" attract each other in the same way that two "positive g-charge" do, while a negative g-charge and a positive g-charge repel each other.

Now the Extended-Newton's law, Eq. (4.9), is perfect dual to the Coulomb's law. Thus, following the same procedure, we can apply UMFT and the Extended-Newton's law to derive vector field equations for gravitational fields, with moving sources.

For a point g-charge as a source, the solutions of Eq. (4.5), Eq. (4.7) and Eq. (4.9) are respectively,

$$\mathbf{g}_+ = -\frac{Q_{g+}}{r^2}\hat{\mathbf{r}}, \tag{4.13}$$

$$\mathbf{g}_- = -\frac{Q_{g-}}{r^2}\hat{\mathbf{r}}, \tag{4.14}$$

$$\mathbf{g} = -\frac{Q_{g+}+Q_{g-}}{r^2}\hat{\mathbf{r}}, \tag{4.15}$$

where $\hat{\mathbf{r}}$ is the unit vector and points radially outward.

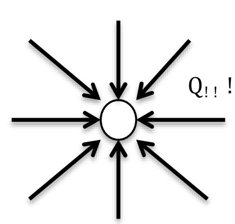

Fig. 4.1. Field line of Q! "

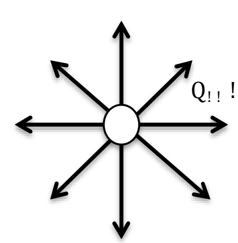

Fig. 4.2. Field line of Q! #

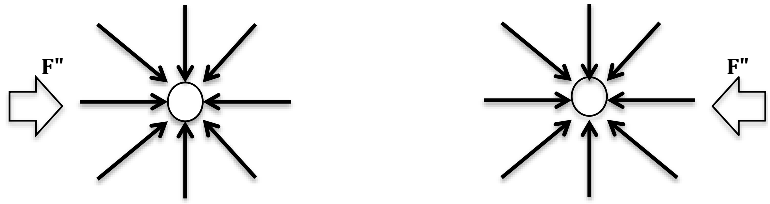

Fig. 4.3: two positive g-charges are attractive to each other

The field lines of gravitational fields, $\mathbf{g}_+$ and $\mathbf{g}_-$, are shown in Fig. 4.1 and Fig. 4.2, respectively [12].

Thus, two positive g-charges are attractive to each other (Fig. 4.3):

$$\mathbf{F}_+ = q_{g+}\,\mathbf{g}_+ = q_{g+}(-\frac{Q_{g+}}{r^2}\hat{\mathbf{r}}). \tag{4.16}$$

Two negative g-charges are attractive to each other (Fig. 4.4):

$$\mathbf{F}_- = q_{g-}\,\mathbf{g}_- = q_{g-}(-\frac{Q_{g-}}{r^2}\hat{\mathbf{r}}). \tag{4.17}$$

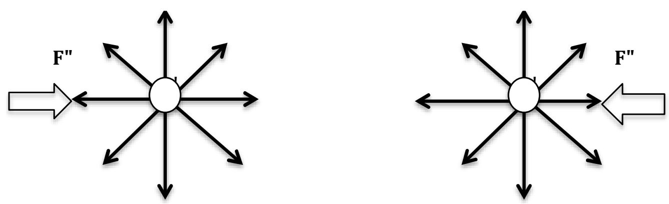

Fig. 4.4: two negative g-charges are attractive to each other

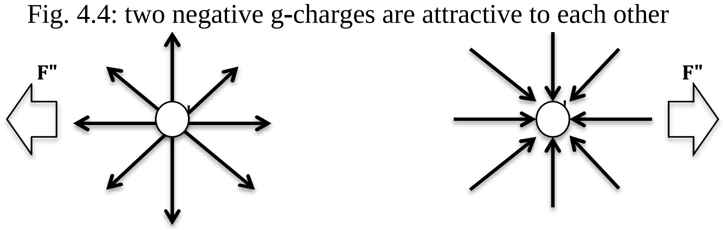

Fig. 4.5: A positive and a negative g-charges repel each other

A positive and a negative g-charges repel to each other (Fig. 4.5):

$$\mathbf{F}_+ = q_{g+}\,\mathbf{g}_- = q_{g+}(-\frac{Q_{g-}}{r^2}\hat{\mathbf{r}}), \tag{4.18}$$

$$\mathbf{F}_- = q_{g-}\,\mathbf{g}_+ = q_{g-}(-\frac{Q_{g+}}{r^2}\hat{\mathbf{r}}). \tag{4.19}$$

Indirect Evidence of Negative g-Charges: Accelerating and Jerking Expansion of Universe:
Extended-Newton's Theory with Negative g-Charges:
It has been shown that, by introducing negative g-charge, the Extended-Newton's theory and Extended-Einstein's equation can explain the accelerated expansion of the Universe and predict that the acceleration of universe is accelerating, which distinguishes negative g-charge model from that of dark energy model and cosmology constant model.

Let's assume that objects in our Universe: (1) have positive mass; (2) carry either positive or negative g-charges, and form positive or negative sub-Universe, respectively.

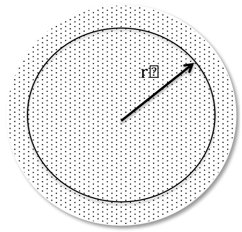

Fig. 4.6: Accelerating Expansion of Universe

Let's consider gravitational fields of a spherical distribution (Fig. 4.6) of mixed negative and positive g-charges. Eq. (4.9) gives

$$\nabla \cdot \mathbf{g} = -\sqrt{G}[\rho_{m0+}(r,t) - \rho_{m0-}(r,t)]. \tag{4.20}$$

Where $\rho_{g-} \equiv -\sqrt{G}\,\rho_{m0-}$ and $\rho_{g+} \equiv \sqrt{G}\,\rho_{m0+}$ are negative g-charge density and positive g-charge density respectively. And ρ_{m0-} (>0) and ρ_{m0+} (>0) are the mass densities of objects with negative g-charge in negative sub-Universe and with positive g-charge in positive sub-Universe, respectively.

Eq. (4.19) gives the gravitational field at **r**,

$$\mathbf{g} = -\frac{\sqrt{G}\int[\rho_{m0+}(r,t) - \rho_{m0-}(r,t)]d^3x}{r^2}\hat{\mathbf{r}}. \tag{4.21}$$

For uniform distribution of positive and negative sub-Universes, the gravitational field **g** at r is,

$$\mathbf{g} = -\frac{4\pi\sqrt{G}}{3}[\rho_{m0+}(t) - \rho_{m0-}(t)]\,\mathbf{r}. \tag{4.22}$$

The motion of a non-relativistic object carrying positive g-charge at a given r is described by Eq. (4.6). Substituting Eq. (4.15) into Eq. (4.6), we obtain

$$\frac{1}{r}\frac{d^2r}{dt^2} = -\frac{4\pi G}{3}\rho_{m0+}(t) + \frac{4\pi G}{3}\rho_{m0-}(t). \tag{4.23}$$

To solve Eq. (4.23), we consider three different situations:

First situation: $\rho_{m0+} > \rho_{m0-}$, Eq. (4.23) yields,

$$\frac{1}{r}\frac{d^2r}{dt^2} = \frac{4\pi G}{3}[\rho_{m0-}(t) - \rho_{m0+}(t)] < 0, \tag{4.24}$$

Which implies that the positive (negative) sub-Universe is in accelerated collapse (expansion).
Second situation: $\rho_{m0-} = \rho_{m0+}$, $\mathbf{g} = 0$, Eq. (4.23) yields,

$$\frac{1}{r}\frac{d^2r}{dt^2} = \frac{4\pi G}{3}[\,\rho_{m0-}(t) - \rho_{m0+}(t)] = 0. \qquad (4.25)$$

This case corresponds to static positive and negative sub-Universes.

Third situation: $\rho_{m0-} > \rho_{m0+}$, we obtain

$$\mathbf{g} = \frac{4\pi G}{3}\,\mathbf{r}\,[\,\rho_{m0-}(t) - \rho_{m0+}(t)]\,\mathbf{r} > 0. \qquad (4.26)$$

For a non-relativistic object of positive g-charge, Eq. (4.23) give,

$$\frac{1}{r}\frac{d^2r}{dt^2} = \frac{4\pi G}{3}[\,\rho_{0m-}(t) - \rho_{m0+}(t)] > 0. \qquad (4.27)$$

This result agrees with observation data of the accelerating expansion of the Universe, without needing of negative pressure and cosmological constant. Eq. (4.26) and Eq. (4.27) implies that an object carrying positive g-charge is pushed away from the distribution with acceleration, i.e., the negative g-charge provides physics mechanism for the accelerating expansion of positive sub-Universe.

For a non-relativistic object of negative g-charge, for the third situation, Eq. (4.23) and Eq. (4.26) give,

$$\frac{1}{r}\frac{d^2r}{dt^2} = -\frac{4\pi G}{3}\rho_{m-}(t) + \frac{4\pi G}{3}\rho_{m+}(t) < 0. \qquad (4.28)$$

Eq. (4.28) implies that an object carrying negative g-charge is attracted toward to the distribution, which leads to the accelerated collapse of the negative sub-Universe.

The mass density, ρ_{m0+}, of objects carrying positive g-charges in the ball of radius r decreases continuously, while the mass density, ρ_{m0-}, of objects carrying negative g-charges in the ball of radius r increase continuously. As the consequence, the gravitational field $\mathbf{g}_+$ decreases and the gravitational field $\mathbf{g}_-$ increases. The net effects of the combination of decreasing gravitational field $\mathbf{g}_+$ and increasing gravitational field $\mathbf{g}_-$, is that the acceleration of the expansion of objects with positive g-charge is increase, namely the acceleration of the expanded regular observed Universe is accelerating, so we have a jerking universe,

$$\frac{1}{r}\frac{d^3r}{dt^3} = \frac{4\pi G}{3}\left[\frac{d\rho_{m-}(t)}{dt} - \frac{d\rho_{m+}(t)}{dt}\right] > 0. \qquad (4.29)$$

Eq. (4.20) to Eq. (4.29) form a dynamical model that explains the accelerated expansion of our observed positive sub-Universe and predict that the acceleration is increase with time, which may distinguish negative g-charge model from dark energy/cosmological constant model.

We argue that this dynamical model plays a role of indirect evidence of existence of negative g-charge.

Extended-Einstein's Theory (GR) with Negative g-Charges:

It has been shown that, similar to the dark energy or Einstein's cosmology constant Λ, the negative g-charges can naturally explain the accelerating expansion of the universe. Moreover, the negative g-charges can resolve the fine-tuning problem encountered by Einstein's cosmology constant Λ when explaining the accelerating expansion of the universe.

Einstein's equation,

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \frac{8\pi G}{C^4} T^{\mu\nu}, \tag{4.30}$$

describes spacetime, $R^{\mu\nu}$, curved by sources, $T^{\mu\nu}$, of only positive gravitation mass.

Introducing the concept of g-charge, for a system containing positive g-charges only, Einstein's equation, Eq. (4.30) can be rewritten as,

$$R_+^{\mu\nu} - \frac{1}{2} g_+^{\mu\nu} R_+ = \frac{8\pi\sqrt{G}}{C^4} T_{g+}^{\mu\nu}. \tag{4.31}$$

Writing Einstein's equation in this form, Eq. (4.31), gives us an indication how to generalize Einstein's theory to describe spacetime curved by negative g-charge. For this aim, based on Hypothesis 2, we generalize Einstein's theory to contain additional equations having the same form as that of Eq. (4.31) to describe spacetime curved either by negative g-charge alone or by a combination of positive and negative g-charges.

For source(s) carrying negative g-charge, $T_{g-}^{\mu\nu}$, we propose Einstein-type equation,

$$R_-^{\mu\nu} - \frac{1}{2} g_-^{\mu\nu} R_- = \frac{8\pi\sqrt{G}}{C^4} T_{g-}^{\mu\nu}. \tag{4.32}$$

It is obvious that the Einstein's equations, Eq. (4.31) and Einstein-type equation, Eq. (4.32), have g-Charge conjugation. Note objects having positive mass, $T_{m+}^{\mu\nu}$, might carry either positive $T_{g+}^{\mu\nu}$ or negative $T_{g-}^{\mu\nu}$.

We postulate that for a source of two-compound system, by analogy to electrodynamics, its gravitational field should be determined by net g-charges. In geometric term, even there are positive and negative g-charges, spacetime is still described by one metric. Next let's consider a two-compound system containing first and second kinds of object(s). The former has positive energy-momentum pseudo-tensor $T_1^{\mu\nu}$ and carrying positive g-charge $T_{g+}^{\mu\nu}$; the latter has positive energy-momentum pseudo-tensor $T_2^{\mu\nu}$ and carrying negative g-charge $T_{g-}^{\mu\nu}$. To describe this two-compound system, we propose Extended-Einstein equation,

$$R_{net}^{\mu\nu} - \frac{1}{2} g_{net}^{\mu\nu} R_{net} = \frac{8\pi\sqrt{G}}{C^4} T_{g,net}^{\mu\nu} = \frac{8\pi G}{C^4}(T_1^{\mu\nu} - T_2^{\mu\nu}). \tag{4.33}$$

Where

$$T_{g,net}^{\mu\nu} \equiv T_{g+}^{\mu\nu} + T_{g-}^{\mu\nu}, T_{g+}^{\mu\nu} \equiv +\sqrt{G}T_1^{\mu\nu}, T_{g-}^{\mu\nu} \equiv -\sqrt{G}T_2^{\mu\nu}.$$

There are also three situations:

(1) $T_{g+}^{\mu\nu} > T_{g-}^{\mu\nu}$, spacetime $R_{net}^{\mu\nu}$ is the same as $R_+^{\mu\nu}$, thus Eq. (4.33) can be written as

$$R_+^{\mu\nu} - \frac{1}{2} g_+^{\mu\nu} R_+ = \frac{8\pi G}{C^4}(T_1^{\mu\nu} - T_2^{\mu\nu}); \tag{4.34}$$

(2) $T_{g+}^{\mu\nu} < T_{g-}^{\mu\nu}$, spacetime $R_{net}^{\mu\nu}$ is the same as $R_-^{\mu\nu}$, thus Eq. (4.33) can be written as

$$R_-^{\mu\nu} - \frac{1}{2} g_-^{\mu\nu} R_- = -\frac{8\pi G}{C^4}(T_2^{\mu\nu} - T_1^{\mu\nu}). \tag{4.35}$$

(3) $T_{g+}^{\mu\nu} = T_{g-}^{\mu\nu}$, spacetime is flat Minkowski spacetime.

Now let's apply the Extended-Einstein equation to explain the accelerated expansion of Universe. Assume that there are negative g-charge and positive g-charges in Universe, the former forms negative sub-Universe, and the latter forms positive sub-Universe. Eq. (4.33) gives

$$R_{net}^{\mu\nu} - \frac{1}{2} g_{net}^{\mu\nu} R_{net} = 8\pi\sqrt{G}(T_{g+}^{\mu\nu} + T_{g-}^{\mu\nu}). \tag{4.36}$$

Applying the FRW metric,

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2\right], \tag{4.37}$$

where a(t) is the scale factor; K is a constant. Substituting Eq. (4.37) into Eq. (4.36), we obtain

$$\frac{\ddot{a}}{a} = -\frac{4\pi\sqrt{G}}{3}\left[\left(\rho_{g+} + 3p_{g+}\right) - (\rho_{g-} + 3p_{g-})\right], \tag{4.38}$$

and Hubble parameter,

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\sqrt{G}}{3}\left[\rho_{g+} - \rho_{g-}\right] - \frac{K}{a^2}. \tag{4.39}$$

In the case of $3p_{g+} \ll \rho_{g+}$ and $3p_{g-} \ll \rho_{g-}$, then we obtain,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}[\rho_{m+}(t) - \rho_{m-}(t)]. \tag{4.40}$$

Where p_{g+} and p_{g-} are the pressures due to the positive and negative g-charges respectively. Eq. (4.40) implies that for $\rho_{g-} > \rho_{g+}$, the positive g-charges carried by regular objects will be repelled away. With negative g-charge, the Extended-Einstein equation can explain the accelerated expansion of the universe without the need of either "negative pressure" or "dark energy" or the cosmological constant that has issue of fine-turn.

As in Extended-Newton's law, the density of negative g-charge is increasing, $\frac{d\rho_{m-}(t)}{dt} > 0$, and the density of positive g-charge is decreasing, $\frac{d\rho_{m-}(t)}{dt} < 0$, therefore the acceleration of the expansion of the universe is increasing with time, i.e., the universe is a jerking universe,

$$\frac{1}{a}\frac{d^3a}{dt^3} = \frac{4\pi G}{3}\left[\frac{d\rho_{m-}(t)}{dt} - \frac{d\rho_{m+}(t)}{dt}\right] \; > \; 0. \tag{4.41}$$

To compare with Hubble's Constance, H, we calculate the velocity,

$$\dot{r} \; = \int \ddot{r}\, dt \; = \frac{G\,\rho_{m-}}{3} \int r(t)dt$$

Then

$$H = \frac{\dot{r}}{r} = \frac{G\,\rho_{m-}}{3r} \int r(t)dt$$

$$\dot{H} = \frac{G\,\rho_{m-}}{3} - \frac{G\,\rho_{m-}}{3r^2}\,\dot{r} \int r(t)dt = \frac{G\,\rho_{m-}}{3} - \frac{1}{r^2}\left[\frac{G\,\rho_{m-}}{3}\int r(t)dt\right]^2.$$

To summarize, by introducing the negative g-charge, both Extended-Newton's theory and Extended-Einstein's theory can explain the acceleration of the expansion of the universe and predict the jerking universe.

Thus, we argue that the accelerating expansion of the universe is indirect evidence of the existence of negative g-charge.

Extended-Dirac Sea: Negative g-Charge Carrier:

In order to explain the negative energy states, Dirac introduced the concept of Dirac Sea. An interpretation is that the positive operators add a positive energy particle and the negative operators annihilate a positive energy particle. Now we generalize this concept to include the g-charges, we denote it as the Extended-Dirac Sea, which states that the positive operator adds a positive rest mass particle and the negative operator annihilates a positive rest mass particle, i.e., Extended-Dirac Sea exchanges g-charges with particles during the processes of creation and annihilation.

An example: let's consider electron-positron annihilation,

$$e^- + e^+ \rightarrow \gamma + \gamma.$$

We divide this process into 3 separate processes relating with mass/energy, e-charge, and g-charge, correspondingly,

$$\left.\begin{array}{c} m_{electron} + m_{positron} \rightarrow \gamma + \gamma \\ Q_e^- + Q_e^+ \rightarrow \text{Extended} - \text{Dirac Sea} \\ Q_g^- + Q_g^+ \rightarrow \text{Extended} - \text{Dirac Sea} \end{array}\right\}, \tag{4.42a}$$

where $m_{electron}$, Q_e^-, Q_g^+, and $m_{positron}$, Q_e^+, Q_g^-, are the rest mass, e-charge, and g-charge of electron and positron, respectively. In this process,

(1) Electron carries positive g-charge, while positron carries negative g-charge, such that the net g-charge is zero;
(2) the electron's and positron's rest masses convert to radiation energy of gamma ray;
(3) their negative and positive e-charges transport to and store in
(4) Extended-Dirac Sea;
(5) their negative and positive g-charges transport to and store in Extended-Dirac Sea, which implies that the g-charge and inertial rest mass are two different entities.

Another example: the process of the creation of electron-positron pair, which is expressed as

$$\gamma \to e^- + e^+.$$

We divide this process into 3 separate processes,

$$\left.\begin{array}{c} \gamma \to m_{electron} + m_{positron} \\ \text{Extended} - \text{Dirac Sea} \to Q_e^- + Q_e^+ \\ \text{Extended} - \text{Dirac Sea} \to Q_g^- + Q_g^+ \end{array}\right\}. \tag{4.42b}$$

The gamma ray's energy converts to electron and positron's rest masses, respectively. The created electron and positron gain e-charge and g-charges from Extended-Dirac Sea, respectively.

Maxwell-type Gravity (Gravito-Electromagnetics or Gravito-EM): Vector Theory Introduction:

Historically, based on the similarity between the Coulomb's and Newton's laws and, by close analogy to electromagnetism (EM), Maxwell, Heaviside and others proposed the vector theory of gravity which has the form same to that of EM without either mathematical derivation or experimental confirmation, as earlier as year 1865 [13-19] after Maxwell's theory was proposed (1861). However, they stopped further investigation, because they concluded that there was an issue that the potential energy density of static Newtonian gravitational field was negative. In spite of the issue, many scientists still work along this line [20-24].

In 1905, SR was established, which predicted that the inertial mass is a function of velocity, $m_{inertial} = \gamma m_0$. Where m_0 is the rest mass, $\gamma = 1/\sqrt{1 - v^2/c^2}$ is the Lorentz factor.

Now those vector theories faced another issue that is related with none-complying with SR. According to the Weak Equivalence Principle (WEP), the gravitational mass, m_g, is equal to the inertial mass, $m_{inertial}$,

$$m_g = m_{inertial} = \gamma m_0.$$

The gravitational mass "current" of a point g-charge has the form,

$$\mathbf{j}_g = m_g v = \gamma m_0 \mathbf{v}.$$

Therefore, the gravitational mass density and the gravitational current density cannot construct a four-current. Namely, unlike Electromagnetic theory in which e-charge Q_e is a constant and $\mathbf{j}_e = Q_e \mathbf{v}$, those vector theories of gravity do not satisfy Lorentz transformation.

Based on the Original Weak Equivalent Principle (Original-WEP), the U (1) gauge theory of gravity is proposed [25], which is consistent with Gravito-EM. As a vector field theory, Gravito-EM therefore needs to address:

(1) the negative energy issue and
(2) the lack of Lorentz invariance issue.

Indeed, it has been shown that the exchanged energy of gravitation fields is always positive, i.e., transported energy in and out of gravitation field is always positive [12]. Therefore, the total energy of gravitation fields being defined as either negative or positive is only a matter of bookkeeping. The energy issue of the vector field theory of gravity is addressed. There is no negative energy issue in Gravito-EM.

On the other hand, theory of gravity should inevitable contain magnetic-type gravitation. Actually, in 1983 [26], Einstein's equation has been written in Maxwell-type form with second rank tensor fields,

$$\frac{\partial G^{\mu\nu\lambda}}{\partial x^{\lambda}} = -(T^{\mu\nu} + t^{\mu\nu}), \tag{4.43}$$

$$G^{\alpha\mu\nu,\lambda} + G^{\alpha\nu\lambda,\mu} + G^{\alpha\lambda\mu,\nu} = 0, \tag{4.44}$$

where $t^{\mu\nu}$ is non-linear term represent self-interaction of gravitation field.

Following EM, scalar potential $V_{g/GR}$, vector potential $\mathbf{A}_{g/GR}$, field strengths $\mathbf{g}_{GR}$ and $\mathbf{B}_{g/GR}$, or G^{00i} and G^{0ij}, were introduced,

$$V_{g/GR} \equiv \frac{1}{4}\bar{h}^{00}, \tag{4.45}$$

$$A^{i}_{g/GR} \equiv \frac{1}{4}\bar{h}^{0i}, \mathbf{A}_{g/GR} = (A^1_g, A^2_g, A^3_g), \tag{4.46}$$

$$G^{0ij} = A^{i,j}_{g/GR} - A^{j,i}_{g/GR}, \tag{4.47}$$

and

$$\mathbf{g}_{GR} = (g^1_{GR}, g^2_{GR}, g^3_{GR}), \qquad g^{i}_{GR} = G^{00i}, \tag{4.48}$$

$$\mathbf{B}_{g/GR} = (B^1_{g/GR}, B^2_{g/GR}, B^3_{g/GR}), \tag{4.49}$$

$$\mathbf{B}_{g/GR} = \nabla \times \mathbf{A}_{g/GR}, \tag{4.50}$$

$$B^1_{g/GR} = G^{023}, B^2_{g/GR} = G^{031}, B^3_{g/GR} = G^{012}. \quad (4.51)$$

For the situation of slow motion of a point source, ignore non-linear terms,

$$T^{00} \approx \rho_g, \quad (4.52)$$

$$T^{0i} \approx \rho_g v^i, \quad (4.53)$$

$$T^{ij} \approx 0.$$

Eqs. (4.43) and (4.44) give

$$\nabla \cdot \mathbf{g}_{GR} = -\rho_g, \quad (4.54)$$

$$\nabla \times \mathbf{B}_{g/GR} = \mathbf{v}(\nabla \cdot \mathbf{g}_{GR}) + \frac{\partial \mathbf{g}_{GR}}{\partial \mathbf{t}}, \quad (4.55)$$

$$\nabla \times \mathbf{g}_{GR} = -\mathbf{v}(\nabla \cdot \mathbf{B}_{g/GR}) - \frac{\partial \mathbf{B}_{g/GR}}{\partial t}, \quad (4.56)$$

$$\nabla \cdot \mathbf{B}_{g/GR} \equiv 0, \quad (4.57)$$

The subscript "GR" represents the quantities related with "General Relativity", which will distinguish above equations from the Maxwell-type Gravito-EM. Eq. (4.57) indicates that, mathematically, there is no Gravito-magnetic monopole charge in GR of gravity.

Many effects of magnetic-type gravitational field have been proposed [27-33]. The Lense-thirring effect [32] has been explained as the effect of magnetic-type g-field of the rotating Earth, and the positive results have been obtained experimentally [33]. Most significant result is that the gravitational wave, denoted as g-Wave, is detected [34,35]. We argue that, theoretically, the detection of g-Wave indicates the existence of time varying gravito-magnetic fields [36], just as "electromagnetic wave is equivalent to time varying magnetic field".
Experimentally, gravito-magnetic field exists.

Gravito-EM Field Equations:
Let's derive Gravito-EM [37]. Starting from UMFT, Eq. (2.12),

$$\oint(\mathbf{v} \times \mathbf{G}) \cdot d\mathbf{l} + \frac{d}{dt}\iint \mathbf{G} \cdot ds = \iint \left[\mathbf{v}(\nabla \cdot \mathbf{G}) + \frac{\partial \mathbf{G}}{\partial t} - \mathbf{G}(\nabla \cdot \mathbf{v}) + (\mathbf{G} \cdot \nabla)\mathbf{v}\right] \cdot ds, \quad (2.12)$$

where $\mathbf{G}$ can be any vector or axial vector. Following what we did in section 3, let $\mathbf{G} = \mathbf{g}$, then Eq. (2.12) gives Extended-Ampere-Maxwell-type Gravito-magnetic field equation for a moving g-charge,

$$\oint(\mathbf{v} \times \mathbf{g}) \cdot d\mathbf{l} + \frac{d}{dt}\iint \mathbf{g} \cdot ds = \iint \left[\mathbf{v}(\nabla \cdot \mathbf{g}) + \frac{\partial \mathbf{g}}{\partial t} - \mathbf{g}(\nabla \cdot \mathbf{v}) + (\mathbf{g} \cdot \nabla)\mathbf{v}\right] \cdot ds. \quad (4.58)$$

The gravito-magnetic field equation, Eq. (4.58), is type-1 dual to the extended-Ampere-Maxwell equation,

$$\oint(\mathbf{v}\times\mathbf{E})\cdot d\mathbf{l}+\frac{d}{dt}\iint\mathbf{E}\cdot ds=\iint\left[\mathbf{v}(\nabla\cdot\mathbf{E})+\frac{\partial\mathbf{E}}{\partial t}-\mathbf{E}(\nabla\cdot\mathbf{v})+(\mathbf{E}\cdot\nabla)\mathbf{v}\right]\cdot ds. \quad (3.12)$$

When we convert between $\mathbf{g}$ and $\mathbf{E}$, $\mathbf{g}\leftrightarrow\mathbf{E}$, gravitation Eq. (4.58) converts to EM Eq. (3.12), and vice versa.

Next let's start from the Faraday-type UMFT, Eq. (2.20),

$$\oint(\mathbf{v}\times\mathbf{M})\cdot d\mathbf{l}+\frac{d}{dt}\iint\mathbf{M}\cdot ds=\iint\left[\mathbf{v}(\nabla\cdot\mathbf{M})+\frac{\partial\mathbf{M}}{\partial\mathbf{t}}-\mathbf{M}(\nabla\cdot\mathbf{v})+(\mathbf{M}\cdot\nabla)\mathbf{v}\right]\cdot ds. \quad (2.20)$$

Let the $\mathbf{M}$ field is a gravito-magnetic field $\mathbf{B}_g$, $\mathbf{M}=\mathbf{B}_g$, Eq. (2.20) gives,

$$\oint-\left(\mathbf{v}\times\mathbf{B}_g\right)\cdot d\mathbf{l}-\frac{d}{dt}\iint\mathbf{B}_g\cdot ds=-\iint\left[\mathbf{v}\left(\nabla\cdot\mathbf{B}_g\right)+\frac{\partial\mathbf{B}_g}{\partial\mathbf{t}}-\mathbf{B}_g(\nabla\cdot\mathbf{v})+\left(\mathbf{B}_g\cdot\nabla\right)\mathbf{v}\right]\cdot ds. \quad (4.59)$$

The Faraday-type gravitation field equation, Eq. (4.59), is type-1 dual to the extended-Faraday EM equation, Eq. (3.1),

$$\oint-(\mathbf{v}\times\mathbf{B})\cdot d\mathbf{l}-\frac{d}{dt}\iint\mathbf{B}\cdot ds=-\iint\left[\mathbf{v}(\nabla\cdot\mathbf{B})+\frac{\partial\mathbf{B}}{\partial\mathbf{t}}-\mathbf{B}(\nabla\cdot\mathbf{v})+(\mathbf{B}\cdot\nabla)\mathbf{v}\right]\cdot ds. \quad (3.1)$$

When we convert between $\mathbf{B}_g$ and $\mathbf{B}$, $\mathbf{B}_g\leftrightarrow\mathbf{B}$, gravitation Eq. (4.59) converts to EM Eq. (3.1), and vice versa.

Eq. (2.9) gives mathematically,

$$\nabla\cdot\mathbf{B}_g\equiv 0, \quad (4.60)$$

Which is the type-1 dual to Eq. (3.2).

We have shown that the Extended-Newton's law, Newton's force law, Eq. (4.58), Eq. (4.59) and Eq. (4.60) are mathematical type-1 duals of Coulomb's law, Coulomb's force law, Eq. (3.12), Eq. (3.1) and Eq. (3.2) respectively. Also Eq. (3.12) and Eq. (3.1) are identical to Eq. (3.14) and Eq. (3.7) respectively.

By transferring law of dualities, mathematical duality can be transferred to physical duality. Thus, by transferring $\mathbf{E}\leftrightarrow\mathbf{g}$ and $\mathbf{B}\leftrightarrow\mathbf{B_g}$, the Extended-Maxwell equations, Eq. (3.14), Eq. (3.7) and Eq. (3.25), convert to Gravito-EM equations, Eq. (4.62) and Eq. (4.63) and Eq. (4.65) respectively, denoted as the Extended-Gravito-EM equations,

$$\nabla\cdot\mathbf{g}=-\rho_g, \quad (4.61)$$

$$\nabla\times\mathbf{B_g}=\mathbf{v}(\nabla\cdot\mathbf{g})+\frac{\partial\mathbf{g}}{\partial\mathbf{t}}-\mathbf{g}(\nabla\cdot\mathbf{v})+(\mathbf{g}\cdot\nabla)\mathbf{v}, \quad (4.62)$$

$$\nabla \times \mathbf{g} = -\mathbf{v}(\nabla \cdot \mathbf{B_g}) - \frac{\partial \mathbf{B_g}}{\partial t} + \mathbf{B_g}(\nabla \cdot \mathbf{v}) - (\mathbf{B_g} \cdot \nabla)\mathbf{v}, \tag{4.63}$$

$$\nabla \cdot \mathbf{B_g} = 0, \tag{4.64}$$

$$\mathbf{F} = q_g \mathbf{g} + q_g \mathbf{v} \times \mathbf{B}_g, \tag{4.65}$$

where $\mathbf{g}$ and $\mathbf{B_g}$ are the Extended-Newton gravitational field and gravito-magnetic field respectively, and

$$\mathbf{g} = \mathbf{g}_{\text{vector}} + \mathbf{g}_{\text{axial-vector}}, \tag{4.66}$$

$$\nabla \cdot \mathbf{g} = \nabla \cdot \mathbf{g}_{\text{vector}}, \tag{4.67}$$

$$\nabla \times \mathbf{g} = \nabla \times \mathbf{g}_{\text{axial-vector}}. \tag{4.68}$$

$$\mathbf{B_g} = \mathbf{B}_{\text{g/axial-vector}}, \tag{4.69}$$

$$\nabla \cdot \mathbf{B_g} = \nabla \cdot \mathbf{B}_{\text{g/axial-vector}} = 0, \tag{4.70}$$

$$\nabla \times \mathbf{B_g} = \nabla \times \mathbf{B}_{\text{g/axial-vector}}. \tag{4.71}$$

Therefore, it is $\mathbf{g}_{\text{axial-vector}}$ that is type-2 dual to $\mathbf{B}_{\text{g-axial-vector}}$. The $\mathbf{g}_{\text{vector}}$ is the regular vector Newton gravitational field.

Note Gravito-EM field equations, Eq. (4.61) to Eq. (4.65), need empirical confirmation. The Extended Newton's field described by Eq. (4.61) contain two parts, vector field and axial vector field, Eq. (4.66).

The interpretations of terms of the right-hand side of Eq. (4.62) are,

(1) The term, $\mathbf{g}(\nabla \cdot \mathbf{v})$, describes stretching of the $\mathbf{g}$ field due to source velocity compressibility;
(2) The term, $(\mathbf{g} \cdot \nabla)\mathbf{v}$, describes the stretching or tilting of the $\mathbf{g}$ field due to the velocity gradients.
(3) Extended-Ampare-Maxwell-type gravitational law, Eq. (4.62), predicts that the spatially-varying velocity, such as particles distributing and moving in space, terms $\mathbf{g}(\nabla \cdot \mathbf{v})$ and $(\mathbf{g} \cdot \nabla)\mathbf{v}$ create axial vector $\mathbf{B_g}$ fields.
(4) To detect the effects of the terms, $\mathbf{g}(\nabla \cdot \mathbf{v})$ and $(\mathbf{g} \cdot \nabla)\mathbf{v}$, will confirm whether Extended-Maxwell-type gravitation equations are correct.

Similarly, the interpretations of terms of the right-hand side of Eq. (4.63) are,

(1) The term, $\mathbf{B}_g(\nabla \cdot \mathbf{v})$, describes stretching of the $\mathbf{B}_g$ field due to source velocity compressibility;
(2) The term, $(\mathbf{B}_g \cdot \nabla)\mathbf{v}$, describes the stretching or tilting of the $\mathbf{B}_g$ field due to the velocity gradients.

(3) Extended-Faraday-type gravitational law, Eq. (4.63), predicts that the spatially-varying velocity, such as particles distributing and moving in space, terms $\mathbf{B}_g(\nabla\cdot\mathbf{v})$ and $(\mathbf{B}_g\cdot\nabla)\mathbf{v}$ induce axial vector $\mathbf{g}$ fields.
(4) To detect the effects of the terms, $\mathbf{B}_g(\nabla\cdot\mathbf{v})$ and $(\mathbf{B}_g\cdot\nabla)\mathbf{v}$, will confirm whether Extended-Maxwell-type gravitation equations are correct.

For non-spatially-dependent velocity,

$$(\nabla\cdot\mathbf{v}) = (\mathbf{g}\cdot\nabla)\mathbf{v} = (\mathbf{B_g}\cdot\nabla)\mathbf{v} = 0,$$

the Extended-Gravito-EM (Eq. (4.61) to Eq. (4.64)) is simplified to Gravito-EM as

$$\nabla\cdot\mathbf{g} = -\rho_g, \tag{4.61}$$

$$\nabla\times\mathbf{B_g} = \mathbf{v}(\nabla\cdot\mathbf{g}) + \frac{\partial\mathbf{g}}{\partial\mathbf{t}}, \tag{4.72}$$

$$\nabla\times\mathbf{g} = -\mathbf{v}(\nabla\cdot\mathbf{B_g}) - \frac{\partial\mathbf{B_g}}{\partial t}, \tag{4.73}$$

$$\nabla\cdot\mathbf{B_g} = 0. \tag{4.64}$$

Due to the duality between EM equation and Gravito-EM equation, the Gravito-EM field strengths can be expressed in terms of scalar and vector potentials,

$$\mathbf{B}_g = \boldsymbol{\nabla}\times\mathbf{A_g}, \tag{4.74}$$

$$\mathbf{g} = -\nabla V_g - \frac{\partial\mathbf{A}_g}{\partial t}, \tag{4.75}$$

Type-1 Duality between Field Equations of Positive and Negative g-Charges:
Due to the positive and negative g-charge conjugate, the Gravito-EM equations derived above can be applied to Gravito-EM fields created by either positive g-charge/g-current or negative g-charges/g-current with the same forms:

$$\nabla\cdot\mathbf{g}_\pm = -\rho_{g\pm}, \tag{4.76}$$

$$\nabla\times\mathbf{B_{g\pm}} = \mathbf{v}(\nabla\cdot\mathbf{g}_\pm) + \frac{\partial\mathbf{g}_\pm}{\partial t} - (\mathbf{g}_\pm)(\nabla\cdot\mathbf{v}) + (\mathbf{g}_\pm\cdot\nabla)\mathbf{v}, \tag{4.77}$$

$$\nabla\times\mathbf{g}_\pm = -\mathbf{v}(\nabla\cdot\mathbf{B_{g\pm}}) - \frac{\partial\mathbf{B_{g\pm}}}{\partial t} + (\mathbf{B_{g\pm}})(\nabla\cdot\mathbf{v}) - (\mathbf{B_{g\pm}}\cdot\nabla)\mathbf{v}, \tag{4.78}$$

$$\nabla\cdot\mathbf{B_{g\pm}} = 0, \tag{4.79}$$

$$\mathbf{F} = (q_{g+})\mathbf{g}_\pm + (q_{g+})(\mathbf{v}\times\mathbf{B}_{g\pm}), \tag{4.80}$$

$$\mathbf{F} = (q_{g-})\mathbf{g}_{\pm} + (q_{g-})(\mathbf{v} \times \mathbf{B}_{g\pm}), \tag{4.81}$$

where $\mathbf{g}_{\pm}$ and $\mathbf{B}_{g\pm}$ are the Extended-Newton gravitational field and magnetic-type gravitational field created by either positive or negative g-charge and either positive or negative g-current respectively. We have

$$\mathbf{g}_{\pm} = \mathbf{g}_{\pm/\text{vector}} + \mathbf{g}_{\pm/\text{axial-vector}}, \tag{4.82}$$

$$\nabla \cdot \mathbf{g}_{\pm} = \nabla \cdot \mathbf{g}_{\pm/\text{vector}}, \tag{4.83}$$

$$\nabla \times \mathbf{g}_{\pm} = \nabla \times \mathbf{g}_{\pm/\text{axial-vector}}. \tag{4.84}$$

$$\mathbf{B}_{g\pm} = \mathbf{B}_{g\pm/\text{axial-vector}}, \tag{4.85}$$

$$\nabla \cdot \mathbf{B}_{\mathbf{g}\pm} = \nabla \cdot \mathbf{B}_{g\pm/\text{axial-vector}} = 0, \tag{4.86}$$

$$\nabla \times \mathbf{B}_{\mathbf{g}\pm} = \nabla \times \mathbf{B}_{g\pm/\text{axial-vector}}. \tag{4.87}$$

$$\mathbf{B}_{g\pm} = \boldsymbol{\nabla} \times \mathbf{A}_{\mathbf{g}\pm}, \tag{4.88}$$

$$\mathbf{g}_{\pm} = -\nabla V_{g\pm} - \frac{\partial \mathbf{A}_{g\pm}}{\partial t}. \tag{4.89}$$

Therefore, it is $\mathbf{g}_{\pm/\text{axial-vector}}$ that is type-2 dual to $\mathbf{B}_{g\pm}$. The $\mathbf{g}_{\pm/\text{vector}}$ is the vector Newton gravitational field.

For the situations of non-spatially-varying velocity, the extended-Gravito-EM equation become

$$\nabla \cdot \mathbf{g}_{\pm} = -\rho_{g\pm}, \tag{4.90}$$

$$\nabla \times \mathbf{B}_{\mathbf{g}\pm} = \mathbf{v}(\nabla \cdot \mathbf{g}_{\pm}) + \frac{\partial \mathbf{g}_{\pm}}{\partial t}, \tag{4.91}$$

$$\nabla \times \mathbf{g}_{\pm} = -\mathbf{v}\left(\nabla \cdot \mathbf{B}_{\mathbf{g}\pm}\right) - \frac{\partial \mathbf{B}_{\mathbf{g}\pm}}{\partial t}, \tag{4.92}$$

$$\nabla \cdot \mathbf{B}_{\mathbf{g}\pm} = 0. \tag{4.93}$$

For simplicity, hereafter, we will drop the subscript sign "$\pm$".

Type-2 Duality between Newton Gravitation and Magnetic-type Gravitational Fields:
Based on the Transfer Rules of Section 1.2, there is a duality between Eq. (4.91) and Eq. (4.92), i.e., under transformation $\mathbf{g} \leftrightarrow \mathbf{B}_g$ and $\mathbf{B}_g \leftrightarrow -\mathbf{g}$, Eq. (4.91) and Eq. (4.92) convert to each other. With distinguishable feature of "type-1 duality" and "type-2 duality", the duality between induced gravitational field determined by the Faraday-type gravitational law, Eq. (4.92), and magnetic-type gravitational field determined by Ampere-Maxwell-type gravitational equation,

Eq. (4.91), is actually a type-2 duality, as well as Eq. (4.77) and Eq. (4.78). UMFT provides the mathematical origin of the type-2 duality between axial vector induced gravitational field and magnetic-type axial vector gravitational field.

Type-1 Duality between Maxwell-EM and Gravito-EM:
So far, we have encountered and proposed several dualities:

(1) positive and negative e-charges Conjugate;
(2) positive and negative g-charges conjugate (proposed);
(3) duality between e-charge and g-charge;
(4) duality between Coulomb's law and Newton's law, which is perfect after introducing duality (3);
(5) duality between EM and Gravito-EM (proposed based on UMFT and duality (3).

Based on the above dualities, the phenomena in EM, such as wave, quantization, renormalization and unification, can be transferred into Gravito-EM directly without complex calculation.

Table 4.2: Significant Different between Theories of Gravity

	Field equation	Source	Four-current
EM	$\frac{\partial F_e^{\alpha\beta}}{\partial x^\beta} = -J_e^\alpha$	Q_e: Q_e is invariant	$j_e^\mu = (\rho_e, \mathbf{j}_e)$
Gravito-EM	$\frac{\partial F_{g\pm}^{\alpha\beta}}{\partial x^\beta} = -J_{g\pm}^\alpha$	$Q_{g\pm} = \pm\sqrt{G}m_0$: m_0: rest mass. $Q_{g\pm}$ is invariant	$j_{g\pm}^\mu = (\rho_{g\pm}, \mathbf{j}_{g\pm})$
Linearized GR	$\frac{\partial G_g^{\alpha\beta\mu}}{\partial x^\mu} = -T_g^{\alpha\beta}$	$m_g = \gamma m_0$	No
Other vector theories	$\frac{\partial F_g^{\alpha\beta}}{\partial x^\beta} = -m_g U^\alpha$	$m_g = \gamma m_0$:	No

Type-1 Duality between EM and Gravito-EM allows us to transfer the phenomena in EM directly to Gravito-EM. In the following sections, we present those phenomena of gravity.

Origin of Differences between Newton Field and Gravito-Magnetic Field:
A magnetic-type gravitational field is completely different from a Newton field in the following senses:

(1) sources: "g-charge" vs. "g-current";
(2) way the fields are generated: "$\nabla \cdot \mathbf{g}$" vs. "$\nabla \times \mathbf{B}_g$";
(3) nature of fields: "vector field $\mathbf{g}$" vs. "axial vector field $\mathbf{B}_g$";
(4) forces acting on a test g-charge: "$q_g\mathbf{g}$" vs. "$q_g\mathbf{v} \times \mathbf{B}_g$".

The origin of those differences is the same as that of differences between Coulomb electric field and magnetic field. All of above-mentioned differences between Newton field and magnetic-type gravitation field have a mathematical origin that is Eq. (2.1).

Gravito-EM shows that gravity is a local physical field like the EM field and thus, is quantizable and renormalizable [3].

Moreover, Gravito-EM have the same form as that of linearized GR, Eq. (4.54) to Eq. (4.57), which implies that all of results of linearized GR can also be obtained from Gravito-EM, including gravitation wave (G-Wave). The existences of time-varying-gravito-magnetic field and G-Wave are equivalent. So now, there are either magnetic or magnetic-type fields in all of four interactions in nature.

Equation of Continuity:

g-Current Creating Gravito-Magnetic Field:

Utilizing Newton's law, $\nabla \cdot \mathbf{g} = -\rho_g$, let's define a "current", $\mathbf{j}_{\mathbf{v}-\mathbf{B}_g}$, that generates magnetic-type gravitational field $\mathbf{B}_g$,

$$\mathbf{j}_{\mathbf{v}-\mathbf{B}_g} = -\rho_g \mathbf{v} - \mathbf{g}(\nabla \cdot \mathbf{v}) + (\mathbf{g} \cdot \nabla)\mathbf{v}. \tag{4.94}$$

Where the subscripts "$\mathbf{v} - \mathbf{B}_g$" represent the quantity generating the gravit-magnetic field $\mathbf{B}_g$. Then Eq. (4.62) becomes,

$$\nabla \times \mathbf{B}_g = \mathbf{j}_{\mathbf{v}-\mathbf{B}_g} + \frac{\partial \mathbf{g}}{\partial t}. \tag{4.95}$$

The current $\mathbf{j}_{\mathbf{v}-\mathbf{B}_g}$ satisfies the equation of continuity,

$$\nabla \cdot \mathbf{j}_{\mathbf{v}-\mathbf{B}_g} + \frac{\partial(\nabla \cdot \mathbf{g})}{\partial t} = 0. \tag{4.96}$$

For the situation of the non-spatially-varying velocity, $\mathbf{g}(\nabla \cdot \mathbf{v}) = (\mathbf{g} \cdot \nabla)\mathbf{v} = 0$, we have

$$\mathbf{j}_{\mathbf{v}-\mathbf{B}_g} = -\rho_g \mathbf{v}. \tag{4.97}$$

Extended-Ampere-Maxwell-type gravitational equation, Eq. (4.62), becomes the Ampere-Maxwell-type gravitational law,

$$\nabla \times \mathbf{B}_g = \mathbf{v}(\nabla \cdot \mathbf{g}) + \frac{\partial \mathbf{g}}{\partial t}. \tag{4.98}$$

The Ampere-Maxwell-type gravitation law Eq. (4.98) provides a mathematical origin of why and how e-current, $\rho_g \mathbf{v}$, and displacement-current, $\frac{\partial \mathbf{g}}{\partial \mathbf{t}}$, create inevitably gravito-magnetic fields.

g-Current Creating Induced-Gravito-Electric Field:

Let's define a "current" generating induced gravitation field, as

$$\mathbf{j}_{\mathbf{v}-\mathbf{g}} \equiv \mathbf{v}(\nabla \cdot \mathbf{B}_g) - \mathbf{B}_g(\nabla \cdot \mathbf{v}) + (\mathbf{B}_g \cdot \nabla)\mathbf{v}. \tag{4.99}$$

Where the subscripts "v-g" represent the quantity generating the induced axial vector gravitational field.

Then Extended-Faraday-type gravitational law, Eq. (4.63), may be rewritten as,

$$\nabla \times \mathbf{g} = -\mathbf{j}_{\mathbf{v-g}} - \frac{\partial \mathbf{B}_{\mathrm{g}}}{\partial \mathrm{t}}. \tag{4.100}$$

Eq. (4.100) shows the equation of continuity as

$$\nabla \cdot \mathbf{j}_{\mathbf{v-g}} + \frac{\partial(\nabla \cdot \mathbf{B}_{\mathrm{g}})}{\partial \mathrm{t}} = 0. \tag{4.101}$$

For the situation in which, the velocity is non-spatially-varying, i.e., $\mathbf{B}_{\mathrm{g}}(\nabla \cdot \mathbf{v}) = \left(\mathbf{B}_{\mathrm{g}} \cdot \nabla\right)\mathbf{v} = 0$, Extended-Faraday-type gravitation law, Eq. (4.100), becomes,

$$\nabla \times \mathbf{g} = -\frac{\partial \mathbf{B}_{\mathrm{g}}}{\partial \mathrm{t}}, \tag{4.102}$$

where the **g** field is an axial vector field and

$$\nabla \cdot \mathbf{B}_{\mathrm{g}} = 0, \mathbf{j}_{\mathbf{v-g}} = 0. \tag{4.103}$$

Gravito-Magnetic Field of Steady g-Current

Gravito-magnetic Field Line:

The gravito-magnetic field lines of a steady g-current are determined by Eq. (4.72), which can be written as the following,

$$\mathbf{B}_{\mathbf{g}} = \mathbf{B}_{\mathbf{g+}} + \mathbf{B}_{\mathbf{g-}}, \nabla \times \mathbf{B}_{\mathbf{g+}} = -\rho_{\mathrm{g+}}\mathbf{v} = -\rho_{\mathrm{m0+}}\mathbf{v}, \tag{4.104}$$

$$\nabla \times \mathbf{B}_{\mathbf{g-}} = -\rho_{\mathrm{g-}}\mathbf{v} = \rho_{\mathrm{m0-}}\mathbf{v}, \tag{4.105}$$

where $\rho_{\mathrm{m0-}} > 0$, $\rho_{\mathrm{m0+}} > 0$.

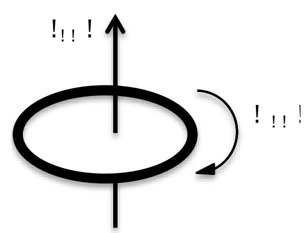

Figure. 4.7: “The left-hand rule” for gravito-magnetic field +

To determine the directions of the steady positive g-current and its gravito-magnetic field, Eq. (4.104) indicates to use “the left-hand rule”. Similarly, Eq. (4. 105) requires to apply the “left-hand rule” on determining the gravito-magnetic field line of a negative g-current. The “left-hand rule” states: if one's thumb of left-hand points to the direction of a g-current, for example a positive g-current, the fingers point to the direction of the gravito-magnetic field, say $\mathbf{B}_{\mathbf{g+}}$ (Fig. 4.7).

Interaction between Two Steady g-Currents:

Two positive g-currents $\mathbf{I}_{g+}$ in same directions repel to each other (Fig. 4.8). Two positive g-currents $\mathbf{I}_{g+}$ in opposite directions attract to each other (Fig. 4.9).

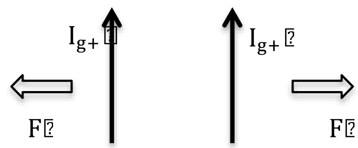

Fig. 4.8

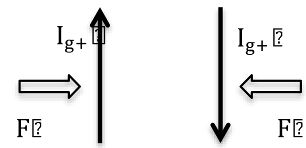

Fig. 4.9

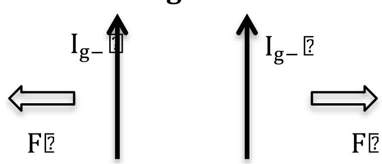

Fig. 4.10

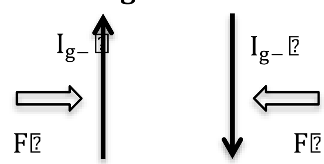

Fig. 4.11

Two negative g-currents $\mathbf{I}_{g-}$ in same directions repel to each other (Fig. 4.10). Two negative g-currents $\mathbf{I}_{g-}$ in opposite directions attract to each other (Fig. 4.11).

A negative g-current $\mathbf{I}_{g-}$ and a positive g-current$\mathbf{I}_{g+}$ in same directions attract to each other (Fig. 4.12). A negative g-current and a positive g-current in opposite directions repel to each other (Fig. 4.13).

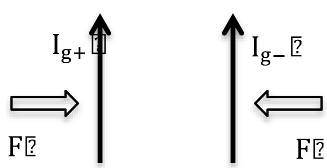

Fig. 4.12

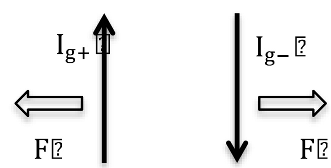

Fig. 4.13

Rotation Curve: Dark Matter

Considering a spiral galaxy with a rotation symmetry. The rotating motion of the galaxy generates a gravito-magnetic field $\mathbf{B}_g$**.** A moving star with g-charge, q_g, at a radius R to the center of the galaxy, will experience a Lorentz-type gravitational force,

$$\mathbf{F} = q_g\mathbf{g} + q_g\mathbf{v} \times \mathbf{B}_g,$$

which consists of two parts, the Newtonian force, $g = \frac{\sqrt{G}M}{R^2}$, and the gravito-magnetic force and $\mathbf{v} \perp \mathbf{B}_g$, both have the same direction.

The Lorentz-type force keeps the star moves around the center of the galaxy,

$$\frac{v^2}{R} = \frac{GM}{R^2} + \sqrt{G}vB_g, \tag{4.106}$$

Or

$$v^2 - \left(\sqrt{G}B_gR\right)v - \frac{GM}{R^2} = 0, \tag{4.107}$$

where M(R) is the mass interior to the radius R, $\mathbf{B}_g$ is determined by the model of distribution of stars in the galaxy.

Predictions of Gravito-EM

The Gravito-EM introduces/predicts the following [37]:

(1) Perihelion Precession of Mercury
(2) Extended-Virial Theorem in Gravito-Magnetics Field
(3) Poynting-type Theorem in Gravito-EM
(4) Energy-Momentum Tensor of Gravito-EM Fields
(5) Retarded Gravitational Potentials
(6) Gravitational Radiation (g-wave): Lienard-type formula
(7) Bremsstrahlung-type radiation
(8) Synchrotron-type radiation
(9) Transformation Law of Energy Density of g-Wave
(10) Doppler-type Effect of g-Wave
(11) Hubble Constant and Redshift of g-Wave
(12) Tully-Fisher-type law of g-Wave
(13) Dipole Radiation of Non-Relativistic Binary System
(14) Dipole Radiation of Relativistic Binary System
(15) Quadrupole Radiation of Relativistic Binary System
(16) Gravito-Bremsstrahlung-type Radiation of Two Relativistic Stars Collision
(17) Energy Loss by Radiation: Non-Relativistic Binary System
(18) Energy Loss by Radiation: Relativistic Binary System
(19) Resolving Negative Energy Issue of Vector Gravity
(20) Lorentz Transformation of Gravito-EM Field
(21) Extended-Doppler Effect of g-Wave of Accelerating Source
(22) Extended-Hubble Law for Accelerating Universe

Discussion and Summary

We have systematically study G-Wave in the framework of Gravito-EM theoretically and experimentally and shown the following:

(1) The physical characteristics of G-WAVE have been studied by following Electrodynamics and, especially, in terms of the field strengths, $\mathbf{g}$ and $\mathbf{B}_g$; thus, the directions of looking for new effects of G-WAVE are conceptually clear, and the related calculations are simple.
(2) The concept of gravitational field strength has been introduced into GR. For the components of T^{00} and T^{0i}, the linearized Einstein equation has been written as [28],

$$\nabla \cdot \mathbf{g} = -4\pi\rho_g, \nabla \cdot \mathbf{B_g} = 0,$$

$$\nabla \times \mathbf{g} = -\frac{1}{C}\frac{\partial \mathbf{B_g}}{\partial \mathbf{t}}, \nabla \times \mathbf{B_g} = -\frac{4\pi}{C}\mathbf{J}_g + \frac{1}{C}\frac{\partial \mathbf{g}}{\partial \mathbf{t}}.$$

$$\mathbf{F} = Q_g\mathbf{g} + 4Q_g\mathbf{V} \times \mathbf{B}_g.$$

This set of linearized Einstein equations has the form same to that of Gravito-EM. All of conclusions in this section are derived from Gravito-EM. The linearized GR, therefore, will give the same results, except a factor of 4 in the expression of gravitational force.

1. There is gravitational synchrotron radiation from a relativistic star moving in the gravitomagnetic field of a rotating massive object, such as black hole.
2. There is gravitational Bremsstrahlung-type radiation from a head-on collision of two relativistic stars.
3. The gravitational Doppler-type effect is derived, which needs to be considered in the detection of G-Wave, especially for a faraway source.
4. The correlation between the redshifts of both G-Wave and EM-Wave is derived.

Table 4.3: Summary on Extended Hubble Law and Red Shift of G-Waves

	Distance-Movement Relation	Distance-Redshift-Movement Relation: $1 + Z = \frac{r(t_0)}{r(t)}$
Uniformly Expanding Universe	$r = \frac{\dot{r}}{H}$	$Z = Z_{H0} = \frac{H_0 r(t_0)}{c}$
Accelerating Universe	$r = \frac{\dot{r}}{H} - \frac{\ddot{r}}{2H^2}$	$Z = Z_{H0} - \frac{\ddot{r}(t_0)r(t_0)}{2\dot{r}^2(t_0)} Z_{H0}^2$ $Z = Z_{H0} + \frac{1}{2} q_{R0}^2 Z_{H0}^2$
Jerking Universe	$r = \frac{\dot{r}}{H} - \frac{\ddot{r}}{2H^2} + \frac{\dddot{r}}{3!\,H^3}$	$Z = Z_{H0} - \frac{\ddot{r}(t_0)r(t_0)}{2\dot{r}^2(t_0)} Z_{H0}^2 + \frac{\dddot{r}(t_0)r^2(t_0)}{3!\,\dot{r}^3(t_0)} Z_{H0}^3$ $Z = Z_{H0} + \frac{1}{2} q_{R0}^2 Z_{H0}^2 - \frac{1}{3!} q_{R0}^3 Z_{H0}^3$

SPIN-ELECTROMAGNETICS (SPIN-EM) DERIVED FROM UMFT, COULOMB'S LAW, AND SPIN OF SOURCE

UMFT is the math theory and thus can be applied to more situations. In this section let's apply it to the combination of spin and electromagnetics, namely to establish Spin-EM and apply it to study phenomena related with spin systematically.

Spin-EM of Resting-Spin-Electric-Charge Derived from UMFT, Coulomb's Law and Spin

Definitions of Spin-electric Field $\mathbf{E_s}$ and Spin-magnetic Field $\mathbf{B_s}$:

First let's consider a resting-spin-e-charge characterized by electric charge Q_e and spin $\mathbf{S}_e$, such as an electron at rest. The charge Q_e produces a Coulomb field, $\nabla \cdot \mathbf{E} = Q_e$. **UMFT** shows that the spin of the e-charge Q_e induces axial vector fields. To applying UMFT formular, Equation (2.4b), to the spin-e-charge, let

(1) $\mathbf{S_c} \equiv \mathbf{S_e}$, and $\mathbf{T} = \mathbf{E}$, where $\mathbf{E}$ is the electric field:
(2) $\mathbf{S_c} \equiv \mathbf{S_e}$, and $\mathbf{T} = \mathbf{B}$, where $\mathbf{B}$ is the magnetic field:

We show in this article that:

(1) a resting-spin-e-charge Q_e with spin $\mathbf{S}_e$ produces both an electric field $\mathbf{E}$, and a spin-magnetic field $\mathbf{B_s}$;

(2) a moving-spin-e-charge Q_e with velocity $\mathbf{v}$ and spin $\mathbf{S}_e$ produces the superposition of a magnetic field $\mathbf{B}$, an electric field $\mathbf{E}$, the spin-magnetic field $\mathbf{B_s}$, and the spin-electric field $\mathbf{E_s}$, where,

$$\mathbf{B_s} \equiv \mathbf{S}_e \times \mathbf{E}, \tag{5.1}$$

$$\mathbf{E_s} \equiv \mathbf{S}_e \times \mathbf{B}. \tag{5.2}$$

The $\mathbf{B_s}$ and $\mathbf{E_s}$ are "Second order axial vector field" (SAF) and "Third order axial vector field" (TAF) respectively, and mathematically satisfy,

$$\nabla \cdot \mathbf{B_s} = 0, \tag{5.3}$$

$$\nabla \cdot \mathbf{E_s} = 0. \tag{5.4}$$

Subscript "s" indicates the quantity related to spin. The $\mathbf{B_s}$ and $\mathbf{E_s}$ are type-2 dual to the $\mathbf{B}$ and $\mathbf{E}$ fields, respectively**.** We keep those divergence terms in some of equations of Spin-EM, as well the magnetic monopole term $\nabla \cdot \mathbf{B} = \mathbf{0}$, which shows the nature and the breaking mechanism of duality.

The definitions, Eq. (5.1) and Eq. (5.2), predict two categories of phenomena:

(1) by interacting with the electric field $\mathbf{E}$ or the magnetic field $\mathbf{B}$**,** the spin of e-charges induces a spin-magnetic field $\mathbf{B_s}$ or a spin-electric field $\mathbf{E_s}$, respectively.
(2) the spin of an e-charge in the electric field $\mathbf{E}$ or in the magnetic field $\mathbf{B}$ will experience a spin-magnetic field $\mathbf{B_s}$ or a spin-electric field $\mathbf{E_s}$ respectively.

Note: the definitions of $\mathbf{B_s}$ and $\mathbf{E_s}$ are conceptually different from that of Rashba-induced-Spin-EM.

Spin-EM derived from UMFT, Coulomb law and Spin:
Spin-EM can be derived with three approaches: (1) from Eq. (2.1); (2) from Eq. (2.2); (3) Eq. (2.4b); and (4) based on duality, from Extended EM. The so derived three Spin-EMs are mathematically equivalent.

We start with Eq. (2.4b) of **UMFT** and le**t** $\mathbf{T} = \mathbf{E}$ and $\mathbf{T} = \mathbf{B}$**,** respectively. Substituting Eq. (5.1) and Eq. (5.2) into Eq. (2.4b) respectively, we obtain Spin-EM equations for fields induced by spin of the e-charge,

$$\nabla \times (\mathbf{B_s}) = \mathbf{S}_e(\nabla \cdot \mathbf{E}) - \mathbf{E}(\nabla \cdot \mathbf{S}_e) + (\mathbf{E} \cdot \nabla)\mathbf{S}_e - (\mathbf{S}_e \cdot \nabla)\mathbf{E}, \tag{5.5}$$

$$\nabla \times (\mathbf{E_s}) = \mathbf{S}_e(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{S}_e) + (\mathbf{B} \cdot \nabla)\mathbf{S}_e - (\mathbf{S}_e \cdot \nabla)\mathbf{B}. \tag{5.6}$$

Eq. (5.5) is the type-2 dual of Eq. (5.6) under the transformation:

$$\mathbf{E} \leftrightarrow \mathbf{B}, \mathbf{B_s} \leftrightarrow \mathbf{E_s}.$$

Eq. (5.5) and Eq. (5.6) are the basic equations of Spin-EM, the Ampere-type equation and Faraday-type equations. For a e-charge with a constant spin $\mathbf{S}_e$, Eq. (5.5) and Eq. (5.6) become

$$\nabla \times (\mathbf{B_s}) = \mathbf{S}_e(\nabla \cdot \mathbf{E}) - (\mathbf{S}_e \cdot \nabla)\mathbf{E}, \tag{5.7}$$

$$\nabla \times (\mathbf{E_s}) = \mathbf{S}_e(\nabla \cdot \mathbf{B}) - (\mathbf{S}_e \cdot \nabla)\mathbf{B}, \tag{5.8}$$

$$\nabla \cdot \mathbf{B_s} = 0, \tag{5.3}$$

$$\nabla \cdot \mathbf{E_s} = 0, \tag{5.4}$$

The Equation (5.7), Equation (5.8), Equation (5.3) and Equation (5.4) are the complete set of Spin-EM equations for constant spin. Maxwell equations describe the behavior of the e-charge. **Note**: We still keep the term of $\nabla \cdot \mathbf{B}$, for showing the perfect duality between Equation (5.7) and Equation (5.8).

Equations of Continuity of Spin Currents:

By analogy to "electric current", let's define the "spin-magnetic-current $\mathbf{j}_{\mathbf{s}-\mathrm{B}}$" and the "spin-electric-current $\mathbf{j}_{\mathbf{s}-\mathbf{E}}$", which induce the spin-magnetic field $\mathbf{B_s}$ and the spin-electric field $\mathbf{E_s}$ respectively, as

$$\mathbf{j}_{\mathbf{s}-\mathbf{B}} \equiv \mathbf{S}_e(\nabla \cdot \mathbf{E}) - (\mathbf{S}_e \cdot \nabla)\mathbf{E}, \tag{5.9}$$

$$\mathbf{j}_{\mathbf{s}-\mathbf{E}} \equiv \mathbf{S}_e(\nabla \cdot \mathbf{B}) - (\mathbf{S}_e \cdot \nabla)\mathbf{B}. \tag{5.10}$$

Eq. (5.7) and Eq. (5.8) become respectively,

$$\nabla \times \mathbf{B_s} = \mathbf{j}_{\mathbf{s}-\mathbf{B}}, \quad (5.11$$

$$\nabla \times \mathbf{E_s} = \mathbf{j}_{\mathbf{s}-\mathbf{E}}. \tag{5.12}$$

The $\mathbf{B_s}$ field and $\mathbf{j}_{\mathbf{s}-\mathbf{B}}$ are type-2 dual of the $\mathbf{E_s}$ field and $\mathbf{j}_{\mathbf{s}-\mathbf{E}}$ respectively.

Note: Eq. (5.9) to Eq. (5.12) shows that the spin $\mathbf{S}_e$ plays the role of "velocity" generating spin-magnetic field, which is conceptually different from that of Rashba-induced-Spin-EM.

Lagrangian and Hamiltonian:

For a non-relativistic non-spinning e-charge Q_e in the EM field, the regular Lagrangian and Hamiltonian are respectively,

$$\mathcal{L}_{\mathrm{REG}} = \frac{1}{2}\mathrm{MV}^2 + \mathrm{Q_E}\mathbf{A} \cdot \mathbf{v} - \mathrm{Q_E}\Phi,$$

$$\mathrm{H_{REG}} = \frac{1}{2\mathrm{M}}(\mathrm{P} - \mathrm{Q_E}\mathbf{A})^2 + \mathrm{Q_E}\Phi. \tag{5.13}$$

For a rotating mass, the Lagrangian contains its rotation energy, $\mathrm{KE_{rotation}} = \frac{1}{2}\mathrm{I}\omega^2$.

We define the rotation energy of a spinning e-charge as,

$$\mathrm{KE_{SPIN}} \equiv \frac{1}{2}\alpha_1(\mathbf{S}_\mathrm{E})^2 = \frac{1}{2\alpha_1}(\mathbf{L}_\mathrm{S})^2, \tag{5.14}$$

where the spin angular momentum $\mathbf{L}_\mathrm{s} \equiv \alpha_1\mathbf{S}_\mathrm{e}$. Where a_1 is the coefficient.

By the duality between Extended EM and Spin-EM, let's introduce Lagrangian,

$$\mathcal{L}_\mathrm{SPIN} = \frac{1}{2}\alpha_1(\mathbf{S_e})^2 + \mathrm{A}_2\mathrm{Q}_\mathrm{E}\mathbf{A_s}\cdot\mathbf{S_e}. \tag{5.15}$$

Note: For the Spin-EM, there is no scalar potential φ_s.

Taking into account the interaction between the velocity and spin-vector-potential, and between the spin and vector potential, we obtain

$$\mathcal{L}_\mathrm{interaction} = \mathrm{a}_3\mathrm{Q}_\mathrm{e}\mathrm{A}_\mathrm{s}\cdot\mathrm{v} + \mathrm{a}_4\mathrm{Q}_\mathrm{e}\mathrm{A}\cdot\mathrm{S}_\mathrm{e}. \tag{5.16}$$

Where a_2 to a_4 are coupling coefficients. Hereafter we ignore those coefficients.

The total Lagrangian of a spinning e-particle in EM fields and Spin-EM fields is

$$\mathcal{L}_\mathrm{total} = \frac{1}{2}\mathrm{mv}^2 + \mathrm{Q_eA}\cdot\mathrm{v} - \mathrm{Q_e}\varphi + \frac{1}{2}(\mathrm{S_e})^2 + \mathrm{Q_eA_s}\cdot\mathrm{S_e} + \mathrm{Q_eA_s}\cdot\mathrm{v} + \mathrm{Q_eA}\cdot S_e. \tag{5.17}$$

In the derivation of Spin-EM, we have replaced the velocity v by spin S_e in UMFT. Now we use spin as a "generalized velocity", substituting it into Hamiltonian,

$$\mathrm{H} = \sum \dot{q}^i\frac{\partial\mathcal{L}_\mathrm{total}}{\partial\dot{q}^i} - \mathcal{L}_\mathrm{total} = \mathrm{v}\frac{\partial\mathcal{L}_\mathrm{total}}{\partial\mathrm{v}} + (\mathrm{S_e})\frac{\partial\mathcal{L}_\mathrm{total}}{\partial\mathrm{S_c}} - \mathcal{L}_\mathrm{total}, \tag{5.18}$$

we obtain the Hamiltonian for spinning e-particles,

$$\mathrm{H} = \frac{(\mathrm{p}-\mathrm{Q_eA}-\mathrm{Q_eA_s})^2}{2m} + \frac{(\mathrm{p_s}-\mathrm{Q_eA_s}-\mathrm{Q_eA})^2}{2\mathrm{a}_\mathcal{L}} + \mathrm{Q_e}\varphi, \tag{5.19}$$

which describes dynamics of spinning e-particles in both Extended EM and Spin-EM fields. Where the $\mathrm{p_s}$ is a conjugate momentum corresponding to the spin $\mathrm{S_e}$,

$$\mathrm{p_s} = \frac{\partial\mathcal{L}_\mathrm{total}}{\partial\mathrm{e}}. \tag{5.20}$$

For the situation, both uniform magnetic field B and uniform spin-magnetic field $\mathrm{B_s}$ are in z-direction, vector potential A and spin-vector-potential $\mathrm{A_s}$ have similar form,

$$\mathrm{A} = \frac{1}{2}\mathrm{B} \times \mathrm{r}, \tag{5.21}$$

$$\mathrm{A_s} = \frac{1}{2}\mathrm{B_s} \times \mathrm{r}. \tag{5.22}$$

Eq. (5.22) shows the relation between spin-vector potential and spin-magnetic field induced by spin, which has the same form as that induced by magnetic momentum.
Substituting Eq. (5.21) and Eq. (5.22) into Eq. (5.19), we obtain,

$$\mathrm{H} \approx -\frac{\mathrm{Q_e}\mathbf{p}\cdot\mathbf{A}}{m} - \frac{\mathrm{Q_e}\mathbf{p}\cdot\mathbf{A_s}}{m} - \frac{\mathrm{Q_e}\mathbf{p_s}\cdot\mathbf{A_s}}{\mathrm{a}_\mathcal{L}} - \frac{\mathrm{Q_e}\mathbf{p_s}\cdot\mathrm{A}}{\mathrm{a}_\mathcal{L}} = -\frac{\mathrm{Q_e}}{2\mathrm{m}}\mathbf{B}\cdot\mathbf{L} - \frac{\mathrm{Q_e}}{2\mathrm{m}}\mathbf{B_s}\cdot\mathbf{L} - \frac{\mathrm{Q_e}}{2\mathrm{a}_\mathcal{L}}\mathbf{B}\cdot\mathbf{L_s} - \frac{\mathrm{Q_e}}{2\mathrm{a}_\mathcal{L}}\mathbf{B_s}\cdot\mathbf{L_s}. \tag{5.23}$$

Note: With Hamiltonian, Spin-EM can be converted to its quantum version. The Hamiltonian not only provides classical counterparts/origins of several quantum phenomena, but also predicts several classical effects that may be converted to quantum effects.

Effects and Predictions of Hamiltonian:

With the Hamiltonian, the following effects/phenomena have been predicted [37]:

(1) the Spin-Zeeman Effect;
(2) Extended-Rashba-SOC;
(3) Conjugate Angular Momentum Coupling to Magnetic Field;
(4) Total Angular Momentum Coupling to Magnetic Field B;
(5) Total Angular Momentum Coupling to Spin-Magnetic Field;
(6) Spin-Aharonov–Bohm Effect and Experiment;
(7) Spin-Lorentz-type Force and Effects;
(8) Extended Landau–Lifshitz (LL) and Landau–Lifshitz-Gilbert (LLG) Equations;
(9) Dual-Hall Effect/Topological Insulator;
(10) Extended-Hall Effect/Topological Insulator, Extended-Hall effect Contributing to GMR/TMR Effect, Extended-Hall Effect Contributing to Spin Hall Effect, Extended-Hall effect Contributing to Anomalous-Hall Effect;
(11) Temperature Dependence of Extended-Hall Effect;
(12) Magnetic Aharonov-Casher-type Effect and Extended Spin-Orbit-Electric field Coupling;
(13) Lagrangian-Lorentz-type Force and effects;
(14) Spin-Potential-Coupling-Induced Force and Aharonov–Bohm Effect;
(15) Lagrangian-Hall-type Effect/Topological Insulator;
(16) Spin-Larmor-type Precession;
(17) the spin-magnetic force is the mechanism of Rashba Effect;
(18) Spin-magnetic-Rashba-type SOC;
(19) Effects of Lorentz Force on Spin;
(20) Spin-Stark Effect and Magnetic-Rashba-type SOC.

Summery and Discussion:

Combining UMFT, Coulomb’s law and spin, we derived Spin-EM in the perspective of fundamental physics.

Spin-EM is powerful and fruitful, and achieves the following:

(1) Derives spin wave.
(2) Derives Spin-Lorentz-type force and Lagrangian-Lorentz-type force, which, for 3D model, cause Dual-Hall Effect, Extended-Hall effect, Lagrangian-Hall-type Effect, and Temperature Dependence of Extended-Hall effect; also cause Extended Rashba SOC.
(3) Extended-Hall effect contributes universally to zero longitudinal Hall coefficient/resistivity, GMR/TMR, Anomalous Hall effect, Spin Hall effect, and topological insulator.
(4) Predicts Spin-Potential-Coupling-Induces force that contributes to Aharonov–Bohm Effect.
(5) Provides classical counterparts of Aharonov-Bohm effect; Aharonov-Casher effect; Stark effect; Larmor Precession.
(6) Proposes several experiments to test proposed effects, such as, definition of spin-electric and spin-magnetic fields, Spin-Aharonov–Bohm effect, Dual-Hall Effect/Topological Insulator, whether $\rho_{ext-xx}(\mathbf{B}) \approx 0$ of GMR/TMR, Spin-Potential-Coupling-Induced force, Zeeman effect/Extended-Rashba SOC.

We argue that the powerfulness and fruitfulness are evidences supporting Spin-EM. The Spin-EM predicts experimental results, which need to be tested.

The mathematical identities lead to the physical dualities. UMFT provides mathematical origins of physical dualities between Maxwell-EM, Gravito-EM and Spin-EM.

Spin-EM of Moving-Spin-e-Charge

Maxwell-EM shows that a resting and a moving-e-charge q_e respectively produce an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$.

We show in this article that:

(1) a resting spin-e-charge produces the combination of an electric field $\mathbf{E}$, a spin-magnetic field $\mathbf{B_s} = \mathbf{S}_e \times \mathbf{E}$;
(2) a moving spin-e-charge produce the combination of an electric field $\mathbf{E}$, a magnetic field $\mathbf{B}$, a spin-electric field $\mathbf{E_s} = \mathbf{S}_e \times \mathbf{B}$, and a spin-magnetic field $\mathbf{B_s}$, i.e., the superposition of $\mathbf{B}$, $\mathbf{E}$, $\mathbf{B_s}$ and $\mathbf{E_s}$.

Lorentz-type Force

By analog to the Lorentz force, we propose the Lorentz-type force on a moving-spin-e-charge q_e:

$$\mathrm{F} = q_e\mathbf{E} + q_e\mathbf{v} \times \mathbf{B} + q_e\mathbf{E}_s + q_e\mathbf{v} \times \mathbf{B_s} + q_e\mathbf{S}_s \times \mathbf{B} + q_e\mathbf{S}_e \times \mathbf{B_s}. \quad (5.24)$$

Effects of Spin-Lorentz-type Force

The predictions/effects of Spin-Lorentz-type Force are the following:

(1) Dual-Hall Effect/Topological Insulator;
(2) Extended-Hall Effect/Topological Insulator;
(3) Extended-Hall Effect Having Zero Longitudinal Resistivity;
(4) Extended-Hall Effect Contributing to GMR/TMR Effect;

(5) Extended-Hall Effect Contributing to Spin-Hall Effect;
(6) Extended-Hall Effect Contributing to Anomalous-Hall Effect;
(7) Temperature Dependence of Extended-Hall effect;
(8) Magnetic Aharonov-Casher-type effect;
(9) Spin-Stark Effect and Magnetic-Rashba-type SOC;
(10) Classical Origin of Aharonov–Casher effect;
(11) Spin-magnetic-Rashbe-type SOC;
(12) Mechanisms of Rashba SOC;
(13) Spin-Larmor-type Precession;
(14) Lagrangian-Hall-type Effect/Topological Insulator;
(15) Spin-Potential-Coupling Contributing to Aharonov–Bohm Effect.

SPIN-GRAVITY DERIVED FROM UMFT, NEWTON'S LAW AND SPIN-g-CHARGE

Spin-gravity of Resting-Spin-g-Charge Derived from UMFT, Newton's Law and Spin

We have shown in the Section 5, the spin of an e-charge produces Spin-EM theory. In this section we show that the spin of a g-charge produces Spin-gravity fields.

Definitions of Spin-gravity Fields:

We have shown that a g-charge, Q_g, produces a gravito-electric-field $\mathbf{g}$, $\nabla \cdot \mathbf{g} = -Q_g$, while the moving velocity $\mathbf{v}$ of the g-charge produces a gravito-magnetic-field $\mathbf{B_g}$, $\nabla \times \mathbf{B_g} = -Q_g\mathbf{v}$.

Now let us consider a spinning g-charge, for example a neutron, characterized by both g-charge Q_g and spin $\mathbf{S}_g$. Placing the spin-g-charge in an external gravito-electric-field $\mathbf{g}$ and an external gravito-magnetic field $\mathbf{B_g}$. We propose that the spin of the g-charge produces both the "spin-gravito-magnetic field $\mathbf{B_{gs}}$" and "spin-gravito-electric-field $\mathbf{g_s}$", both are axial vector fields. Starting with Equation (2.4b) of **UMFT**, let $\mathbf{S}_c \equiv \mathbf{S}_g$,

$$\nabla \times (\mathbf{S}_g \times \mathbf{T}) = \mathbf{S}_g(\nabla \cdot \mathbf{T}) - \mathbf{T}(\nabla \cdot \mathbf{S}_g) + (\mathbf{T} \cdot \nabla)\mathbf{S}_g - (\mathbf{S}_g \cdot \nabla)\mathbf{T}. \tag{2.4b}$$

In the external gravito-electric-field $\mathbf{g}$ and the external gravito-magnetic field $\mathbf{B_g}$, we define the spin-gravito-electric-field $\mathbf{g_s}$ and the spin-gravito-magnetic field $\mathbf{B_{gs}}$:

$$\mathbf{B_{gs}} \equiv \mathbf{S}_g \times \mathbf{g}, \tag{6.1}$$

$$\mathbf{g_s} \equiv \mathbf{S}_g \times \mathbf{B_g}. \tag{6.2}$$

The definitions, Eq. (6.1) and Eq. (6.2), state that: by interacting with a gravitation field $\mathbf{g}$ and a gravito-magnetic field $\mathbf{B_g}$, the spin of a g-charge produces a spin-gravito-magnetic field $\mathbf{B_{gs}}$ and a spin-gravitation field $\mathbf{g_s}$, respectively.

Subscript "s" indicates that the quantity related to spin. The $\mathbf{B_{gs}}$ and $\mathbf{g_s}$ are type-2 dual to the $\mathbf{B_g}$ and $\mathbf{g}$ fields, respectively. The $\mathbf{B_{gs}}$ and $\mathbf{g_s}$ are "Second order axial vector field" (SAF) and "Third order axial vector field" (TAF) respectively. The $\mathbf{g_s}$ and $\mathbf{B_{gs}}$ mathematically satisfy,

$$\nabla \cdot \mathbf{B_{gs}} = 0, \tag{6.3}$$

$$\nabla \cdot \mathbf{g_s} = 0. \tag{6.4}$$

We still keep those divergence terms in some of equations of Spin-gravity for showing the perfect duality.

Spin-gravity Derived from UMFT, Newton's Law and Spin:

Utilizing the Newton law, $\nabla \cdot \mathbf{g} = -\mathrm{Q_g}$, and assuming that $\mathbf{S}_\mathrm{g}$ is constant. Let the "**T**" is the external gravito-electric-field $\mathbf{g}$ and the external gravito-magnetic field $\mathbf{B_g}$ respectively, Equation (2.4b) gives

$$\nabla \times (\mathbf{S}_\mathrm{g} \times \mathbf{g}) = \mathbf{S}_\mathrm{g}(\nabla \cdot \mathbf{g}) - (\mathbf{S}_\mathrm{g} \cdot \nabla)\mathbf{g}, \tag{6.5}$$

$$\nabla \times (\mathbf{S}_\mathrm{g} \times \mathbf{B_g}) = \mathbf{S}_\mathrm{g}(\nabla \cdot \mathbf{B_g}) - (\mathbf{S}_\mathrm{g} \cdot \nabla)\mathbf{B_g}, \tag{6.6}$$

Or

$$\nabla \times (\mathbf{B_{gs}}) = -\mathrm{Q_g}\mathbf{S}_\mathrm{g} - (\mathbf{S}_\mathrm{g} \cdot \nabla)\mathbf{g}, \tag{6.7}$$

$$\nabla \times (\mathbf{g_s}) = -(\mathbf{S}_\mathrm{g} \cdot \nabla)\mathbf{B_g}, \tag{6.8}$$

$$\nabla \cdot \mathbf{B_{gs}} = 0, \tag{6.3}$$

$$\nabla \cdot \mathbf{g_s} = 0. \tag{6.4}$$

Substituting the external gravito-electric-field g ($\mathbf{T} = \mathbf{g}$), the external gravito-magnetic field ($\mathbf{T} = \mathbf{B_g}$), Eq. (4.72) and Eq. (4.73) into Eq. (2.5b), respectively,

$$\nabla \times (\mathbf{S}_\mathrm{g} \times \mathbf{T}) = \mathbf{S}_\mathrm{g}(\nabla \cdot \mathbf{T}) - \mathbf{T}(\nabla \cdot \mathbf{S}_\mathrm{g}) + \nabla(\mathbf{S}_\mathrm{g} \cdot \mathbf{T}) - 2(\mathbf{T} \cdot \nabla)\mathbf{S}_\mathrm{g} - \mathbf{S}_\mathrm{g} \times (\nabla \times \mathbf{T}) - \mathbf{T} \times (\nabla \times \mathbf{S}_\mathrm{g}). \tag{2.5b}$$

we can also derive Spin-gravity in a different form:

$$\nabla \times (\mathbf{g_s}) = \mathbf{S}_\mathrm{g}(\nabla \cdot \mathbf{B_g}) - \frac{\partial \mathbf{B_{gs}}}{\partial \mathbf{t}} + [\nabla(\mathbf{S}_\mathrm{g} \cdot \mathbf{B_g}) - 2(\mathbf{S}_\mathrm{g} \cdot \nabla)\mathbf{B_g} - \mathbf{S}_\mathrm{g} \times \mathbf{v}(\nabla \cdot \mathbf{g})]. \tag{6.9}$$

$$\nabla \times (\mathbf{B_{gs}}) = \mathbf{S}_\mathrm{g}(\nabla \cdot \mathbf{g}) + \left(\frac{\partial \mathbf{g_s}}{\partial \mathrm{t}}\right) + [\nabla(\mathbf{S}_\mathrm{g} \cdot \mathbf{g}) - 2(\mathbf{S}_\mathrm{g} \cdot \nabla)\mathbf{g}]. \tag{6.10}$$

Spin-gravity in this form has the same form as that of Spin-EM.

Eq. (6.7), Eq. (6.8), Eq. (6.3), and Eq. (6.4) are the complete set of the spin-gravity. Also Eq. (6.9), Eq. (6.10), Eq. (6.3), and Eq. (6.4) are the complete set of the spin-gravity.

Lorentz-type Gravitational Force

By analog to the Lorentz force, we propose the Lorentz-type gravitational force on a moving spin-g-charge q_g:

$$F = q_g\mathbf{g} + q_g\mathbf{g}_s + q_g\mathbf{v} \times \mathbf{B_g} + q_g\mathbf{v} \times \mathbf{B}_{gs} + q_g\mathbf{S}_g \times \mathbf{B}_g + q_g\mathbf{S}_g \times \mathbf{B}_{gs}. \tag{6.9}$$

SUMMARY

We apply the same mathematical identity formular (UMFT) to derive the physical field equations for describe different physical forces, including the known electromagnetic force, the known Newtonian gravitational force, the new gravito-magnetic force, the new spin-electromagnetic force, and the new spin-gravitational force. We argue that those physical forces are the physical dual to each other. UMFT provides mathematical origins of physical dualities between Maxwell EM, Gravito-EM, Spin-EM, and Spin-gravity. They are all derived from UMFT and thus, they have the same symmetry, such as U (1) symmetry of Maxwell-EM, and can be unified in the frame of UMFT.

Appendix:

A. Duality

In Mathematics, duality is one of the most fruitful ideas, is a ‘principle’ and is very powerful and useful [38]. In physics, the concept of duality has played an important role in the development of physical theories. The underlying idea is that the analogy between different phenomena in Nature is not a mere coincidence.

Moreover, duality gives one point of view of looking at the different objects, which, in principle, are all duals. When equations/theories are mathematically equivalent, then they are dual to each other. We argue that: “Mathematical identities lead to mathematical duality that lead to physical duality. Duality discloses the similarity of intrinsic nature of apparently different physical interactions and leads to unification”. We show that this point of view of duality is powerful, fruitful and guidance.

‘Duality’ may be advanced to ‘symmetry’, e.g., the duality between EM and Gravito-EM leads to the e-charge/g-charge symmetry, ultra-symmetry.

Let’s study mathematical duality between different quantities.

A1. Type-1 duality and Type-2 duality

For studying dualities clearly and conveniently, let’s introduce different level axial vector fields:
First level axial vector field (abbreviated FAF): “**c**” is defined as the cross product of **a** and b,

$$\mathbf{c} \equiv \mathbf{a} \times \mathbf{b},$$

where both the “**a**” and “**b**” are *vector field*, denote “**c**” as FAF.

Second level axial vector field (abbreviated SAF): “**d**” is defined as the cross product of **e** and f,

$$\mathbf{d} \equiv \mathbf{e} \times \mathbf{f},$$

where the "**e**" is a *vector field* and "**f**" is a *first level axial vector field*, denote "**d**" as SAF.

Third level axial vector field (abbreviated TAF): "**n**" is defined as the cross product of **q** and p,

$$\mathbf{n} \equiv \mathbf{q} \times \mathbf{p},$$

where both the "**q**" and "**p**" are *first level axial vector fields*, denote "**n**" as TAF.

Now let's introduce two categories of dualities as following:

Type-1 duality: for an axial field $\mathbf{W}_{\text{before}} \equiv \mathbf{S} \times \mathbf{T}$, under transformation(s) of either **S** or **T** or both **S** and **T**, the field $\mathbf{W}_{\text{before}}$ transfers to $\mathbf{W}_{\text{after}}$. Under two conditions: (1) $\mathbf{W}_{\text{after}}$ and $\mathbf{W}_{\text{before}}$ are same level axial vector field; (2) the field equations describing respectively $\mathbf{W}_{\text{after}}$ and $\mathbf{W}_{\text{before}}$ have either the same form or are mathematically equivalent; then there is a Type-1 duality between $\mathbf{W}_{\text{after}}$ and $\mathbf{W}_{\text{before}}$.

Type-2 duality: for an axial field $\mathbf{W}_{\text{before}} \equiv \mathbf{S} \times \mathbf{T}$, under transformation(s) of either **S** or **T** or both **S** and **T**, the field $\mathbf{W}_{\text{before}}$ transfers to $\mathbf{W}_{\text{after}}$. Under two conditions: (1) $\mathbf{W}_{\text{after}}$ and $\mathbf{W}_{\text{before}}$ are different level axial vector fields; (2) the field equations describing respectively $\mathbf{W}_{\text{after}}$ and $\mathbf{W}_{\text{before}}$ have either the same form or are mathematically equivalent; then there is a Type-2 duality between $\mathbf{W}_{\text{after}}$ and $\mathbf{W}_{\text{before}}$.

Where the "**T**" is either a vector field $\mathbf{T}_{\text{vector}}$ (abbreviated $\mathbf{T}_{\mathbf{v}}$) or an axial vector field $\mathbf{T}_{\text{axial-vector}}$ (abbreviated $\mathbf{T}_{\mathbf{av}}$) or a field $\mathbf{T}_{\mathbf{combination}}$ (abbreviated $\mathbf{T}_{\mathbf{c}}$) that is the combination of a vector and an axial vector $\mathbf{T}_{\mathbf{c}} = \mathbf{T}_{\mathbf{v}} + \mathbf{T}_{\mathbf{av}}$. The "**S**" may be either a vector field $\mathbf{S}_{\text{vector}}$ (abbreviated $\mathbf{S}_{\mathbf{v}}$) or an axial vector field $\mathbf{S}_{\text{axial-vector}}$ (abbreviated $\mathbf{S}_{\mathbf{av}}$).

Note: A special situation is that, during the transformation, one term in equation becomes zero. For keeping the same form of equations, we still keep the zero-term for the purpose of discussing duality. Then, in later calculation, ignore those zero-terms.

Specific examples of type-1 duality: In the following examples, "n" is an integer and n = 1, 2, 3….

Example 1: Corresponding to different axial vector fields $\mathbf{T}_{\mathbf{av}/\mathrm{n}}$, the fields $\mathbf{v} \times \mathbf{T}_{\mathbf{av}/\mathrm{n}}$ are SAF, where **v** is a vector. Dualities between those SAFs are type-1 duality, i.e., under transformation,

$$\mathbf{T}_{\mathbf{av}/1} \leftrightarrow \cdots \leftrightarrow \mathbf{T}_{\mathbf{av}/\mathrm{n}}$$

there are conversions between the SAFs,

$$\mathbf{v} \times \mathbf{T}_{\mathbf{av}/1} \leftrightarrow \cdots \leftrightarrow \mathbf{v} \times \mathbf{T}_{\mathbf{av}/\mathrm{n}},\ (\mathrm{n} \neq 1)$$

and between UMFT describing them.

Example 2: Corresponding to different axial vector fields $\mathbf{T_{av/n}}$, the fields $\mathbf{S_v} \times \mathbf{T_{av/n}}$ are TAF. Dualities between those TAFs are type-1 dualities, i.e., under transformation,

$$\mathbf{T_{av/1}} \leftrightarrow \cdots \leftrightarrow \mathbf{T_{av/n}}$$

we have conversions between the TAFs,

$$\mathbf{S_v} \times \mathbf{T_{av/1}} \leftrightarrow \cdots \leftrightarrow \mathbf{S_v} \times \mathbf{T_{av/n}},\ (n \neq 1),$$

and between UMFT describing them.

Specific examples of type-2 duality:

- **Example 1:** Corresponding to a vector field $\mathbf{T_{v/n}}$ and an axial vector field $\mathbf{T_{av/n}}$, the fields $\mathbf{v} \times \mathbf{T_{v/n}}$ and fields $\mathbf{v} \times \mathbf{T_{av/n}}$ are FAF and SAF respectively. Duality between $\mathbf{v} \times \mathbf{T_{v/n}}$ and $\mathbf{v} \times \mathbf{T_{av/n}}$ is type-2 duality.
- **Example 2**: same as Example 1, but replace vector $\mathbf{v}$ with vector $\mathbf{S_v}$.
- **Example 3**: Corresponding to a vector field $\mathbf{T_{v/n}}$, the field $\mathbf{v} \times \mathbf{T_{v/n}}$ and field $\mathbf{S_{av}} \times \mathbf{T_{v/n}}$ are FAF and SAF respectively. Duality between $\mathbf{v} \times \mathbf{T_{v/n}}$ and $\mathbf{S_{av}} \times \mathbf{T_{v/n}}$ is type-2 duality.
- **Example 4**: same as Example 3, but replace $\mathbf{T_{v/n}}$ with $\mathbf{T_{av/n}}$.

A2. Transferability between Dualities of Same Type

The mathematical type-1 duality and type-2 duality can be transferred. We propose Transfer Rules:

1. There are type-1 dualities between A and B and between C and D. If the duality between A and C is type-1, then the duality between B and D is type-1, and vice versa;
2. There are type-2 dualities between A and B and between C and D. If the duality between A and C is type-1, and if B and D are the same lever axial fields, then the duality between B and D is type-1, and vice versa;
3. There are type-1 (or type 2) dualities between A and B; and C is mathematical equivalent to A. There is D that is mathematical equivalent to B, and thus is type-1 (or type 2) dual to C.
4. There is type-1 (or type 2) duality between A and B; and C is mathematical equivalent to A. There is a type-1 (or type 2) dual of C, which is mathematical equivalent to B.

A3. Duality and Quasi-Duality between Physical Theories

The conventional duality is a one-to-one mapping and equivalence between two physical theories, which have similar formulations but have different values of corresponding physical quantities, so that we should describe different interactions with one theory, rather than with two theories.

To study the above topics further, we restudy the concept of duality:

Duality: between two theories should satisfy the following requirements:

- ***Requirement* 1**: both theories have corresponding symmetries.
- ***Requirement* 2**: both theories have the formulas of same forms and corresponding quantities of different values, e.g., linear-to-linear and non-linear-to-non-linear theories.
- ***Requirement* 3**: each theory has charge conjugation.
- In physics, duality with same kind of symmetries will disclose deeper relation between two theories. Based on Noether's theorem, Requirement 1 leads to Requirement 2. But Requirement 2 does not necessarily lead to Requirement 1 before proving.

In physics of gravity, the conventional Gauge/Gravity Duality doesn't satisfy Requirement 1. The gauge theories are dictated by internal and CPT symmetries. On the contrary, there are no such symmetries in conventional Einstein theory of General Relativity (GR).

Thus, we introduce a new less-restrict concept of Quasi-Duality:
Quasi-Duality: two theories satisfy part of Requirements of Duality, e.g., both theories with either non-corresponding symmetries or one of theories with unknown symmetry of the kind that other theory has.

A4. Theory-independent and Transferability of Duality and Quasi-Duality

With theory-to-theory Duality (Quasi-Duality), we postulate following conjectures:

- ***Conjecture* 1**: the intrinsic nature of quantities disclosed by a Duality (Quasi-Duality) must be *theory-independent*.
- ***Conjecture* 2**: Dualities (Quasi-Dualities) are transferable.
- ***Conjecture* 3**: Quasi-Duality should advance to Duality eventually; otherwise, at least, one of theories needs to be modified.

Duality reflects intrinsic nature of the real world, while Quasi-Duality reflects only partial nature of the real world. A theory attempting to describe Nature is limited by present knowledge. Thus, a Duality, as well as a Quasi-Duality, should not depend on theories, since theory will be modified eventually. Conjecture 1 reflects this fact. An example of Conjecture 2 is that if there are theory-A/theory-B Duality (Quasi-Duality) and theory-B/theory-C Duality (Quasi-Duality), we must have theory-A/theory-C Duality (Quasi-Duality). Conjecture 3 implies that our limited understanding of Nature will gradually progress and reach, hopefully, full understanding ultimately.

The general goals of understanding gravity are: (1) Find out the internal symmetry associates with gravity; (2) Suggest a gravitational charge conjugate; (3) Reformulate gravity in the same terms as that of other forces. We have completed the above tasks, so gravity and other three force are on the same footing. We have accomplished the tasks: (1) Quantize gravity; (2) unify gravity with other forces. Gravity is a physical field as other three interactions in real world 4-dimention spacetime.

A. Inver-square Law for Spin

In this article we have assumed that there is no spin-charge, i.e.,

$$\nabla \cdot \mathbf{E_s} = \mathbf{0}.$$

Since, in the nature, the spin has only one value, 1/2, which is the property of a charge. So, we postulate a physically hypothetical "spin-charge" Q_{spin} (abbreviated Q_s), which is a spin counterpart of e-particle, and satisfies an inverse-square law,

$$\nabla \cdot \mathbf{E_s} = Q_s.$$

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