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Bose Statistics of Random Walks on Graphs

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ABSTRACT

We build probabilistic process models of motions on a finite graph for particles moving randomly from vertex to vertex along the edges. For particle statistics, at first we select a single particle and then we build motions of the particle by randomly selected but consecutive edges of the graph. For Bose statistics, at first we build arbitrary random sequences of edges as requests for motions of particles along single edges. The edges are randomly selected by probability weights, or they are triggered by circuits of courses in small steps on circles. It happens that a motion along a selected edge is impossible, a facility not provided in particle statistics and inhibited in quantum mechanics, because there the annihilation of the zero state is zero. We present examples for these process models. We obtain some results, which follow from the balance equation, a refinement of the corresponding balance equation for Markov chains.

Keywords: Random walk, Bose statistics, Fock states, Markov chains, balance equation, harmonic oscillator.

INTRODUCTION

We build probabilistic process models of motions on a finite graph Γ(V, E) for particles moving randomly from a vertex j ∈ V to a vertex k ∈ V along the edge e = (j→k) ∈ E. Particle statistics occur, when we distinguish the particles: We select a single particle and then we build motions of this particle by randomly selected edges. We obtain a Markov chain. It is a sequences of consecutive edges of the graph. Bose statistics is obtained from arbitrary random sequences of edges, therefore independent of particles. The edge selection process uses edge weights ρ(e), or small step sizes ε(e) for each edge e ∈ E for walks on a torus, which trigger edges by circuits, as in [4]. In this context, Hamiltonians or Lagrangians offer special methods for determining edge weights or step sizes based on energy considerations, as in [14].

We obtain Bose statistics via random sequences of edges. The objects of motion are Fock states q: V → $\mathbb{N}_0$. A move along an edge e = (j→k) from vertex j to vertex k is accepted when q(j) > 0. It then alters the Fock state q by a transfer from j to k by q(j) → q(j) – 1 and q(k) → q(k) + 1. When q(j) = 0, a change of the state q is rejected. Along such a walk, the particle number N := $\sum q(j)$ is preserved. The special case, where $\sum q(j)$ = 1, is a Markov chain supplemented by rejections

We consider some counting results for the random walks. The balance equation (4.3) between ingoing and outgoing acceptance at each vertex shows, that the first eigenvector of the weight matrix of the edge weights ρ(e) contains the probability of “q(j) > 0” at each vertex j along the sequence of q’s of a random walk. It holds for all particle numbers N= $\sum q(j)$. Statistical results are comparable with QM (quantum mechanics as introduced in [2]). Creation and annihilation correspond to exchanges between poor vertices and rich vertices of a graph, defined in chapter 4.2.

The Z-function of a quantum grand canonical ensemble contains probability weights of Markov chains through Fock states [6]. Our walks through Fock states by random edge sequences are not Markovian because of the rejections. In this context, the Z-function contains probably weights as limit case of several independently running poor vertices.

EXAMPLES

Example 1: Metropolis Algorithm

Given vertex weights $\mu: V \to \mathbb{R}_+$, we define edge weights ρ for edges $e = (j \to k)$ by $\rho(e) := \min\{ \frac{\mu(k)}{\mu(j)}, 1 \}$. ρ is in equilibrium (3.8), which is useful for building densities via Markov chains (see [13]). In terms of energy, where $\mu(j) = \exp(-\beta \cdot \varepsilon(j))$ with energy values $\varepsilon(j)$, there is a preference for transitions to lower energies.

Example 2: Employers and Employees

We consider a model of fluctuations of employees $\nu \in \{1 \ldots N\}$ between employers, which are the vertices of a graph. A request for a transfer along an edge $e = (j \to k)$ from employer j to employer k has preference $\rho(e) > 0$.

If each transfer is requested by single employees $\nu \in \{1 \ldots N\}$, we obtain a Markov chain for each single employee ν and independently running Markov chains for all employees together.

If each transfer is requested by the employers, regardless of any employees, we obtain a random walk through Fock states, which consist of the number of employees at each employer. Therefore, we obtain Bose statistics. A transfer along an edge $e = (j \to k)$ may be requested, when there is no employee at employer j. Then nothing happens. Such a result is not realistic, but it is a consequence of the assumption of fixed rates $\rho(e)$ for randomly selected edges.

Example 3: Trading

Given a step function $\varepsilon: E \to \mathbb{R}_+$. We consider a random sequence of exchange of goods between trading partners $j, k \in V$ along an edge $e = (j \to k)$, where $\varepsilon(e)$ is the value of money of a single transfer of goods from j to k. At each pair (e,-e) of a directed edge e and its inverse (-e), the exchange is balanced by adding $\varepsilon(e)$ when e is selected and subtracting $\varepsilon(-e)$ when (-e) is selected. Now, the exchange of value of money is supervised by an exchange of coins with a coin value $w \in \mathbb{R}_+$. When the balance at (e,-e) exceeds w, a coin is transferred from k to j, and w is subtracted from the balance value. When the balance value exceeds $(-w)$, a coin is transferred from j to k, and w is added to the balance value. The process of the exchange of coins runs as in Example 2. The number of employees at an employer is replaced by the number of coins available to a trading partner.

Schrödinger in [11] claims, that coins are distinguishable. However, they are not distinguished in this process model. When a coin is requested, but no coin is available, the request is rejected. When fixed transfer rates are taken for granted, rejections are unavoidable. We have to introduce something like a dept service to handle these situations.

Example 4: Harmonic Oscillator

The Hamiltonian of a harmonic oscillator is

$$\mathcal{H}(x,p) = x^2 + p^2$$

From the Hamilton-Jacobi differential equations in small steps We obtain

$$(x, p) \quad \rightarrow \quad (x + \Delta t \cdot p, p) \tag{2.1}$$

$$(x, p) \quad \rightarrow \quad (x, p - \Delta t \cdot x) \tag{2.2}$$

with a small number $\Delta t > 0$. Both ordered steps together perform steps on a circle in the (x,p)-plane with nearly equal step sizes in one direction. The steps in the opposite direction on the circle are ordered pairs of steps

$$(x, p) \quad \rightarrow \quad (x, p + \Delta t' \cdot x) \tag{2.3}$$

$$(x, p) \quad \rightarrow \quad (x - \Delta t' \cdot p, p) \tag{2.4}$$

with another small number $\Delta t' > 0$.

A random sequence of forward steps (2.1), then (2.2), with step size Δt, and backward steps (2.3), then (2.4), with step size $\Delta t'$, serves as a model for an exchange of energy between two vertices, instead of an exchange of the value of money in Example 3. We observe circuits around the (x,p)-circle, where forward and backward circuits are possible. The process generates edge sequences: There is one circle for each pair (e,-e). A forward circuit triggers edge e, and a backward circuit triggers edge -e. For a single pair (e,-e) we obtain a graph with two vertices. When $\Delta t \neq \Delta t'$ one vertex follows mode statistics (compare [4] Resonator), the other follows a mirrored version of mode statistics. We obtain the special case $\Delta t = \Delta t'$ as high temperature limit $\beta \rightarrow 0$, see chapter 4.4.

METHODS

Transition Potentials

Preliminaries:

We consider connected directed simple finite graphs $\Gamma(V, E)$ with vertices V and edges E (see [3]).

<u>Precondition</u> (forward and backward motion)

$$\textit{When } e \in E\textit{, then its inverse } (-e) \in E \tag{3.1}$$

There are two methods for building random sequences of edges $(e_t)_{t=0,1,\ldots}$, using edge weights or small steps. These are stochastic counterparts of the operators considered in ([1] chapter 8.1) in the context of QM:

$$\exp(-\beta \cdot \mathcal{H}) \tag{3.2}$$

$$\exp(-i \cdot \frac{t}{\hbar} \cdot \mathcal{H}) \tag{3.3}$$

derived from a Hamilton operator $\mathcal{H}$, which measures the energy. (3.2) corresponds to edge weights. (3.3) corresponds to small steps on a torus.

Edge Weights:

Definition (random sequence)

A transition potential is a weight function of edges:

$$\rho\colon E \to \mathbb{R}_+ \tag{3.4}$$

A sequences of edges $(e_t)_{t=0,1,\ldots}$ is a random sequence, when

$$e_t = rand(\rho) \text{ for each } t = 0,1,\ldots \tag{3.5}$$

where rand(ρ) returns a randomly selected edge, based upon edge probabilities proportional to the weight function ρ.

Delayed Edge Selection in Small Steps:

Instead of different edge weights we introduce different step sizes at the edges.

Definition (small steps, see [4] for single edge pairs $\{e,-e\}$)

Given a step function

$$\varepsilon\colon E \to (0, ½)$$

and an orientation, i. e.

$$\sigma\colon E \to \{+1,-1\} \text{ with } \sigma(-e) = -\sigma(e)$$

Each pair $\{e,-e\}$ has its own circle $\mathbb{R}/\mathbb{Z}_{\{e,-e\}}$, where it acts upon by

$$e(x) := x + \sigma(e)\cdot\varepsilon(e) \text{ for } x \in \mathbb{R}/\mathbb{Z}_{\{e,-e\}}, \tag{3.6}$$

Definition (sequences of circuits)

The state space is the torus

$$T := \bigoplus_{\{e,-e\}} \mathbb{R}/\mathbb{Z}_{\{e,-e\}}$$

Each sequence of edges $(e_s)_{s=0,1,\ldots}$ together with an initial element $x_0 \in T$ defines a walk $(x_s)_{s=0,1,\ldots}$ through T by

$$x_{s+1} := (e_s)(x_s) \tag{3.7}$$

Because of (3.6), it alters only one circle per step.

At each circle $\mathbb{R}/\mathbb{Z}$ we observe paths trough the zero point 0. Two consecutive paths through 0 contain a total circuit around the circle in positive or negative direction or it are loops from 0 to 0 without a full circuit.

Positive circuits trigger edge e, negative circuits trigger -e. The sequence of circuits at all circles of the torus generated by the random walk (3.7) triggers a sequence $(e_t)_{t=0,\ 1,\dots}$ of the corresponding edges.

From such a sequence $(e_t)_{t=0,\ 1,\dots}$ we obtain probabilities of the edges, which may serve as edge weights for random sequences generated by these weights as defined in (3.5). The result is slightly different. For randomness like (3.5), the step sizes must be small, for example $\varepsilon(e) \approx 0.01$. There remains a waiting time between two selections of edges in {e,-e}, which is not provided in (3.5). However, the counting results are equal. Example 3, trading, makes use of delayed edge selection. It promotes processes with edge selection independent of states.

Equilibrium:

Equilibrium is a property of the weight function ρ. It occurs when there is a function $\mu: V \to \mathbb{R}_+$ that satisfies for each edge $e = (j \to k)$

$$\frac{\rho(e)}{\rho(-e)} = \frac{\mu(k)}{\mu(j)} \tag{3.8}$$

μ is a weight function, frequently represented in a physical context by

$$\mu(j) = \exp\left(-\beta \cdot \mathcal{H}(j)\right)$$

with a common inverse temperature β and energy values $\mathcal{H}(j)$.

Trees are always in equilibrium. Loops tend to be not in equilibrium. Example 3, trading, is not in equilibrium ("the circuit flow of money").

State Transitions

<u>Definition</u>

For $N \in \mathbb{N}$, the state space of the graph Γ (E, V) is

$$\mathcal{F}(V, N) := \{ q: V \to \mathbb{N}_0 \mid \textstyle\sum_{j \in V} q(j) = N\} \tag{3.9}$$

Each edge $e = (j \to k) \in E$ defines state transitions $e: \mathcal{F}(V, N) \to \mathcal{F}(V, N)$, acceptance, when $q(j) > 0$, defined by

$$\begin{aligned} e(q)(j) &:= q(j) - 1 \\ e(q)(k) &:= q(k) + 1 \\ e(q)(l) &:= q(l) \text{ for } l \neq j, k \end{aligned} \tag{3.10}$$

rejection, when q(j) = 0, defined by *(3.11)*

$$e(q) := q$$

The states (3.9) are the pure states of the corresponding bosonic Fock space. In QM rejections (3.11) are not visible because of $\boldsymbol{\mathcal{A}}|0\rangle = 0$ for the QM-specific annihilation $\boldsymbol{\mathcal{A}}$ of the state $|0\rangle$.

Definition

Each sequence of edges $(e_t)_{t=0,1,\ldots}$ together with an initial element $q_0 \in \mathcal{F}(V,N)$ defines a walk $(q_t)_{t=0,1,\ldots}$ through the elements of $\mathcal{F}(V,N)$ by

$$q_{t+1} := (e_t)(q_t) \tag{3.12}$$

Statistical properties of $(q_t)_{t=0,1,\ldots}$ follow Bose statistics, because we consider Fock states of N elements and therefore any individual properties of single $\nu \in \{1\ldots N\}$ are ignored.

IMPLICATIONS

Balance Equation

Each edge in the sequence $(e_t)_{t=0,1,\ldots}$ determines one step in time. For a finite but long random sequence of edges $(e_t)_{t=0,1,\ldots}$ and its corresponding sequence $(q_t)_{t=0,1.,\ldots}$ in $\mathcal{F}$(V,N) (see (3.12)) we define probabilities based on this time scale:

Definition

For each $j \in V$ and for each $n \in \mathbb{N}_0$

$$p(j,n) := p(\, t \mid q_t(j) = n\,) \tag{4.1}$$

For each $j \in V$ and for each $n \in \mathbb{N}_0$

$$p(acc,j) := p(\, t \mid q_t(j) > 0\,)$$
$$p(rej,j) := p(\, t \mid q_t(j) = 0\,)$$

For each edge $e = (j \rightarrow k)$

$$p(e) := p(\, t \mid e_t = e)$$
$$p(acc,e) := p(\, t \mid e_t = e \text{ and } q_t(j) > 0)$$
$$p(rej,e) := p(\, t \mid e_t = e \text{ and } q_t(j) = 0)$$

p(acc,…) and p(rej,…) are the probabilities for acceptance and rejection, restricted to a single edge or restricted to all t with an edge e_t which starts at vertex j, because edge selection is independent of states.

Proposition

$$p(acc,j) + p(rej,j) = 1 \tag{4.2}$$

$$p(e) = \frac{\rho(e)}{\sum_{e' \in E} \rho(e')}$$

Proposition (balance equation)
At a single vertex j, there is

$$\sum_{e=(k\to j)} p(e) \cdot p(acc,k) = \left(\sum_{e=(j\to k)} p(e) \right) \cdot p(acc,j) \qquad (4.3)$$

This holds, because the probabilities of ingoing and outgoing acceptance at a single vertex j must be equal.

Definition
We define $\boldsymbol{p} \in \mathbb{R}^V$ and a VxV-matrix ρ, where for each $e = (j\to k) \in E$

$$\boldsymbol{p}_j := p(acc,j)$$
$$\boldsymbol{\rho}_{jk} := \rho(e),\ if\ e = (j\to k) \in E,\ 0\ otherwise$$
$$\boldsymbol{\rho}_{kk} := -\sum_{j\neq k} \rho_{jk}$$

Proposition
$\boldsymbol{p} \in \mathbb{R}^V$ is an eigenvector of the matrix $\boldsymbol{\rho}$ with eigenvalue 0, unique up to a constant factor. All other eigenvalues have a real part < 0.

The vector $\boldsymbol{p} \in \mathbb{R}^V$ satisfies the balance equation (4.3). Then the proposition follows from the standard application of Perron-Frobenius theory.

Proposition (4.4)
For $\mathcal{F}$(V,1) (i. e. N = 1), *p(acc,j)* is the probability, that a particle is at vertex j.

This holds, because the states $q_j(k) = \delta_{j,k}$ with $j \in V$ are the only possible Fock states for N = 1. Please note, that the same probabilities are obtained, when the weights ρ(e) are used to build transition probabilities for a Markov chain through V.

Proposition
In equilibrium (3.8), $\mu(j) = \lambda\cdot p(acc,j)$ for some $\lambda > 0$ independent of j

This holds, because from (3.8) it follows, that the values μ(j) fulfill the balance equation (4.3).

Poor Vertices and Rich Vertices
Now we consider fixed edge weights (3.4), but different Fock spaces $\mathcal{F}$(V, N), because N will vary, especially N will run to infinity. It alters the counting results. Therefore, the probabilities defined in the previous chapter get an additional parameter "N", i. e. in (4.1) p (j, n, N).

Definition
For $j \in V$

$$p(acc,j,\infty) := \lim_{N\to\infty} p(acc,j,N)$$

p(acc,j,N) increases with N and there is p(acc,j,N) ≤ 1. Therefore, the limit exists.

<u>Definition</u> (not standard, but self-evident in example 3: trading)

$$j \text{ is a poor vertex}, \quad \text{when } p(acc,j,\infty) < 1$$
$$j \text{ is a rich vertex}, \quad \text{when } p(acc,j,\infty) = 1 \tag{4.5}$$

There is always at least one rich vertex because of:

<u>Proposition</u>

$$\sup \{ p(acc,j,N) \mid j \in V, N \in \mathbb{N} \} = 1$$

It holds, because V is finite.

Now we assume independency between the states at vertices j and k, where $j \neq k$, which makes sense for $n,m \ll N$:

<u>Precondition</u>

$$p(t \mid q_t(j) = n \text{ and } q_t(k) = m) = p(t \mid q_t(j) = n) \cdot p(t \mid q_t(k) = m)$$

Then for poor vertices j, where $p(acc,j,\infty) < 1$, we obtain mode statistics as defined in [7] p.100:

<u>Proposition</u> (mode statistics)
For each poor vertex $j \in V$ and $n \in \mathbb{N}_0$ with $p := p(acc,j,\infty)$, there is

$$p(j,n,\infty) = (1 - p) \cdot p^n \tag{4.6}$$

The mean value of n is

$$\langle n_j \rangle = \frac{p}{1-p} \qquad = \frac{p(acc,j,\infty)}{p(rej,j,\infty)}$$

For the mean value please note (4.2). We will derive (4.6) from the precondition, not from Boltzmann factors as in [7]:

$$p(t \mid (q_t(j)=n) \rightarrow (q_{t+1}(j)=n+1)) \text{ and } p(t \mid (q_t(j)=n+1) \rightarrow (q_{t+1}(j)=n))$$

are the probabilities, that an edge triggers a state change from $(q(j)=n)$ to $(q(j)=n+1)$ and vice versa. The first term is

$$p(t \mid (q_t(j)=n) \rightarrow (q_{t+1}(j)=n+1)) = \sum_{e=(k\rightarrow j)} p(e) \cdot p(t \mid q_t(j) = n \text{ and } q_t(k) > 0) =$$
$$\sum_{e=(k\rightarrow j)} p(e) \cdot p(t \mid q_t(j) = n) \cdot p(t \mid q_t(k) > 0) = p(j,n) \cdot \sum_{e=(k\rightarrow j)} p(e) \cdot p(acc,k)$$

using the precondition of (4.6). The second term is

$$p(t \mid (q_t(j)=n+1) \rightarrow (q_t(j)=n)) = \sum_{e=(j\rightarrow k)} p(e) \cdot p(j,n+1)$$

because "q(j)=n+1" inhibits rejections.

The terms on the left side, which are the probabilities of transitions n->n+1 and n+1->n, must be equal. For the rights sides it means

$$\frac{p(j,n+1)}{p(j,n)} = \frac{\sum_{e=k\to j} p(e)\cdot p(acc,k)}{\sum_{e=j\to k} p(e)}$$

and from the balance equation (4.3) we obtain

$$\frac{\sum_{e=k\to j} p(e)\cdot p(acc,k)}{\sum_{e=j\to k} p(e)} = \text{p(acc,j)}$$

Therefore, $\frac{p(j,n+1)}{p(j,n)}$ = p(acc,j), independent of n. It holds for each N, therefore, it holds in the limit N -> ∞, which is (4.6). Please note, that large N-values promote the independency in the precondition of proposition (4.6).

Let n be the number of elements in all poor vertices. Then the number of elements in all rich vertices is (N – n). Therefore, the corresponding probabilities are equal:

Proposition

$$P_{poor}(n) = p_{rich}\,(N - n) \tag{4.7}$$

Because of the limits (4.6) for poor vertices, further enlargement of a large N has a statistical effect only in the rich part. It offers an alternative approach to [8], together with energy considerations also to a Bose-Einstein condensate.

Figure 1 shows the counting results

$$Q(j,n) := \#\{\, t \mid q_t(j) = n \,\}$$

at each j ∈ V of a sequence of length ca. 10^9, generated by asymmetric edge weights, where N=100, therefore n=0 … 100. The corresponding probabilities (4.1) of this sequence are

$$p(j,n) = \frac{Q(j,n)}{\sum_{m=0}^{N} Q(j,m)} \tag{4.8}$$

The weights are

ρ(from→to)	**from [0]**	**from [1]**	**from [2]**	**from [3]**
to[0]		645,661	450,819	425,557
to[1]	354,339		598,051	0,45
to[2]	549,181	401,949		474,854
to[3]	574,443	0,55	525,146	

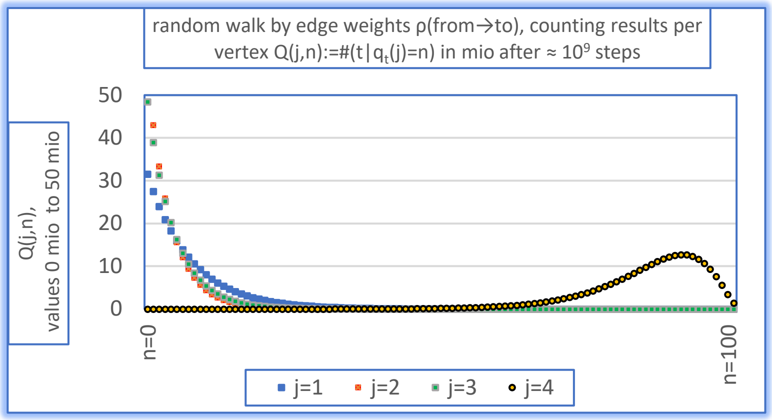

Figure 1

It illustrates the exceptional behavior of the rich vertex j=4.

Figure 2 shows values

$$\log(Q(j,n) \,/\, Q(j,n\text{-}1))$$

derived from counting results Q(j,n). The values are constant, whenever

$$Q(j,n) = \exp(\alpha+\beta\cdot n) \text{ for some } \alpha,\beta \in \mathbb{R}.$$

It fits to mode statistics (4.6), when

$$\alpha = \log\,(1 - p(acc,j,\infty)),\ \beta = \log(p(acc,j,\infty))$$

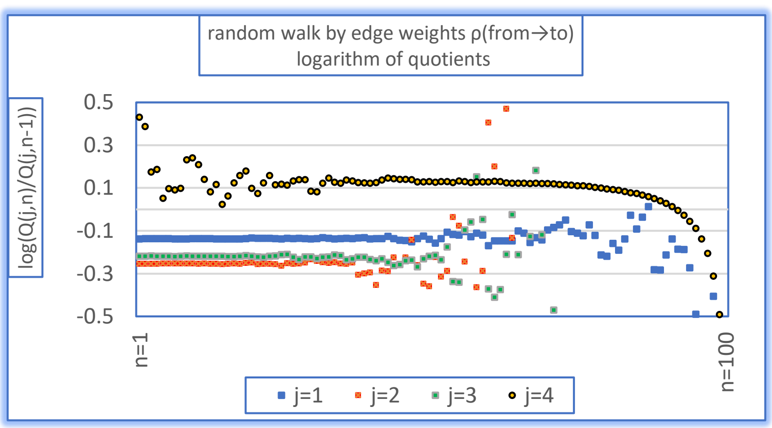

Figure 2

It confirms mode statistics for the poor vertices j = 1,2,3 for n << N, although N = 100 is rather small, which is an obstacle for the precondition of perfect mode statistics.

Figure 3 illustrates the difference between the acceptance rates p(acc,j,N) of poor vertices (j = 1,2,3) and rich vertex (j = 4) as defined in (4.5).

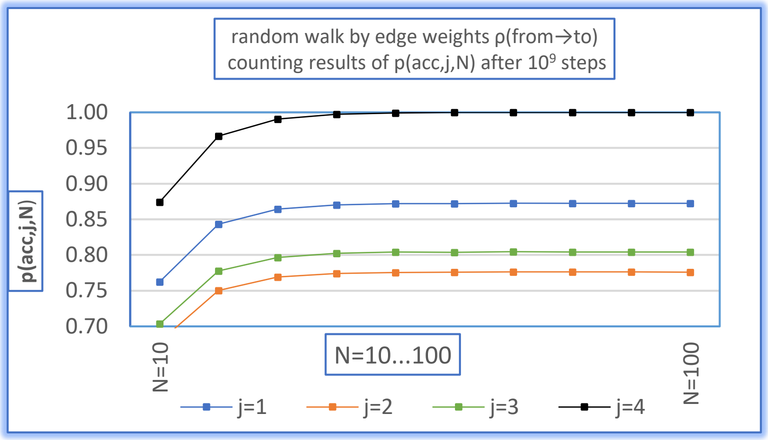

Figure 3

Because of **Figure 1** we expect a limit value p(acc,j,∞) < 1 for j = 1,2,3 and p(acc,j,∞) = 1 for j = 4. It is confirmed already for small values of N.

Z-function

We consider the Z-function of a quantum grand canonical ensemble

$$Z = \prod \frac{1}{1-\mu(j)} = \sum_{N\in\mathbb{N}_0} h_N(\mu_1, \ldots, \mu_{|V|}) \tag{4.9}$$

where h_N is the complete homogeneous symmetric polynomial of degree N in |V| variables μ(j), j ∈ V. It holds for systems, where single monomials of the symmetric polynomials are probability weights.

In [5] there is μ(j) = exp(-β·(ε(j) – μ)) and probability weights occur as Boltzmann factors, as derived in [7] p. 70 ff. In [6] it is derived from probabilities for increment and decrement steps. (4.9) is the Z-function of a corresponding Markov chain.

We obtain a single factor $\frac{1}{1-\mu(1)}$ as Z-function of a graph Γ (V, E), where |V| = 2, with unique poor vertex j = 1 and rich vertex j=2 as limit case (4.6), where p(acc,1, ∞) = μ(1) < 1.

High Temperature Limit

In equilibrium we obtain families of transition potentials by varying the inverse temperature β in (3.2):

$$\rho(\mathcal{H}, \beta) := \exp(-\beta \cdot \mathcal{H}(e))$$

where $\mathcal{H}$(e=(j→k)) = $\mathcal{H}$(k) – $\mathcal{H}$(j). In the limit β → 0 there is ρ(e,β) = 1, and we obtain the incidence matrix of the graph, which is symmetric because of (3.1). Then, the balance equation (4.3) is fulfilled by equal acceptance rates p(acc,j) at the vertices j. There exists one rich vertex j, i. e. p(acc,j) = 1. Therefore all vertices j have p(acc,j) = 1.

In the limit case where β = 0, all elements of $\mathcal{F}$(V, N) are equally probable. Their number is $\frac{(N+K-1)!}{N!(K-1)!}$, with K := |V|. This is the statistical base used by Planck when he derived his radiation law [10], also treated in [9] and [12].

Figure 4 shows the counting results of a limit case of β = 0, therefore edge weights 1, which emerges from weight table (4.8).

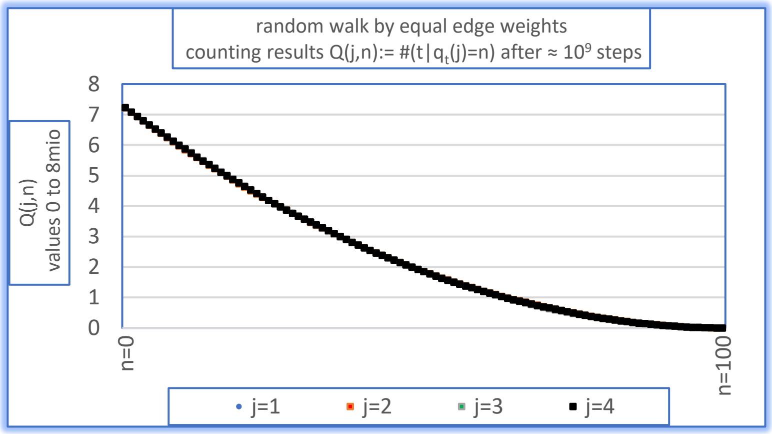

Figure 4

The course is equal for all vertices. It is like a parabola, which is confirmed by the differences in **Figure 5**.

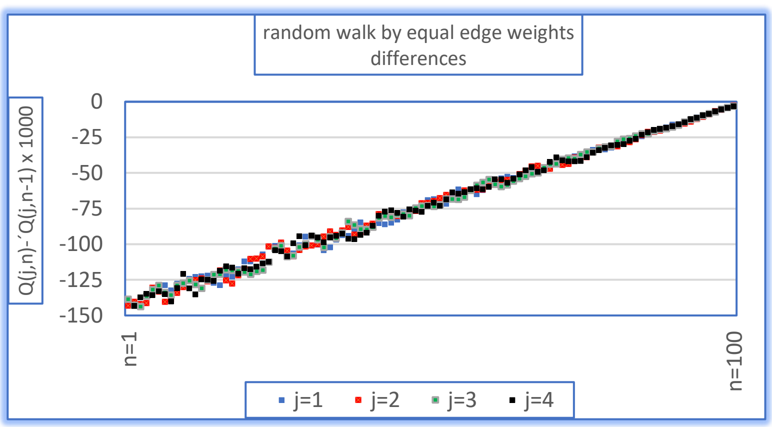

Figure 5

CONCLUSION

We considered two methods to create random sequences of edges, one using edge weights, and the other using small steps. Sequences of the edges generate sequences of vertex states, which are base elements of a Fock space.

We defined probabilities for Fock states q ∈ $\mathcal{F}$ (V, N) in (4.1), which follow particle statistics for N = 1 and Bose statistics for N > 1. We considered two kinds of process steps of single edges, acceptance and rejection.

Rejections occur because of free edge selection by fixed rates, which is independent of current states. At least, we obtain a sequence of the rejected edges and therefrom a secondary sequence through Fock states by (3.12).

In QM rejections are not visible because of $\boldsymbol{\mathcal{A}}|0\rangle = 0$ for the QM-specific annihilation $\boldsymbol{\mathcal{A}}$ of the state $|0\rangle$.

The Z-Function of a quantum grand canonical ensemble describes a Markov chain running through Fock states, where it is possible to ignore rejections.

When we assume, that the number of process steps is a time scale, then for a Markov chain one counts changes of states, but we counted steps of edge selections.

A special case occurred as a limit at high temperature, where all weights are 1 (incidence matrix). Then all vertices are rich vertices.

As a result, for a fixed weight function of edges we obtain a family of random walks through Fock states $\mathcal{F}$ (V, N), parametrized by the number of elements N, with a common balance equation, fulfilled by the first eigenvector of the common weight matrix. For N = 1 we obtain a Markov chain, supplemented by rejections. For $N \to \infty$ we obtain mode statistics, where the poor vertices correspond to single factors of the Z-function of a quantum grand canonical ensemble.

Data Availability

Data of the line diagrams in Figures 1 to 4 available as an Excel sheet on demand

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Conflict of Interest

All authors have no conflicts of interest.

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