

Planet Mercury may have Retained a Retrograde Satellite

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ABSTRACT

The limiting direct and retrograde orbits around the planet Mercury have been calculated. Synthesizing these concepts with the concepts of Roche limit, synchronous orbit around the planet and the tidal drags acting within it, it is shown here that the planet Mercury might have been able to retain a retrograde satellite. Observers are therefore requested to discover it through infrared telescope of wavelength of a few microns.

Keywords: Mercury, its satellites.

INTRODUCTION

In the solar system, Earth has a satellite, our Moon. It also has Trojan asteroids associated with its orbit. Similarly, Mars has two satellites named Phobos and Deimos; as well as other planets have extensive satellite systems. They may also have Trojan asteroids associated with their orbits. One may have a question; what about the planets Mercury? Does Mercury has a satellite system or atleast one or two satellites moving around it in either prograde or retrograde orbits? Is it that Mercury never had any satellites of its own? Or perhaps it had one or two satellites moving in either prograde or retrograde orbits, but somehow were destroyed? Or maybe it has, yet, one or two small satellites orbiting around, probably in prograde orbit at comparatively small distance or in retrograde orbit at relatively large distance; either of which we haven't been able to discover.

Being in the close vicinity of the Sun, Mercury satellite system might not have developed mainly because of two reasons: one is the strong gravity of the Sun and the second is the heat of the Sun due to which planetary material might have evaporated very soon even before it could get together to form a satellite system. However, there is a likelihood that one or two satellites may have formed before the planetary material would have evaporated and hence survived till date. To know this, works of King(1962) and Innanen(1979) come to our rescue. King considered a

system of the Milky Way galaxy, a star cluster and a star and calculated the limiting direct orbit around the star cluster. Following King, Innanen used the same system with the equation of acceleration in a rotating coordinate frame and took into account the Coriolis force of magnitude $2\Omega v_r$, where v_r is the velocity of the star relative to the star cluster and thus calculated the limiting direct orbit as well as the limiting retrograde orbit around the star cluster. Following Innanen, we consider here a system of the Sun, Mercury and a satellite as shown in Figure 1 and calculate the limiting direct and limiting retrograde orbits around the Mercury. Incorporating these concepts with the concepts of Roche limit, synchronous orbit around the planet and tidal forces acting within them, it is shown here that Mercury may not have retained any prograde satellite but there are chances that atleast one retrograde satellite may have been retained orbiting around the planet.

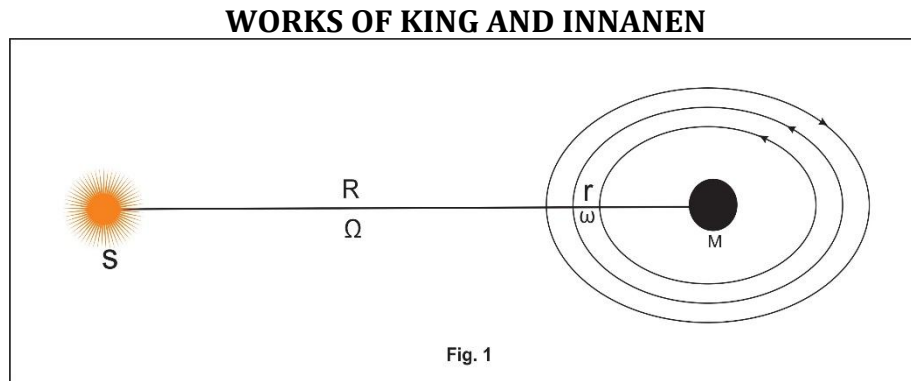


Figure 1: Schematic diagram of showing satellite system of Mercury, if it exists satellites may be going around the planet in retrograde or prograde orbits.

King(1962) calculated the limiting tidal prograde orbit, r_{lim} given by

$$r_{lim} = \frac{Gm}{\Omega^2 - d^2V/dR^2} \quad (1)$$

where r_{lim} is the limiting radial distance of a grand secondary from the secondary; m is the mass of the secondary; Ω , the angular velocity of the secondary around the primary; V , the gravitational potential energy of the primary, and R , the radial distance to the secondary from the center of the primary. If we represent the force field of the primary by an inverse square law due to its mass M then,

$$\frac{d^2V}{dR^2} = -2 \frac{GM}{R^3} \quad (2)$$

Hence

$$r_{lim} = \left[\frac{m}{3M} \right]^{1/3} R \quad (3)$$

But for the elliptical orbit, we have to take eccentricity into consideration and so,

$$\Omega^2 = GMa(1 - e^2)/R^4 \quad (4)$$

where, a is the semi-major axis of the ellipse and e is its eccentricity. At the perihelion point, R takes the value R_p given by

$$R_p = a(1 - e) \quad (5)$$

Therefore, equation (3) in this case becomes,

$$r_{lim} \left[\frac{m}{(3+e)M} \right]^{1/3} R_p \quad (6)$$

A moon revolving around a planet that, in turn, is revolving in the same sense around the Sun, will at some limiting distance from the planet becomes unstable because of the action of the Sun's tidal force. If the same moon, at the same distance, were to revolve in the opposite direction to the planet's revolution, then it could resist the Sun's tidal force better. At greater limiting distance from the planet, this retrograde moving moon would eventually succumb to the Sun's tidal force. This limiting retrograde radius thus defines the true gravitational sphere of influence of a planet. The familiar right hand rule immediately shows that the Coriolis term is always directed radially between the moon and the planet. It counteracts the planet's gravity for the direct motion of the moon, but effectively supplements the planet's gravity for retrograde motion. For the limiting direct and retrograde radii of a moon around a planet, we have the following relation

$$\frac{r_r}{r_d} = 3^{2/3} \quad (7)$$

where, r_d is the limiting direct radius and r_r is the limiting retrograde radius. But we have the value for the limiting direct radius as follows

$$r_d = \left[\frac{m}{3^2 M} \right]^{1/3} R \quad (8)$$

For the case where the planet's orbit has eccentricity e and pericentric distance $R = R_p$, we have

$$\frac{r_r}{r_d} = \left[\frac{5+e+2(4+e)^{1/2}}{3+e} \right]^{2/3} \quad (9)$$

For convenience, we write

$$\left[\frac{5+e+2(4+e)^{1/2}}{3+e} \right]^{2/3} = f(e) \quad (10)$$

From equations (9) and (10), with $R = R_p$, one gets

$$r_d = \left[\frac{1}{f(e)^2} \frac{m}{M} \right]^{1/3} R_p \quad (11)$$

SATELLITES OF MERCURY

We consider here a moon-Mercury-Sun system taking into account the eccentricity. We use formulae (11) and (9) and find the limiting direct r_d and retrograde r_r orbits around Mercury, thus setting the boundary to the satellite system of Mercury. The values are set out in Table 1.

It is a well-known fact that when a secondary, going around a primary, enters the stationary orbit around its primary, the disruptive forces start acting on it. As a result, the secondary starts spiraling in, entering into the Roche limit around the primary and eventually getting itself fragmented due to the tidal forces of the primary. That is, the ultimate fate of a secondary revolving around its primary within the stationary orbit is to destroy itself (see, Brecher et al., 1979[1]; Rawal, 1981[4]). It is, therefore clear that if the limiting prograde or retrograde orbit around primary lies within the stationary orbit around it, then such a primary cannot retain its secondary for a sufficiently longer time through the age of the Solar System. We, therefore calculate first the stationary orbit around Mercury (Table 1).

Table 1 shows that in case of Mercury, the limiting direct orbit does, indeed, lie well within the stationary orbit, however, the limiting retrograde orbit does not. Therefore, Mercury may have a retrograde satellite orbiting around it within a distance 242834 km to 252757 km., or 217989 km to 227132 km depending upon its mass ratio shown in the Table 1, if it has managed to survive the strong gravity and/or the immense heat of the Sun. In calculating the stationary, direct and retrograde orbits around Mercury, we have taken the following values of various quantities:

- Mass of Mercury = $0.0553M_{\oplus}$ (M_{\oplus} is the mass of the Earth)
- rotational period = 58.65 days
- mean orbital radius = 0.387 AU
- eccentricity = 0.206

Lyttleton argues that (see, Roman, 1980) the Sun-Mercury mass ratio which is usually taken as 1:6000000, the corresponding mass of Mercury being $0.0553 \times M_{\oplus}$ is in error by about 50% and suggests it to be 1:8500000 and hence, the corresponding mass of Mercury becoming $0.04 \times M_{\oplus}$. Taking this as the mass of Mercury, we recalculate the values of the stationary, direct and retrograde orbits and find that though, it shifts all the orbits inward, the final conclusion arrived at is not changed.

Table 1: Values for different orbits of Mercury

Mercury mass ratio	Limiting direct radius of a moon around Mercury, r_d (km)	Limiting retro-grade radius of a moon around Mercury, r_r (km)	Stationary orbit around Mercury
1:6,000,000	124,194	252,757	242,834
1:8,500,000	111,603	227,132	217,989

CONCLUSION

We would therefore like to point out the probability of existence of a satellite moving retrograde orbit around Mercury within a distance 242834 km to 252757 km. or 217989 to

227132 km that should be investigated by observers through infrared telescope of wavelength of a few microns to support the theoretical calculations of existence of a satellite mentioned here.

References

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