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Improvement of Multi-criteria Evaluation Tools by AHP-ELECTRE II Hybridization: Application to Drilling

Ouedraogo, Nabonswendé Macaire
Université Joseph KI-ZERBO, Laboratoire d'Analyse Numérique d'Informatique et de Biomathématique,03 BP 7021 Ouaga 03

Savadogo, Zoïnabo
Université Joseph KI-ZERBO, Laboratoire d'Analyse Numérique d'Informatique et de Biomathématique,03 BP 7021 Ouaga 03

Gouba, Elisée
Université Joseph KI-ZERBO, Laboratoire d'Analyse Numérique d'Informatique et de Biomathématique,03 BP 7021 Ouaga 03

ABSTRACT

Elimination and Reality Choices (ELECTRE II) and Hierarchical Decision Process Analysis (HDPA) methods are widely used in decision support. Although they are effective in many cases, they are not without their limitations. The AHP method is sometimes considered to be compensatory. As for the ELECTRE II method, it lacks rigorous tools for setting the weights of the decision criteria. To overcome these limitations, we have developed a hybrid method, AHP-ELECTRE II, which combines these two approaches. This AHP-ELECTRE II hybrid combines the advantages of both methods. In this way, we have been able to put in place a procedure that allows criteria to be weighted appropriately using the technique of binary comparison between criteria in accordance with the requirements of the AHP method. This comparison is made by the decision-maker, enabling him to determine the relative weights of the criteria objectively. We then used the weights of the criteria obtained to aggregate performance using the ELECTRE II method. This enabled us to deal with the interactions between the criteria and to rank the alternatives according to how well they met the criteria defined. In our study, we applied this hybrid method to didactic problems, which enabled us to obtain satisfactory results by taking better account of the decision-maker's preferences. We used it to evaluate areas suitable for drilling, simultaneously taking into account indicators of accessibility, potential and exploitable water. The advantage of this hybrid approach is that it combines the analytical rigour of AHP with ELECTRE II's ability to handle interactions between criteria, providing a more complete method that is suited to complex multi-criteria decision support problems.

Keywords: multi-criteria decision, ELECTRE II, modelling, AHP, Decision Maker, hybridisation.

INTRODUCTION

Making a decision in a complex environment is, and always will be, a challenge for the decision-maker. It requires the use of robust methods to evaluate and rank alternatives or actions. In the literature, several decision support methods have been developed to obtain optimal results.

Some of them have weaknesses when it comes to solving certain multi-criteria problems. When faced with an evaluation model, the decision-maker's preferences are not always rigorously defined a priori and stable [1]. The criterion is the tool used to evaluate an alternative using an aggregation function on various dimensions to objectively express a point of view or a preference. Criterion evaluation is less complicated in a single-criteria analysis where each potential action can be evaluated on a single axis of significance chosen a priori [1]. When the problem incorporates several criteria, the relative assessment of the criteria becomes too complex. Some methods, such as

ELECTRE uses weighted criteria in its aggregation procedure. The weighting of criteria remains problematic for decision-makers, who are unable to express their preferences effectively when there are several criteria.

Faced with this problem of weighting criteria, how can we achieve weights that take account of the relative importance of different criteria for the same problem? In other words, how can the criteria be weighted without being arbitrary? This difficulty in setting weights, which affects multi-criteria aggregation procedures, calls for us to find a scientific way of determining relative weights without sidelining the decision-maker at this stage of the decision-making process. We have therefore opted to use the AHP method to determine the weights of the criteria. The AHP method makes it possible to obtain normalised criteria weights through a calculation process that uses the concept of iteration to find the eigenvalue of the optimal eigenvector [2], [3]. The ELECTRE method and its extensions are still used to solve multi-criteria problems [4]. Hybrid integration models with other methods exist in the literature [5]. Integrating the ELECTREII method with the AHP method will provide a better model of the decision-maker's preferences [5]. This combination leads to stable and objective results. Many of life's multi-criteria problems present criteria that are often conflicting but very important in solving the problem; this is the case for water resource management. Providing everyone with sufficient quality water is one of the Millennium Development Goals (UN 2000), which is a challenge for Sub-Saharan Africa. This is even more complex when we want to make an assessment based on the indicators of accessibility, potentiality and exploitability of groundwater simultaneously [6]. To this end, we have opted for a hybrid approach between the AHP and ELECTRE II methods in order to take full account of all the indicators needed to assess areas suitable for drilling. To do this, we will first present the principle of combining the two methods, and then we will apply this new hybrid approach to solving a problem of multi-criteria evaluation of areas suitable for water drilling. We will also discuss the results obtained by this method, followed by recommendations and, finally, a conclusion and suggestions for possible improvements to the results.

PRESENTATION OF THE AHP AND ELECTRE II METHODS

AHP Method

The Analytic Hierarchy Process (AHP) multi-criteria decision support method created by Thomas Saaty [7] is widely used in decision-making processes in many fields throughout the world. The criteria can be quantitative (objective) or qualitative (subjective), without compromising either perspective [8]. The easy-to-use method is based mainly on pairwise comparisons of alternatives, and of the criteria and sub-criteria themselves. It avoids arbitrary weightings by using a consistency test [9].

Méthode ELECTRE II

The Elimination and Choice Expressing Reality (ELECTRE) method is part of the γ problem, which is a ranking procedure. By using order relations on each of the criteria, it aims to provide the set of potential actions with a total pre-order structure in order to facilitate choice [1].

- Pre-order: this is a reflexive and transitive relationship.
- reflexivity: ; $x\Re x$ for everything $x \in E$ with E the whole in which the pre-order is established.
- Transitivity: $x\Re y$ and $y\Re z \Rightarrow x\Re z$ for everything x; y et z of E.
- The pre-order admits ex-aequo.
- Order: an antisymmetrical pre-order $x\Re y \Rightarrow y \;\; non \;\; y\Re x \;\; (x \neq y)$
- Partial pre-order: pre-order in which incomparability is permitted.

The ELECTRE II method uses the out classification relationship constructed on the basis of concordance and discordance. There are two types of outranking: strong outranking and weak outranking. Each upgrade relationship is represented by a graph called the upgrade graph. The different upgrade graphs are used to rank potential actions based on an algorithm called the upgrade algorithm. This algorithm produces two types of ranking: a direct ranking and an indirect or inverse ranking. The two rankings are used to produce a final stock ranking.

METHODOLOGY

Presentation of the Hybrid Principle AHP- and ELECTRE II

The hybrid approach consists of evaluating the relative importance of the decision criteria. This evaluation is carried out using the technique of peer comparison between criteria belonging to the same level of hierarchy. The hierarchical levels are obtained by decomposing the problem into structures according to the principle of the AHP method. The weights of the criteria are then used in the decision matrix of the problem to be solved. Performance is aggregated using the ELECTRE II method.

Mathematical Formulation of the Problem

Let us solve a multi-criteria decision problem where we have several possible alternatives (n alternatives), with several criteria (m criteria) and each alternative is evaluated according to each criterion, and each criterion has a relative weight according to its contribution to solving the problem. We formulate this problem as follows:

- ✓ $A = \{a_1; a_2; \dots a_n\}$ where A denotes the set of n possible alternatives.
- ✓ $C = \{g_1; g_2; \dots g_n\}$ C designates all the m evaluation criteria.
- ✓ $G = \{g_1(a_1); g_2(a_2); \dots g_m(a_n)\}$ is the total performance of the alternatives by criterion
- ✓ $W = \{w_1; w_2; \dots w_n\}$ Represents all the weights of the decision criteria.

Presentation of the New Hybrid AHP-ELECTRE II Approach

In this approach we use the efficiency of the peer evaluation of the AHP method to determine the weights of the intervening weights in the decision. We then use the ELECTRE II method to aggregate the performance of the alternatives and rank them from best to worst.

Hybridization Steps

Step 1 Weighting the Criteria:

1 This stage of preference modelling involves clearly identifying the potential actions available to the decision-maker and all the criteria that can be used to evaluate the actions. This step enables us to take better account of the decision-maker's needs.

2 Carry out a binary comparison between the criteria identified: At this stage, the analyst asks the decision-maker to compare the criteria in pairs, using the SAATY verbal scale. The comparison is made between criteria belonging to the same level of hierarchy. The decision-maker's assessments are then converted into numerical values using the SAATY numerical scale.

3 Drawing up the judgement matrix: After converting the decision-maker's verbal evaluation expressions into numerical values, we arrange the data in a matrix according to the following principle:

$$A=\left[a_{(i,j)}\right] \quad \begin{cases} a_{(i,i)}=1; \quad for\ i=1\ldots k\ \ and \\ a_{(i,j)}=1;\ if\ \ a_i=a_j \qquad and \\ a_{(i,j)}\succ 1;\ \ if\ a_i\ \ is\ prefered\ to\ \ \ a_j \\ a_{(j,i)}=\dfrac{1}{a_{(i,j)}}\ (reciprocal\ value) \end{cases} \quad (1)\ \ [10]$$

4 Determining the criteria weights principle of the AHP method: The criteria weights are calculated on the basis of the judgement matrix established in step 3. It consists of calculating the eigenvalues of the matrix. The calculation procedure follows the relationships.

5 Calculating priorities. The aim is to find a set of weights $W=\{w_1;w_2;\ldots w_n\}$ such that $\frac{w_i}{w_j}$ corresponds to comparisons $a_{(i,j)}$ in the matrix of pairwise comparisons.

- ❖ The calculation is based on the three steps below. Add up the elements in each column j

$$\sum_{i=1}^{n} a_{(i,j)}\ ;\ \forall j \qquad (2)$$

- ❖ Divide each element of the matrix by the sum of the elements in its column: normalisation $$a'_{(i,j)}=\frac{a_{(i,j)}}{\sum_{i=1}^{n} a_{(i,j)}}\ ;\quad \forall j \quad (3)$$

- ❖ Calculate the arithmetic mean of each line $i;\quad i=1\ldots n$

$$w_i=\sum_{i=1}^{n}\frac{a'_{(i,j)}}{n};\quad \forall j \quad (4)$$

The w_i are the weights of the criteria or factors i

6 Testing the consistency of judgements: The consistency of judgements is tested by calculating the consistency ratio. If this ratio is less than or equal to 0.10, then the decision-maker's judgements are considered to be consistent and, consequently, the weights of the criteria are well determined. Otherwise, the judgements are repeated. To calculate the consistency ratio, we apply the relationships.

$$IC = \frac{\lambda \max - n}{n-1} \qquad (5)$$

Such as n is the dimension of the comparison matrix and $\lambda \max$ is the maximum eigenvalue of the comparison matrix.

$$\lambda \max = \frac{1}{n}\sum_{i=1}^{n}\frac{Vp}{w_i}\ ;\ \ \forall j \qquad (6)$$

Vp Is the sum of the coefficients of rows i of the products of the judgements $a_{(i,j)}$ by weights w_i .

$$V_p = \sum_{i=1}^{n} a_{(i,j)} w_i\ ;\forall j \qquad (7)$$

The Consistency Ratio (CR) is calculated using the formula:

$$RC = \frac{IC}{IA} \qquad (8)$$

such that: RC is the coherence or consistency ratio. IA is the random coherence index determined by Saaty as a function of the number n of factors to be compared.

Table 1: Random coherence index (IA) derived from Saaty (2004) [2]

n	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Random Consistency Index (RCI)	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

IC is the consistency index.

If RC is less than or equal to 0.1 then the matrix is considered to be consistent and acceptable, otherwise the comparisons must be reviewed.

Draw up the action evaluation matrix with the weights of the criteria determined:

$$G = \begin{pmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1} & g_{n2} & \cdots & g_{nn} \\ w_1 & w_2 & \cdots & w_n \end{pmatrix}$$

Step 2 Aggregation of the performance of actions or alternatives:
The steps in the aggregation procedure are those proposed by the ELECTRE II method [1]

Step 2.1: Determination of Preference Sets and Their Weights:
These sets are determined by comparing the performances of the actions in pairs, according to each criterion and taking into account the direction of optimisation of the criteria. $g_j(a_i)$ and $g_j(a_k)$ respective actions a_i et a_k $i,k=1,......,n$. This gives us the following sets, known as preference sets:

$$\begin{cases} J^+(a_i;a_k) = \left\{ j \in F; g_j(a_i) > g_j(a_k) \right\} \\ J^=(a_i;a_k) = \left\{ j \in F; g_j(a_i) = g_j(a_k) \right\} \\ J^-(a_i;a_k) = \left\{ j \in F; g_j(a_i) < g_j(a_k) \right\} \end{cases} \quad (9)$$

$$\begin{cases} P^+(a_i;a_k) = \Sigma P_j; \quad j \in J^+(a_i;a_k) \\ P^=(a_i;a_k) = \Sigma P_j; \quad j \in J^=(a_i;a_k) \\ P^-(a_i;a_k) = \Sigma P_j; \quad j \in J^-(a_i;a_k) \\ P^+(a_i;a_k) + P^=(a_i;a_k) + P^-(a_i;a_k) = P(a_i;a_k) \end{cases} \quad (10)$$

Preference sets and weights are defined as follows:

- $J^+(a_i;a_k)$ the set of criteria for which the action a_i is preferred to action a_k.
- $J^=(a_i;a_k)$ Equivalence of shares.
- $J^-(a_i;a_k)$ l'ensemble des critères pour lesquels l'action a_k is preferred to action a_i.
- $P^+(a_i;a_k)$ Sum of the weights of the criteria belonging to the set J^+.
- $P^=(a_i;a_k)$ Sum of the weights of the criteria belonging to the set $J^=$ \\
- $P^-(a_i;a_k)$ Sum of the weights of the criteria belonging to the set J^- \\

Step 2.2: Indices of Concordance and Discordance:
The concordance and discordance indices apply the principles of majority and respect for minorities respectively, in order to confirm whether or not an action b outranks another a action.

There are three thresholds for the concordance test noted

$$C^{+}\ ;\ C^{0}\ \text{et}\ C^{-}\ \text{où}\ C^{+}..C^{0}..C^{-}$$

NB: The concordance thresholds are set by the expert or designer.

$$C(a_i;a_k)=\begin{cases}0\ ,\ \text{if there are no criteria in favour of}\ \ a_iSa_k\\ 1\ ,\ \text{if all the criteria are in favour of}\ \ a_iSa_k\\ \in\ [0;1]\ \text{if only certain criteria are in favour of}\ \ a_iSa_k\end{cases}\qquad(11)$$

The concordance indices are calculated using the following relationships:

$$C(a_i;a_k)=\begin{cases}\dfrac{P^{+}(a_i;a_k)+P^{=}(a_i;a_k)}{P(a_i;a_k)}\\ 0\le C(a_i;a_k)\le 1\end{cases}\qquad(12)$$

$$C(a_i;a_k)=\begin{cases}\dfrac{P^{+}(a_i;a_k)+\frac{1}{2}P_{ik}^{=}}{P(a_i;a_k)} & ou\\ \dfrac{P^{+}(a_i;a_k)}{P(a_i;a_k)}\end{cases}\qquad(13)$$

The concordance test is accepted only if:

$$C(a_i;a_k)=\begin{cases}C(ai;ak)\ge C^{+}\quad ou\\ C(ai;ak)\ge C^{0}\quad ou\\ C(ai;ak)\ge C^{-}\\ et\quad \dfrac{P^{+}((a_i;a_k))}{P^{-}((a_i;a_k))}\ge 1\end{cases}\qquad(14)$$

$$D(a_i;a_k)=\begin{cases}0\quad si\ \ aucun\ critère\ \ \ n'est\ \ en\ \ \ faveur\ \ de\ \ a_i\not{S}a_k\\ 1\quad si\ \ \ tous\ \ les\ \ critère\ \ sont\ \ en\ \ faveur\ \ de\ \ a_i\not{S}a_k\end{cases}\qquad(15)$$

The mismatch indices are determined by the relationship below:

$$D(a_i;a_k)=\begin{cases}0\qquad si\quad J^{-}(a_i;a_k)=\phi\\ max_{J^{-}}\left[g_j(a_k)-g_j(a_i)\right]\end{cases}\qquad(16)$$

The non-mismatch test can be summarised as follows $j \in J^{-}(a_i, a_k)$ as follows:

- if $g_j(a_k) - g_j(a_i) \leq D_2(j)$; then there is a strong certainty that the criterion does not present a major obstacle to the hypothesis of outclassification.
- if $D_2(j) < g_j(a_k) - g_j(a_i) \leq D_1(j)$; then there is a low degree of certainty that the criterion does not present a major obstacle to the hypothesis of outclassification.

Note that the two discordance thresholds D_2 et D_1 are defined for all criteria g_j with $D_2 \leq D_1$.

Step 2.3: Setting Concordance and Discordance Thresholds:

To begin with, we set three concordance thresholds (c^+ , c^0 , c^-) and two discordance thresholds (D_1 , D_2) for each criterion. [11]

- c^+ : Concordance with strong presumption.
- c^0 : Concordance with median presumption.
- c^- : Concordance with weak presumption; with. $c^- < c^0 < c^+ \leq 1$
- D_1 : most demanding discordance threshold
- D_2 : most lax discordance threshold

The concordance threshold with strong presumption, combined with the demanding discordance threshold, as well as the median concordance threshold, combined with the lax discordance threshold, determine the strong outclassification. The lax concordance threshold, combined with the demanding discordance threshold, determines the weak upgrade.

Step 2.4: Exploiting the Upgrade Relationship:
Strong Upgrade Conditions

- Strong upgrade $a_i S^F a_k$ (a_i strongly outperforms a_k)

$$\begin{cases} C(ai;ak) \geq C^+ \\ et \\ D(ai;ak) \leq D_1 \\ et \\ \dfrac{P^+((a_i;a_k))}{P^-((a_i;a_k))} \geq 1 \qquad et/ou \begin{cases} C(ai;ak) \geq C^0 \\ et \\ D(ai;ak) \leq D_2 \quad et \quad \dfrac{P^+((a_i;a_k))}{P^-((a_i;a_k))} \geq 1 \end{cases} \end{cases} \qquad (17)$$

- The conditions for a weak upgrade are defined as follows.

Weak upgrade $a_i S^f a_k$ (a_i low upgrade a_k)

$$\begin{cases} C(ai;ak) \geq C^- \\ et \\ D(ai;ak) \leq D_1 \quad (18) \\ et \\ \dfrac{P^+((a_i;a_k))}{P^-((a_i;a_k))} \geq 1 \end{cases}$$

Step 2.5: Construction of Outranking Graphs:

The outranking graph visualises the outranking relationship for the set of pairs of actions. [11] In an outranking graph, the actions offered for choice constitute the vertices of the graph and an arrow called an outranking arc is created, directed from the a_i action to the action a_k if the a_i action is judged to be at least as good as the action a_k . This means that a_i upgrade a_k . This does not mean that a_i is better than a_k , but only that a_i there is no well-founded reason to think that is strictly preferable to a_k [1].

Step 2.6: The Ranking Process:

This takes place in three sub-steps:

- sub-step 1 direct ranking or pre-order 1,
- sub-step 2 indirect ranking or pre-order 2,
- sub-step 3: final ranking or partial pre-order.

Application of AHP-ELECTRE II hybridization: Evaluation of areas suitable for water drilling.

In this section, we will apply the AHP-ELECTRE II hybridization principle, as defined in Chapter 3 of this document, to the evaluation of areas suitable for water drilling. In the absence of real field data, we will illustrate our study with a didactic example.

PRESENTATION OF THE PROBLEM

The study consists of evaluating the zones that are favourable for the installation of boreholes. To do this, we will consider three (3) main indicators for modelling the problem, namely: the water potential indicator, the water accessibility indicator and finally the water exploitability indicator [12]. The aim here is to classify the sections by taking all the indicators into account simultaneously. We therefore assume that all the indicators are equally important. In terms of weight, each indicator has a weight of 1/3, giving a total of 1 for the three indicators. In each indicator, the weights of the criteria that contribute to the assessment of the indicator must be normalised to 1. In fact, the sum of all the weights of the criteria must be equal to 1. In each indicator there are a certain number of criteria that are considered to contribute to the assessment of the indicator.

The principle here is to determine the weights of each of the criteria using the AHP method. The study area is then partitioned, and each portion is considered to be an action to be assessed (the actions a_i). The actions will be aggregated using the ElECTRE II method. This will enable us to rank the actions from best to worst.

Mathematical Formulation of the Problem

The problem of multi-criteria evaluation of the zones favourable for drilling is formulated as follows:

- $A = \{a_1; a_2; .....; a_n\}$ with $n \geq 2$ all the zones to be evaluated.
- $C = \{g_1; g_2; .....; g_n\}$ with $n \geq 2$ all the evaluation criteria.
- $I = \{A; B; C\}$ all the indicators used for the evaluation.

Each indicator includes a number of criteria that contribute to its evaluation.

- $G = \{g_{a_1}; g_{a_2}; .....; g_{a_n}\}$ with $n \geq 2$ all the performances of the actions by criteria

Criteria Used:

The evaluation criteria are:

- ✓ Slope (P), effective infiltration (Ie), drainage density (Dd), induced permeability (Pi), fracture density (Df), alteration thickness (Ea), drilling cost (Cf), flow rate (D) and static level (Ns).
- ✓ Probability of success (Ps): To determine the probability of success for the zone, we calculate the rate of positive drillings carried out in the section in question.

Once the numerical values of the criteria have been determined, we will use the criteria classification and standardisation table to determine the evaluation of the alternatives by criterion. Finally, we will draw up the action evaluation matrix with the weights of each criterion determined by the principle of the AHP method.

For the breakdown of the study area, we will use a case with five (5) points to be evaluated: (a_1); (a_2); (a_3); (a_4) et (a_5).

Assessment of Criteria

In the rest of our work, we will consider the following tables: (these tables are taken from the work of YOUAN [12].

Action Evaluation Matrix:

It should be remembered that these data were determined randomly, but taking into account the classification and standardisation tables for the criteria per indicator.

Table 2: Matrix for evaluating study points by criterion

criteria	P	Ie	Dd	Df	Pi	Ea	PS	Cf	De	Ns
a_1	2	65	11	3	7	5	25	2 millions	3.5	10
a_2	3	35	9.5	1.5	13	12	37	1.3 millions	2.5	12

a_3	3.5	58	8	2.1	12	17	16	1.5 millions	5	30
a_4	1.5	22	5	2.7	4	30	71	1.9 millions	1.99	245
a_5	5	5	13	4	6	27	50	1 million	5	40
Sense of optimisation.	↘	↗	↘	↗	↗	↗	↗	↘	↗	↘

1.6 Comparison Matrix by Pair of Criteria and By Indicator

We have drawn on the work of YOUAN and JORDAN [12] to evaluate the criteria.

Groundwater Potential Indicator:

We selected a total of six (6) criteria that contribute to assessing groundwater potentiality. These are: slope, effective infiltration, drainage density, fracture density, induced permeability and thickness of rock alteration.

Table 2: Matrix of peer comparisons of water potential indicator criteria.

critères	**P**	**Ie**	**De**	**Df**	**Pi**	**Ea**
P	1	1/2	2	3	5	7
Ie	2	1	7	5	5	5
De	1/2	1/7	1	½	1/3	1
Df	1/3	1/5	2	1	1	5
Pi	1/5	1/5	3	1	1	3
Ea	1/7	1/5	1	1/5	1/3	1

P: Slope; Ii: Effective infiltration; Dd: Drainage density; Df: Fracture density; Pi: Induced permeability; Ea: Thickness of weathering;

Groundwater Accessibility Indicator:

Water resources can only be used when certain factors combine to make access possible. The criteria used are the probability of success and the cost of drilling. The matrix comparing the criteria gives the following table.

Ps: Probability of success; Cf: Cost of drilling

Table 3: Matrix of comparisons by pair of criteria for the water accessibility indicator.

critères	Ps	Cf
Ps	1	1/7
Cf	7	1

Operability Indicator:

Groundwater operability takes into account the following criteria: flow rate and static water level.

The criteria comparison matrix gives the table below:

Table 4: Matrix of comparisons by pair of criteria for the water exploitability indicator

critères	De	Ns
De	1	1/7
Ns	7	1

De: Operating flow ; Ns: Static level

1.5 Solving the Problem using AHP-ELECTRE II Hybridisation

1.6 Determining Criteria Weights Per Indicator with AHP:

1.5.1 Applying the above steps to calculate the weights gives the following results.

Table 5: Weight matrix.

criteria	P	Ie	De	Df	Pi	Ea	poids
P	0.24	0.22	0.13	0.28	0.39	0.32	0.26
Ie	0.48	0.45	0.44	0.47	0.39	0.23	0.41
De	0.12	0.06	0.06	0.05	0.03	0.05	0.06
Df	0.08	0.09	0.13	0.09	0.08	0.23	0.12
Pi	0.05	0.09	0.19	0.09	0.08	0.14	0.11
Ea	0.03	0.09	0.06	0.02	0.03	0.05	0.05
Total							1

$$\lambda max = 6.47$$

$$IC = \frac{\lambda max - n}{n-1} = \frac{6.47-6}{5} = 0.93.$$

The coherence ratio is then determined by the following relationship $RC = \frac{IC}{IA} = \frac{0.93}{1.25} = 0.07$ with $IA = 1.25$ corresponding to the random index of six criteria. Since the ratio is less than 0.1, the judgements are consistent.

Determining the Weights of Water Accessibility Criteria:

Table 6: Matrix of criteria weights.

critères	Ps	Cf	weights
Ps	0.13	0.13	0.12
Cf	0.88	0.88	0.88
Total			1

Determining the Weights of Groundwater Exploitability Criteria:

Table 7: Matrix of criteria weights

Critères	De	Ns	weights
De	0.75	0.75	0.75
Ns	0.25	0.25	0.25
Total			1

Table 8: Summary of standardised criteria weights

criteria	P	Ie	Dd	Df	Pi	Ea	PS	Cf	De	Ns
Initial weights	0.26	0.41	0.06	0.12	0.11	0.05	0.13	0.88	0.75	0.25
Standard weight	0.08	0.13	0.01	0.03	0.03	0.01	0.04	0.29	0.24	0.08

Performance Aggregation

Direction of Criterion Optimisation:

We have decided to standardise the direction of optimisation of the criteria by maximisation. Consequently, the criteria to be minimised will be weighted by (-1), which transforms them into a maximisation. We will also normalise the weights by dividing each weight by the total weights $\Sigma P = 1$. This gives us the new evaluation matrix that will be used for the rest of our work.

critères	P	Ie	Dd	Df	Pi	Ea	PS	Cf	De	Ns
a_1	-5	5	-3	8	5	1	3	-1	5	-8
a_2	-5	3	-5	3	10	3	3	-5	5	-5
a_3	-3	5	-5	5	10	5	1	-3	8	-1
a_4	-8	1	-8	8	3	8	8	-3	3	-3
a_5	-3	1	-1	10	5	8	5	-10	8	-1
poids	0.08	0.13	0.01	0.03	0.03	0.01	0.04	0.29	0.24	0.08
Sense of optimisation.	↘	↗	↘	↗	↗	↗	↗	↘	↗	↘

By applying the relationships *9* ;10 ;*11* ;12 ;*14 et* 16*.*

We obtain the following results:

Table 9: matrix of concordance coefficients

Alternatives	a_1	a_2	a_3	a_4	a_5
a_1		0.82	0.50	0.81	0.45
a_2	0.48		0.08	0.52	0.45
a_3	0.57	0.9		0.86	0.85
a_4	0.16	0.45	0.37		0.47
a_5	0.52	0.49	0.49	0.6	

Table 10: matrix of mismatch coefficients

Alternatives	a_1	a_2	a_3	a_4	a_5
a_1		5	7	7	7
a_2	5		7	5	7
a_3	3	2		7	6
a_4	5	7	7		9
a_5	9	5	7	7	

Table 11: P+/P- ratio matrix

Alternatives	a_1	a_2	a_3	a_4	a_5
a_1		3.83	0.84	2.22	0.85
a_2	0.26		0.04	1.15	0.91
a_3	1.18	1.5		7.12	5
a_4	0.24	0.91	0.14		0.7
a_5	1.16	1.08	0.2	1.39	

Exploiting the Outranking Relationship

We set the match thresholds as follows $C^{+} = 0.65$; $C^{0} = 0.60$; $C^{-} = 0.50$ The mismatch thresholds per criterion are determined by the table below:

Table 12: discordance threshold matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
D_1	7	7	7	7	7	7	7	7	7	7
D_2	5	5	5	5	5	5	5	5	5	5

Applying relations 17 and 18 we obtain the following results:

Table 14: Matrix of alternative outrankings.

Alternatives	a_1	a_2	a_3	a_4	a_5
a_1		S^F		S^F	
a_2				S^f	
a_3	S^F	S^F		S^F	S^F
a_4					
a_5		S^f			

Direct Outranking Graphs:

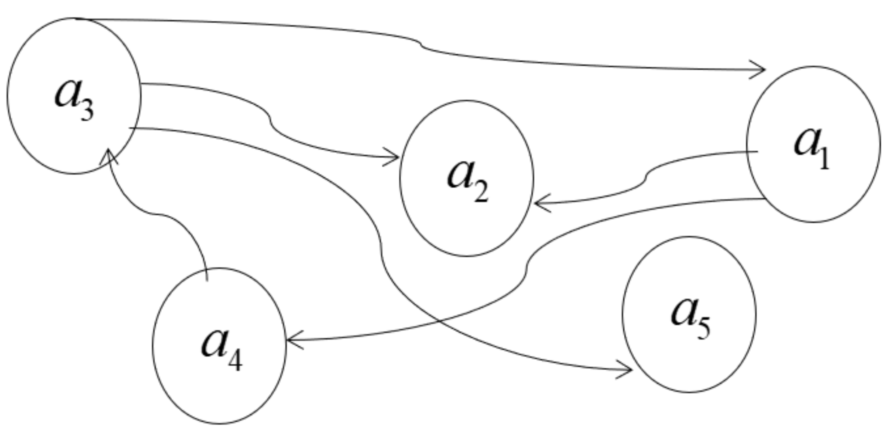

Figure 1: Strong direct outranking graph

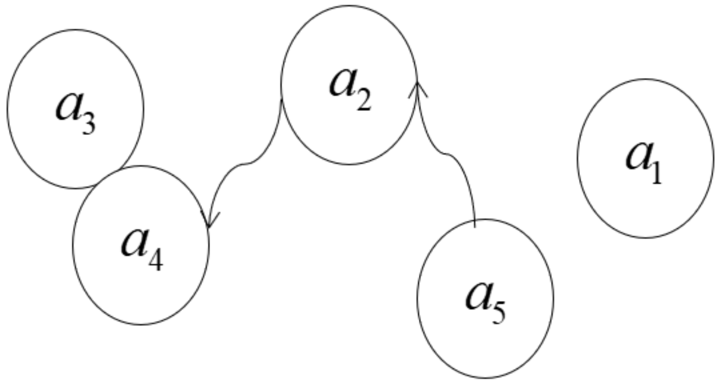

Figure 2: Weak direct outranking graph.

Indirect Outranking Graphs:

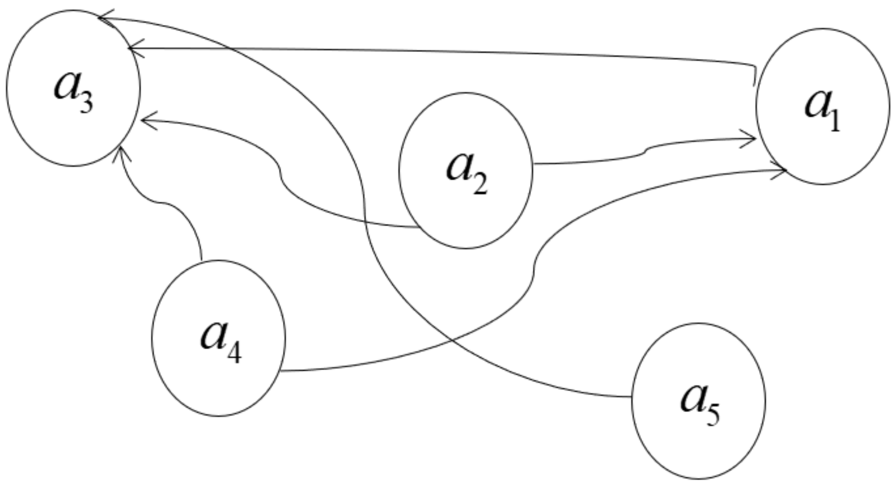

Figure 3: Strong indirect outranking graph

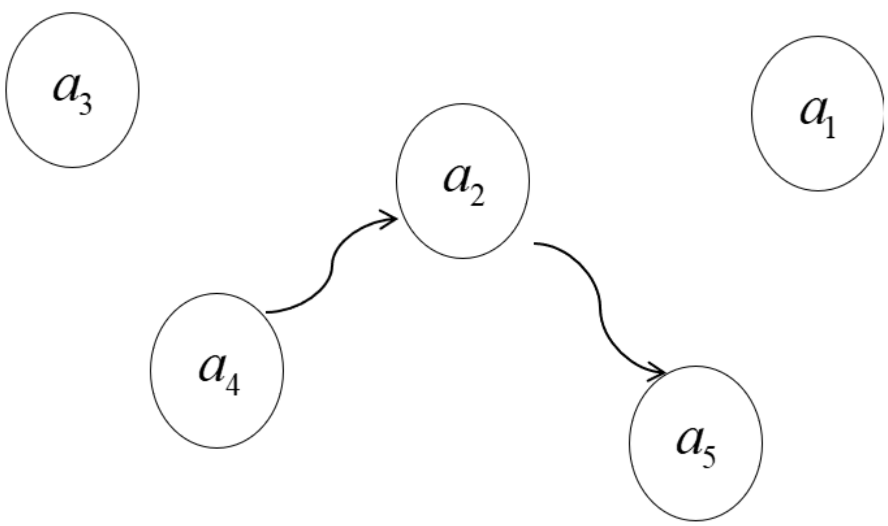

Figure 4: Weak indirect outranking graph.

Table 13: Ranking summaries.

Alternatives	Rank1	Rank2	Final Rank
a_1	2	2	2
a_2	3	3	3
a_3	1	1	1
a_4	4	4	4
a_5	2	2	2

INTERPRETATION OF THE RESULTS

Alternative a_3 is ranked first with a medium class potentiality indicator and an equally medium accessibility indicator, the exploitability of this action is strong. All in all, this makes this action the best among the other four.

Alternatives a_1 and a_5 are considered equivalent with respectively a water potentiality indicator considered medium for a1 and low for a_5 and a low accessibility indicator for a1 (high drilling cost) and good water accessibility for a_5 with a low drilling cost. Exploitability is considered medium for a_1 and strong (good) for a_5.

Action a_2 is ranked third with a low water potential indicator and an average accessibility indicator and a very low exploitability indicator with regard to the static water level. Action a_4 is considered the worst; it is incomparable to action a_5. Its concordance index is 0.64 and its discordance index is 7, making it incomparable with action a_5. It has a very low water potential indicator and a very low water accessibility indicator, given the very high drilling cost. The exploitability indicator is also very low, given the production rate.

RECOMMENDATION

For better use of the AHP-ELCETRE II hybrid method, the criteria used must be real criteria for better aggregation of the decision-maker's preferences. When the problem includes sub-criteria, it is necessary to normalize the weights of the sub-criteria, taking into account the weights of the criteria that generate the different sub-criteria.

LIMITATIONS

The application of this hybrid method is time-consuming and requires numerous calculations. It has weaknesses when the problem includes sub-criteria.

CONCLUSION

In this article, we implemented a multi-criteria decision support method called AHP-ELECTRE II. This method is a hybridization of two well-known methods in the literature: the AHP method is inspired by the single synthesis criterion, and the ELECTRE II method is inspired by the synthesis outranking criterion. The work in this article allowed us, among other things, to conduct a literature review on multi-criteria decision support and the problem of water well siting. Applying this new multi-criteria decision support approach to the problem of water management clearly shows that this issue can be addressed in greater depth by taking into account many other aspects, such as environmental issues, water quality, and many other parameters. As a perspective, in our future research we intend to use the AHP-ELECTRE II hybridization to evaluate multidimensional choices of areas favorable to the installation of water drilling in Africa in general and in BURKINA FASO in particular. We also intend to improve the AHP-ELECTRE II hybridization with a modeling of the decision-maker's preferences that takes into account the interactions between them.

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