

## Portfolio Selection Based on a Volatility Measure Adjusted for Irrationality

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### ABSTRACT

This paper investigates the combination of the two risk measures irrationality and volatility for portfolio selection as well as irrationality as a standalone risk measure. The study is conducted for a period of 20 years, ranging from 1999-01-04 to 2018-12-31, using daily closing prices from 1.295 stocks. The companies are derived from the indices S&P 500, STOXX 600 and a representative index of the German market out of the indices DAX, MDAX, SDAX and TecDAX. The findings indicate a negative relationship between risk and return in terms of irrationality across the three indices. The effect is particularly evident when comparing the portfolios with the lowest and highest values of irrationality. Further, the analysis provides an indication of how an index can be replicated with fewer expenses and complexity, particularly regarding the fact that the volatility of the synthetic portfolio is equivalent.

**Keywords:** Irrationality, Volatility, Portfolio Selection, Portfolio Management

### INTRODUCTION

The linear relationship between the expected risk and return is one of the fundamental assumptions of modern portfolio theory. The basic idea behind this relationship goes back to Markowitz [1] and lies in the fact that investors must accept a corresponding "variance of profits" for their expected returns. If an investment with a better risk-return ratio exists, this will increase the demand for the investment opportunity and thus place the ratio back in the line of all other investment opportunities. Based on the resulting portfolio selection theory, which includes the liquidity preference, the Capital Asset Pricing Model (CAPM) was established by William F. Sharpe [2] and John Lintner [3]. The CAPM assumes that the specific risks of the individual investments are offset by diversification and that only the market risk remains, which cannot be further reduced. This risk is therefore borne by the shareholders who invest via a diversified portfolio along the capital market line, which represents the linear relationship between expected risk and return. Under the assumption of unrestricted borrowing and lending, an investor with maximum risk aversion would theoretically receive exactly the risk-free return.

Assuming that the market portfolio should take into consideration all risk-bearing investment opportunities, the hypothesis of a linear relationship is not empirically verifiable or falsifiable [4]. Empirical studies therefore analyze samples of the market portfolio for example through broad stock indices that cover a large proportion of the total market capitalization of a country or region.

Early empirical studies such as Black, Jensen and Scholes [5] therefore used portfolios instead of individual stocks. The portfolios covering all shares of the New York Stock Exchange (NYSE) for the period from 1926 to 1966 were sorted in ascending order from low to high beta values. The study confirms the linear relationship between risk and return, but with a flatter curve than assumed by the CAPM. The returns for portfolios with lower risk are higher and vice versa lower with higher risk. In a more recent study, Fama and French [6] confirmed the results of Black, Jensen and Scholes for the shares of the NYSE over a period from 1928 to 2003 and the National Association of Securities Dealers Automated Quotations (NASDAQ) from 1972 to 2003. The authors conclude as well that the relationship between risk and return is flatter than assumed by the single factor model and thus the CAPM cannot be verified for the portfolios under consideration. Hence low-risk equities are underpriced by the market and high-risk equities are attributed too high future returns.

All these studies are conducted based on the assumption that risk is fully captured in terms of volatility - the annualized standard deviation of returns - which is a measure of the overall margin of fluctuations. In our analysis we examine the inclusion of irrationality as a measure of risk in addition to volatility as well as irrationality as a standalone risk measure. Irrationality as defined by Schädler [7] measures the ratio of long to short frequency components of historical stock quotes in relation to each other. Therefore, it is a relative measure which in contrast to volatility does not consider the overall fluctuation margin. A possible advantage in the combination of the two risk measures may be that irrationality, unlike volatility, does not assume a standard normal distribution of returns.

The difference to volatility becomes apparent when one considers an ideal company which grows linearly without uncertainties about the future course of business and no exogenous cycles. In this case, under the premise of efficient markets, the price of the shares would move within a narrow corridor around the present value of the company, which results purely from the usual trading of the shares. The irrationality would then tend towards zero. In the next step, market participants are introduced, who, driven by irrational behavior, cause the share price to fluctuate around the actual company value. In the extreme case of very high fluctuations around the actual value, irrationality would tend towards one, as the additional fluctuations would have a much stronger impact on the development of the share price than the usual trading in the above example. Empirical values are somewhere between these two extremes of the idealized company.

Both Shiller [8] and Appel and Grabinski [9] conclude that price fluctuations are three to five times higher than the change in the discounted future cash-flows of the companies considered. To take this into account, it is assumed that equity markets are efficient, meaning that all available information is priced directly into the market. Therefore, it is not possible to generate a constant excess return based on new information. "Efficient" however, does not mean that valuations on the stock exchange should by no means deviate from the actual company value. In order to reflect this in the risk indicator irrationality, it is assumed that the maximum time interval between the release of new information is no more than three months apart and possible price fluctuations accounting for the information are offset within the same period.

This rational part of the fluctuations is compared to fluctuations that are in a range of more than three months to one year, in which no new information is available. Therefore, this irrational part is adjusted for the overall trend of the company's development, which affects fluctuations over a period longer than one year. The historical development of a share price can thus be described by the following model

$$\text{Stock Price} = \text{Trend} + \text{Irrational Fluctuations} + \text{Rational Fluctuations} + \text{Random Trade}$$

whereby solely the rational and irrational fluctuations are considered in the calculation of irrationality.

The next chapter outlines the selection and the way the individual stocks used for the analysis are preprocessed. The calculation of the two risk ratios is then presented in detail. While volatility is calculated using logarithmic returns, irrationality is calculated over three steps. First, the logarithmized time series are detrended with a linear regression model whose parameters are estimated via the ordinary least squares. The resulting weakly stationary time series can then be used to calculate the power spectrum via the discrete Fourier transform. The last step is to accumulate the corresponding frequency bands and to put them in relation to each other.

### RESEARCH DESIGN

We analyzed daily historical stock quotes ex-post on a total return basis for a period of 20 years, ranging from 1999-01-04 to 2018-12-31. The data used were taken from Datastream. Since it is assumed that shares with high market capitalization within developed countries are efficient, the following indices were selected.

For the German market, the performance indices DAX, MDAX, SDAX and TecDAX were combined to form the German Major Indices Index (GMII), adjusted for double listings of individual stocks as provided by Deutsche Boerse AG. The STOXX 600 was used as a proxy for the European market as provided by STOXX AG. The American market is represented by the S&P 500 as provided by the Standard and Poor's Corporation (S&P). All data for the analysis were accessed on 2019-05-29.

The daily values of the time series had to be checked for missing values first. Shares with a history of less than 20 years were excluded from the analysis. Furthermore, price recordings on public holidays were eliminated. In order to exclude systematic errors in the data series, these were cross-checked with the stock price data of Thomson Reuters Eikon. The existing survivorship bias is accepted due to data availability and required history of 20 years.

Table 1 shows the number of shares contained in the respective index compared to the number of shares available for our analysis.

**Table 1. Stock count after cleaning per index**

Indices	Total individual stocks	Individual stocks count after cleaning
GMII	190	73
STOXX 600	600	368
S&P 500	505	380

The next step is to calculate the historical volatility of the time series. Historical stock volatility as a measure for dispersion of returns defined as

$$\text{Volatility} = \sqrt{\frac{1}{q-1} \sum_{p=1}^q (r_p - \bar{r})^2} \sqrt{q/a}, \quad r_p = \ln \frac{S_p}{S_{p-1}} \quad (1)$$

is the annualized square root of the variance, with  $r_p$  being the logarithmic returns of the respective historical stock quotes  $S_p$  and  $\bar{r}$  the expected value. Due to the handling of daily data with the length  $q$  and  $a$  the number of years under consideration, the raw variance is annualized by the square root of  $q/a$ . The annualization is done for consistency and has no influence on the relative risk in sense of a risk ranking as all stocks are handled equally.

As described above, irrationality is calculated over three steps. First, the logarithmized time series are detrended with a linear regression model as the data must be weakly stationary for being applicable to the discrete Fourier transform. In this paper, the adjustment was made using a fourth-degree polynomial regression model, which was found to be a good fit for the analyzed data series.

In the general case, the parameters of a  $k$ -th degree polynomial regression model

$$y_t = \beta_0 + \beta_1 x_t + \beta_2 x_t^2 + \dots + \beta_k x_t^k + \varepsilon_t \tag{2}$$

are estimated via the ordinary least squares in its well-known matrix form

$$\hat{\beta} = (X^T X)^{-1} X^T y_t \tag{3}$$

by minimizing the sum of squares residuals where  $y_t$  represents the logarithmized time series.  $\hat{\beta}$  being the coefficient vector with the parameters  $\beta_0, \dots, \beta_k$  and  $X$  the  $n \times m$  design matrix. To detrend the stock quotes, the values of the regression model will be deducted from the original logarithmized time series. In the next step, the Power spectrum is estimated.

For a finite, stationary time series the power spectrum  $P(n)$  is defined as the squared convolution of the magnitude of the frequency components and its complex conjugate.

$$P(n) = |F(n)|^2, \quad F(n) = \sum_{t=0}^{N-1} f(t) e^{\frac{-2\pi i n t}{N}} \tag{4}$$

The frequency components are derived via the discrete Fourier Transform.  $f(t)$  denoted as the logarithmized, detrended historical stock quotes,  $N$  the number of daily observations of the time series,  $n$  the respective frequency bins and  $i^2 \equiv -1$ . Although stock prices do not hold energy in a physical sense, the use of this analogy is useful to compare the strength or influence of frequency bands relative to each other. It is important to note, that a comparison of the summed absolute magnitudes of frequency bands would be biased towards shorter frequencies. In the final step the values of the corresponding frequency bands are accumulated and put in relation to each other.

The limits of the frequency bands depend on the number of years under observation and the number of trading days per year. While the one-year boundary  $h$  matches the number of years under observation, the three-month  $m$  and the ten days threshold  $j$  are selected as the closest to the corresponding frequency bin. The sum of the power of the frequency components relative to each other returns the value of irrationality.

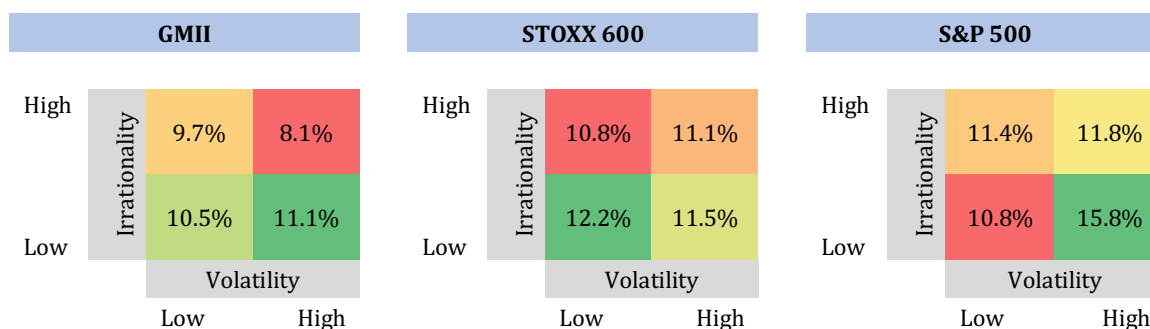
$$Irrationality = \frac{\sum_{n=h}^{n=m} P(n)}{\sum_{n=h}^{n=j} P(n)} \tag{5}$$

Irrationality can thus rise in two ways. On one hand, it increases if the influence of long frequencies on the price development increases. On the other hand, it increases when the influence of short-term frequencies declines if all else is equal. We would like to point out anew that short-term fluctuations reflect the risk inherent in the respective business model and thus form the basis of the calculation which the long-term fluctuations are compared to.

From the calculation logic it can further be concluded that irrationality is less volatile than volatility itself. An increase in very short fluctuations under 10 days will not have a direct effect on the results. Significant fluctuations over a period of more than 10 days increase the signal strength of the short frequencies and only influence long-term frequency ranges over a longer period. In addition, it should be noted that a single data error, even when more than 5.000 measurement points are considered, can have such a strong influence on volatility that a value which is within the market average in terms of volatility may be classified as very risky. By excluding all frequencies below 10 days, irrationality is resistant to such data errors.

## RESULTS

As described above, the individual shares of the indices analyzed were split according to their respective values of irrationality and volatility. The division of the portfolios results from the process of first sorting the individual stocks in ascending order from low to high irrationality. This enables the subdivision into two halves, while in the case of an odd number of shares, the portfolio with a comparatively higher risk receives one more share. Subsequently, each of the two portfolios were separated according to low and high volatility and divided into lower and higher risk portfolios.<sup>1</sup> For each quadrant, the compound annual growth rate (CAGR) of the respective portfolio was calculated. As the market capitalization changes significantly over the period of 20 years, we preferred to measure the CAGR on an equal weighted basis. The total return CAGRs of the equal weighted portfolios are presented in Figure 1. The colors in the plot are scaled from red to green whereby red indicates the lowest, green the highest value in terms of CAGR.



**Figure 1. Total return CAGRs per index and equal weighted portfolios**

Across all three indices a consistent pattern for the CAGRs emerges. High returns can be found in portfolios in the lower half of irrationality regardless of volatility. This is evident for all indexes analyzed. In terms of volatility, the American market differs from the European market. While the European markets have a higher CAGR for the lower half, the American market shows a higher CAGR for the higher half of volatility.

At an individual portfolio level, no definite conclusion can be drawn for the lowest CAGR value. It is noticeable that all the portfolios with the highest CAGR are in the range of low irrationality.

<sup>1</sup> Thresholds	GMI	STOXX 600	S&P 500
Irrationality Threshold	77.8%	77.6%	77.2%
Volatility Threshold low Irrationality	32.1%	30.8%	30.9%
Volatility Threshold high Irrationality	37.8%	34.4%	37.4%

Especially the portfolios with low irrationality and high volatility show higher returns than the market portfolio.

From the results it appears that a strategy with low irrationality is advantageous for the European market as well as the American market, while an investment decision favoring high volatility irrespective of irrationality is most advantageous for the US market. It is interesting to note that low irrationality is associated with higher CAGRs. In other words, the assumption of the CAPM that higher returns are associated with higher risk - defined as volatility - cannot be confirmed for irrationality. Therefore, an analysis of irrationality as a standalone risk measure by dividing it into quarters should confirm the results.

The analysis with irrationality as a standalone risk ratio is presented in the Table 2 based on equal-weighted portfolios. All Shares portfolios contain all evaluated equities within the respective index. The other portfolio names are in ascending order by irrationality whereby the First Quarter contains the shares with the lowest and the Fourth Quarter the stocks with the highest irrationality values. The thresholds<sup>2</sup> result from this classification and represent the upper limit for the respective portfolio.

Sharpe ratios are calculated with a risk-free rate of zero. The maximum Drawdown (MDD) measures the highest loss throughout the whole time series in percentage points from the highest point before the drawdown and the subsequent trough. Calmar Ratios are defined as the CAGR of the portfolio divided by the corresponding MDD. Statistical significance<sup>3</sup> of Sharpe ratios and Calmar ratios of the four portfolios based on irrationality was tested against die All Shares portfolios on a rolling annual basis using the Mann-Whitney U test.

**Table 2. Descriptive statistics of the total return portfolios**

Index	Portfolio	CAGR	Volatility	Sharpe Ratio	MDD	Calmar Ratio
<b>GMI</b>	All Shares	10.0%	18.9%	0.50	59.4%	0.17
	First Quarter	11.8%	19.6%	0.57*	59.9%	0.20***
	Second Quarter	9.8%	21.0%	0.44***	59.9%	0.16
	Third Quarter	9.9%	20.7%	0.46***	63.9%	0.16***
	Fourth Quarter	7.9%	24.7%	0.31***	65.6%	0.12***
<b>STOXX 600</b>	All Shares	11.4%	17.6%	0.62	54.5%	0.21
	First Quarter	12.0%	17.1%	0.66	46.3%	0.26***
	Second Quarter	11.7%	16.4%	0.68	50.7%	0.23
	Third Quarter	11.3%	18.1%	0.60**	55.8%	0.20*
	Fourth Quarter	10.5%	20.7%	0.48***	66.7%	0.16***
<b>S&amp;P 500</b>	All Shares	12.8%	19.9%	0.61	50.6%	0.25
	First Quarter	15.0%	20.0%	0.70	47.0%	0.32***
	Second Quarter	12.4%	18.9%	0.62**	46.2%	0.27***
	Third Quarter	11.1%	21.3%	0.49***	54.6%	0.20***
	Fourth Quarter	12.1%	22.6%	0.51***	58.1%	0.21***

<sup>2</sup> **Thresholds**

	<b>GMI</b>	<b>STOXX 600</b>	<b>S&amp;P 500</b>
First Quarter	75.6%	75.3%	74.3%
Second Quarter	77.8%	77.6%	77.2%
Third Quarter	80.6%	80.1%	79.6%

<sup>3</sup> \*, \*\* and \*\*\* denote statistical significance at the 90%, 95% and 99% levels.

The Sharpe Ratios of the First Quarter portfolios are consistently higher compared to the All Shares portfolios. Both the First Quarter and the All Shares portfolios have higher Sharpe Ratios than the Fourth Quarter portfolios. Besides the lower CAGR, the main reason for the lower Sharpe Ratios is that the highest volatility is measured for the portfolios with the highest risk. Further, the high risk of the Fourth Quarter portfolios is reflected in the highest MDD values over the 20-year period. Rising MDD in accordance with risk from the first to the fourth Quarter portfolios and the CAGR falling simultaneously lead to decreasing Calmar ratios. This effect is evident across all three indices. In summary, we conclude, that higher risk in terms of irrationality is not compensated by higher returns for the analyzed equity universe.

The statistical significance of the Sharpe ratios from the 95% level upwards indicates that the All Shares portfolio can be replicated by the First Quarter portfolio. In this case, an equivalent volatility with statistically significant higher Calmar ratios can be assumed.

### CONCLUSIONS

Due to the exclusion of frequencies below 10 days, irrationality is per design less volatile than volatility itself. Furthermore, irrationality is more robust against erroneous data series. While even a single measurement error may have a significant effect on volatility, the irrationality in this case would hardly change. Irrationality can therefore be a valuable addition to volatility when measuring risk.

Irrationality differs markedly from volatility. Whereas the literature assumes a linear relationship of higher CAGRs and increasing volatility, the expected returns decrease with increasing irrationality. Since irrationality is associated with speculative behavior, it can be concluded that equities with a high speculative component in relation to rationally explainable fluctuations are associated with lower CAGRs.

For the management of equity portfolios, the analysis provides an indication of how an index can be replicated with fewer expenses and complexity, particularly regarding the fact that the volatility of the synthetic portfolio is equivalent.

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