Archives of Business Research – Vol.7, No.5
Publication Date: May. 25, 2019
DOI: 10.14738/abr.75.6493.

Samsa, G. (2019). A Mathematical Illustration of Why It's Good for Long-term Investors to Buy Wonderful Companies at Fair Prices. *Archives of Business Research, 7(5)*, 23-27.

A Mathematical Illustration of Why It's Good for Long-term Investors to Buy Wonderful Companies at Fair Prices

Greg Samsa, PhD
Duke Department of Biostatistics and Bioinformatics

ABSTRACT

One of Warren Buffett's many insights on investment is often rendered as: "It's far better to buy a wonderful company at a fair price than a fair company at a wonderful price." We illustrate some of the mathematics behind this aphorism. More specifically, we illustrate the returns associated with purchasing stock in a company which compounds its earnings per share at a constant annual rate of 15% (i.e., the economic component of total return), at a price-earnings ratio of 20, and sold at different time points at different price-earnings ratios (i.e., the speculative component of total return). The primary result is that, over time, the relative contribution of the economic component of the total return increases relative to the speculative component. The Buffett strategy appropriately focuses on the economic component of long-term returns. This result can be applied to investing in mutual funds as part of a long-term retirement portfolio, and the demonstration used to illustrate the potential benefits and risks of purchasing diversified mutual funds during a bubble.

Key words: Economic return; Investment strategies; Speculative return; Stock market; Warren Buffett

INTRODUCTION

One of Warren Buffett's many insights on investment is often rendered as: "It's far better to buy a wonderful company at a fair price than a fair company at a wonderful price." Like many of Mr. Buffett's aphorisms, it embeds more than one mathematical insight. Our goal is to illustrate the mathematics behind this saying. In doing so, we also address a related saying, often rendered as: "Time is the friend of the wonderful company; the enemy of the mediocre".

METHODS

We illustrate the returns which would be generated from holding a "wonderful" company, operationally defined as one which compounds its earnings per share (EPS) at an annual rate of 15%, across different assumptions about prices and holding periods. For clarity of exposition, we make various simplifying assumptions:

- Earnings are the primary measure of corporate performance of interest to investors, and that these earnings are accurately reported (e.g., we assume that all the potential accounting machinations around earnings can be ignored)
- Earnings compound at a constant annual rate (i.e., we assume that, even though actual earnings are lumpier, they can be treated as if they are smoothed)
- The company doesn't pay a dividend
- The number of shares remains constant
- When trading the stock, the metric of interest to investors is the price-earnings ratio (PE)
- The earnings per share are positive and not near 0 (i.e., so that the PE is interpretable)
- Taxes don't matter

In fact, this illustration can be extended to more realistic scenarios than this, as its fundamental logic holds more generally.

CASE STUDY

Table 1 illustrates the results of holding a stock of a company which has EPS of $1.00 at baseline (i.e., year 0), and then compounds these earnings annually at a rate of 15%. We assume that, at baseline, the stock is purchased at a PE of 20, and thus a price of $20.00 (i.e., price/earnings = $20.00/$1.00 = 20). A PE of 20 is intended to illustrate a "fair price" and "not a bargain". For example, a PE of 20 would be above the historical median PE for stocks as a whole, and also modestly exceeds the annual rate of earnings growth (i.e., the ratio of the PE to EPS growth of 20/15 exceeds a benchmark value of 1).

The left-most column of Table 1 presents selected years: 1, 2, 3, 4, 5, 10, 20, 30 and 40, these latter years representing very long holding periods. The next column presents EPS, compounding at an annual rate of 15% (i.e., $1.00 * (1.15)^t$).

Table 1: Results of compounding earnings at 15% annually, with different assumptions about price-earnings ratios

Year	EPS	P_20	R_20	P_10	R_10	P_40	R_40
1	1.15	23.00	15.0%	11.50	-42.5%	46.00	130.0%
2	1.32	26.45	15.0%	13.23	-18.7%	52.90	62.6%
3	1.52	30.42	15.0%	15.21	-8.7%	60.83	44.9%
4	1.75	34.98	15.0%	17.49	-3.3%	69.96	36.8%
5	2.01	40.23	15.0%	20.11	0.1%	80.45	32.1%
10	4.05	80.91	15.0%	40.46	7.3%	161.82	23.3%
20	16.37	327.23	15.0%	163.67	11.1%	652.66	19.1%
30	66.21	1,324.24	15.0%	662.12	12.4%	2,648.47	17.7%
40	267.86	5,357.27	15.0%	2,678.64	13.0%	10,714.54	17.0%

The next column, labelled P_20, presents the stock price, were the stock to be sold for a PE of 20. For example, in year 1, $23.00/$1.15=20.

The next column, labelled R_20, presents the annual return, derived from the formula

$$R+1 = (X_t/X_0)^{(1/t)},$$

where t is the follow-up period, in years, X_t is the sales price at time t and X_0 is the purchase price at baseline. As expected, since the PE at sale is identical to the PE at purchase, the return is entirely based on the rate at which earnings compound, and thus is 15% per year regardless of the holding period.

The next two columns, labelled P_10 and R_10, respectively, illustrate the results of selling the stock at a PE of 10. PE's vary over time, due to various economic and speculative criteria, and the PE of 10 is intended to illustrate a scenario where investors have (temporarily) become pessimistic. Accordingly, the annual return should be below 15%, and what is of interest is how much less than 15%, and how this annual return varies as the holding period lengthens.

The final two columns, labelled P_40 and R_40, use the same logic as P_10 and R_10, but now investors have (temporarily) become optimistic, and the PE at the time of sale is 40. Accordingly, the annual return should be above 15%, and what is of interest is how much greater than 15%, and how this annual return varies as the holding period lengthens.

In interpreting the results, it can be helpful to note that the price at time t equals the EPS at time t (E_t) multiplied by the PE at time t (PE_t). Thus, the previous formula can be rewritten as

$R+1 = ((E_t/E_0)(PE_t/PE_0))^{(1/t)}$,

where (E_t/E_0) can be termed the "economic component of return", since it is based on the increase in EPS, and (PE_t/PE_0) can be termed the "speculative component of return", since it is based on an "exchange rate" between earnings and stock price, which at any point in time is substantially driven by psychology and other non-economic considerations. For P_10 and R_10 the speculative component of return is negative (i.e., the ratio is less than 1), whereas for P_40 and R_40 the speculative component of return is positive (i.e., the ratio is greater than 1).

RESULTS

By the construction, the economic component of return is constant, and so what is primarily being illustrated is the relationship between the speculative component of the total return and time. Considering the case where the PE=10, as might be the case during a market crash, the investor is losing money until year 5 and is earning a decent annualized return of 7.3% by year 10. By year 30, the annualized return is beginning to approach an asymptote: it is 12.4% in year 30 and 13.0% in year 40. Being on the wrong side of the speculative component of return is uniformly harmful, but the relative impact of the speculative component on the total annualized rate of return decreases over time, and eventually becomes modest.

A caveat to this observation is that, when considered over long time periods, modest absolute differences in the annual compounding rate of return can be associated with large differences in the absolute amount of money earned by the investor. Moving from PE=10 to PE=20, at any point in time, implies that the investor will double their money.

Considering the case where PE=40, similar results are observed in the opposite direction. It is always advantageous to sell in the midst of a bubble. Moreover, in terms of annualized returns, the ideal scenario is for the bubble to occur as soon as possible. For example, if the bubble occurs during year 1, the investor will more than double their money.

One way to estimate the impact of the speculative component of return uses the "rule of 72"; namely, that an investment will double in value by approximately the product of the annual compounding rate of return times the number of years: that is, R*D = 72 (approximately), where R is the annual compounding rate of return and D is the time required for the investment to double in value once. Recognizing that

$D = t / \log_2 (X_t/X_0)$, and also that the number of doublings in the presence of a speculative rate of return is

$\log_2 (E_t/E_0) + \log_2 (PE_t/PE_0)$, the annual compounding rate of return becomes (approximately)

$R = (72 / t) / (\log_2 (E_t/E_0) + \log_2 (PE_t/PE_0))$.

To illustrate this calculation: at year 40, assuming that the PE at the time of sale is 20, $\log_2 (E_t/E_0) = \log_2 (267.86) = 8.065$, $\log_2 (PE_t/PE_0) = \log_2 (1) = 0$, and thus $R = 72 / (40 / (8.065+0)) = 14.51$ (i.e., 15%, approximately).

On the other hand, if the PE at the time of sale is 40, $\log_2 (E_t/E_0) = \log_2 (267.86) = 8.065$, $\log_2 (PE_t/PE_0) = \log_2 (2) = 1$, and thus $R = 72 / (40 / (8.065+1)) = 16.31$ (i.e., 17.0%, approximately).

Here, the impact of the speculative component of the return is to reduce the time to the first doubling from 5 to 4.4, which in turn is derived from increasing the number of doublings from 8 to 9.

DISCUSSION

The main result of this illustration is to demonstrate that, over time, the impact of the economic component of return increases and the speculative component of return decreases. Indeed, this is consistent with the observation by Benjamin Graham that in the short run the stock market is a voting machine (i.e., reflecting temporary popularity and other speculative considerations) but in the long run is a weighing machine (i.e., reflecting corporate performance).

The economic component of return can be quantified by the number of times the EPS doubles during a specific period of time. The speculative component of return can be quantified by the ratio of the PE at the time of purchase to the time of sale. The longer the period of time the stock is held, the more times its EPS doubles, and thus the greater the relative impact of the economic component. Similarly, the greater the rate at which the EPS compounds, the more times it doubles, and thus the greater the relative impact of the economic component.

In essence, what Warren Buffet has done is to begin with this observation, and then ask what are the characteristics of companies which are likely to have dependably excellent rates of compounding earnings growth over time. Implicit in this question is that the company in question will also have a trivial risk of bankruptcy or less complete collapse: for example, if a stock is to be held for 40 years it does no good for it to perform wonderfully during the first 39 and then have its stock price fall suddenly to 0. Identifying "wonderful" companies wasn't ever a trivial task, and has become even more difficult as the pace of business has increased, competition has become global, etc. Whether attempting to replicate Mr. Buffet's investment strategy, as originally implemented, is reasonable or realistically possible is an open question, although the mathematical logic (and genius) behind that strategy is not.

A situation where the mathematical insight behind Mr. Buffet's strategy is most commonly applied is the standard advice provided to those investing in stocks through retirement funds. There, the single "wonderful" company in the illustration is replaced with a diversified mutual fund and, in the extreme, an index fund which replicates (minus trivial fees) the overall return of the market. The compounding rate of earnings growth is based on fundamental economic considerations such as productivity and innovation, and can, barring a profound economic disaster, be assumed to be relatively stable in the long run. Similarly, the PE for an individual stock in the illustration is replaced with the PE for the market as a whole, and the standard advice can be loosely translated as: "Even if the market is currently in a bubble the PE at purchase doesn't (much) matter so long as the investor has a sufficiently long-term perspective". Moreover, in contrast to individual stocks which can go to 0, diversification provides reassurance that complete disaster will be avoided.

Such advice is consistent with the demonstration presented here, albeit with some caveats. One caveat is that, over time, the impact of the speculative rate of return never entirely disappears. Second, the advice doesn't apply to people who are nearing retirement age, or otherwise won't be able to hold stocks for a sufficiently long period of time. Finally, perusal of the first row of Table 1 illustrates that a crash can be a profoundly frightening experience – when a bubble turns into a crash the value of a retirement account can drop by 50% or more, and the observation that the drop in value is likely to be temporary doesn't necessarily provide solace at the time.

At the time of this writing, it is generally agreed that, as a whole, American stocks are overpriced relative to historical norms (whether or not to term this overpricing "a bubble" is mostly semantic). What might be the implications of this overpricing for the stock component of a retirement portfolio? We argue that the primary implication is that two goals should be simultaneously pursued: (1) diversification should be maintained; and (2) bubbles should be avoided. We further argue that one approach to (probably) accomplishing these goals is through sector funds in non-bubble sectors of the market: for example, value funds, international funds, and the like. What is gained is a better initial PE, and thus greater confidence that the speculative component of total return won't be profoundly negative. What is lost is diversification across the entire market, since overpriced sectors are avoided, and it is quite possible that these sectors will continue to outperform others (in the short run). However, the benefits of diversification across stocks within a sector are maintained: for example, the bankruptcy of any single company should have a trivial impact on returns so long as the entire sector doesn't become economically obsolete, the target of government intervention, etc. What is also lost is that the compounding rate of earnings growth in non-bubble sectors might be less than ideal.

Put in other terms, we argue that critical questions to ask during a bubble are (1) in the event of a crash, which stocks are likely to ultimately recover; and (2) in the event of a crash, which stocks are likely to drop less than others. So long as the mutual fund pursues a conservative strategy, diversification within a sector ought to make the first question relatively moot. The second question acknowledges that the impacts of indexing, the unwinding of leverage, herd behavior, etc. imply that if the crash is sufficient severe few to no sectors will be left untouched. Nevertheless, a fall out of a first-story window is preferable to a fall from the roof, and a decrease in PE from 15 to 10 is a shorter fall than a decrease from 50 to 10, and so in general value-based funds, especially those which pay dependable dividends, tend to be good bets. In other words, if stocks must be purchased during a bubble, the goal should be to protect oneself against disaster, even at the cost of decreasing expected returns.