



Quantifying Interactions among European Equity Markets with Time-Varying Conditional Skewness

Cian E. Twomey

J.E. Cairnes School of Business & Economics,
NUI Galway, Ireland

ABSTRACT

This paper explores the influence of global and regional factors on the conditional distribution of daily stock returns in four European markets – the U.K., Germany, France, and Spain - using factor models in which unexpected returns comprise global, regional and local shocks. Besides conditional heteroscedasticity, the model innovates by allowing shocks to incorporate time-varying conditional skewness, which is found to increase the explanatory power of our modelling. The relative importance of the global and regional factors varies across the four markets, with the largest, most international market (the U.K.) being more dependent on global factors as compared to the regional importance for the Spanish market. Also, the global factor is relatively less important for market volatility in models that permit time-varying conditional skewness.

Keywords: Asymmetries, Skewness, Volatility, Spillovers, Stock returns

INTRODUCTION

A good understanding of the origins and transmission intensity of shocks is necessary for many financial decisions, including optimal asset allocation, the construction of global hedging strategies, as well as the development of various regulatory requirements, like capital requirements or capital controls. This paper examines the fundamental forces driving return volatility in four large European markets. Specifically, we focus on how and to what extent volatility in each of the four markets is influenced by foreign shocks from other national markets. We contribute to this understanding by presenting new measurements of the relative importance of global, regional and local components of risk in European equity markets.

This paper has two innovations: first, we re-estimate volatility spillovers using a factor model that, unlike previous models used for this purpose, allows for time-varying conditional skewness in a manner first proposed by Harvey and Siddique [1, 2]; second, we present additional evidence that distinguishes between downside and upside risks. This new approach is motivated by the strong empirical evidence that the standardised residuals from conditionally heteroskedastic models fitted to stock returns are asymmetrically distributed [3, 4]. The evidence we present is from four European equity markets, namely the U.K., Germany, France, and Spain, using daily data from January 2002 to December 2017.

Many early studies on spillovers focus on how a *single* international market (often the US or world market) influences other stock markets but do not distinguish regional versus world market factors [5, 6]. However, evidence of co-movements in the mean and volatility of equity returns suggests that factor models, such as those developed in Bekaert and Harvey [5] and Ng [7], are useful ways of modelling the behaviour of stock returns.

Specifying unexpected returns to depend on a world factor as well as an idiosyncratic shock, Bekaert and Harvey [5] find evidence that emerging market volatility is affected by a world factor, and that the influence of the world factor varies considerably over time. Ng [7]

constructs a volatility spillover model by assuming that there are three sources of shocks — local, regional and world. By considering innovations from the Japanese and US markets as regional and world shocks respectively, she analyses how much of the return volatility of any particular market in the Pacific– Basin is driven by a world factor and how much is left to be explained by a regional force. This is important for evaluating the relative importance of the world's two largest markets on smaller markets, such as the six Pacific–Basin countries examined — Hong Kong, Korea, Malaysia, Singapore, Taiwan, and Thailand. To conclude, Ng [7] finds evidence of spillovers in volatility from the US and Japanese markets to six Asian stock markets, with the US market exerting a stronger influence, although the external shocks appear to explain only a small fraction of volatility in these markets.

In another paper, Forbes and Chinn [8] estimate a factor model in which a country's market returns are a function of global factors (global interest rates, oil prices, gold prices, and commodity prices), sectoral factors (stock returns for 14 sectoral indices), cross-country factors (returns in other large financial markets), and country-specific effects. Forbes and Chinn [8] find that both cross-country factors and sectoral factors are important determinants of stock and bond returns in countries around the world (although it is often difficult to differentiate between these two sets of factors). Not surprisingly, movements in the largest regional economy tend to be the most important cross-country factor for nearby countries (such as the U.S. market for the Americas), although movements in the U.S. market are also important for most regions. In the later half of the 1990s, the U.S. factor and sectoral factors gained importance in most regions, while the Japanese and U.K. factors lost importance.

A large body of research has attempted to identify how the integration of previously segmented markets has changed patterns of cross-country equity correlations and interdependencies. Increased integration with global markets, however, does not necessarily generate increased correlations between domestic and global asset returns. One reason why integration may not generate increased correlations is differences in industrial structures between individual countries and the world average. Subsequent research has seen an active debate on the relative importance of industry effects versus country-specific effects in explaining cross-country correlations and volatility. For example, Heston and Rouwenhorst [9] argue that industrial structure explains very little of the cross-sectional differences in country return volatility in Europe, and that the low correlation between country indices is almost completely due to country-specific sources of return variation.

While there is ample evidence that full convergence in European bond markets and money markets had been achieved by the mid- to late 1990s, it is far less clear whether, and to what extent, European equity markets have become more integrated. This has important implications both for investors' portfolio allocation decisions and for policy-makers in meeting the challenges of European integration and shaping policy responses to more integrated and interdependent financial markets in Europe. Fratzscher [10] analyses the integration process of European equity markets since the 1980s. To address the above questions, the paper builds on an uncovered interest parity condition applied to asset prices to define financial market integration, and it employs a trivariate GARCH model for 16 OECD countries to test the implied integration hypotheses empirically. The paper has a number of important findings. First, European equity markets have become highly integrated with each other only since 1996. And second, the Euro area market has taken over from the US the role as the most important market in explaining equity returns in most individual European markets. It compares the relative importance of the three EMU pillars of exchange rate stability, real convergence, and monetary policy convergence in explaining the time variations of equity market integration in

Europe. Using a GARCH methodology with time-varying coefficients to analyse and compare the role of these three factors, the evidence suggests that EMU has indeed fundamentally altered the nature of financial integration in Europe. Fratzscher [10] found that it was in particular the reduction and elimination of exchange rate volatility, and to some extent also monetary policy convergence, that has played a central role in explaining the increased financial integration among EMU members. Moreover, the shock transmission across equity markets was found to be asymmetric, i.e. negative shocks are more strongly transmitted, large shocks have a stronger impact than small shocks, and these asymmetry and threshold effects have become larger over time.

In this paper, we begin with some evidence outlining why time-varying conditional skewness should be incorporated into the modelling process. Next, we present some preliminary data analysis in section 3. In particular, we document evidence of time-varying asymmetry in the markets that we study. The evidence we present here justifies our use of a time-varying skewness framework for studying spillover effects. The evidence also highlights the importance of studying the extent of spillovers in skewness. The models that we employ for studying spillovers are described in detail in section 4. The models assume that unexpected returns comprise world, regional and local shocks, with the difference that these shocks are now characterised not just by time-varying conditional volatility, but also by time-varying conditional skewness. Empirical results are presented and discussed in section 5, and section 6 concludes.

THE NATURE OF CONDITIONAL SKEWNESS

The second moment of returns, variance, has been the subject of a large literature in finance []. Variance of returns has been widely used as a proxy for risk in financial returns. Therefore, the properties of variance by itself as well as the relation between expected return and variance have been important topics in asset pricing. It is useful to focus on the intertemporal relation between return and risk where risk is measured in the form of variance or covariance. An important concern has been the sign and magnitude of this trade-off.

The generalised autoregressive conditional heteroscedasticity (GARCH) class of models, including the exponential GARCH (EGARCH) specification, have been the most widely used models in modelling time-series variation in conditional variance. Persistence and asymmetry in variance are two stylised facts that have emerged from the models of conditional volatility. Persistence refers to the tendency where high conditional variance is followed by high conditional variance. Asymmetry in variance, i.e., the observation that conditional variance depends on the sign of the innovation to the conditional mean has been documented in asymmetric variance models used in several seminal econometric papers [11-13]. These studies find that conditional variance and innovations have an inverse relation: conditional variance increases if the innovation in the mean is negative and decreases if the innovation is positive. The fourth moment of financial returns, kurtosis, has drawn substantial attention as well. This has been primarily because kurtosis can be related to the variance of variance and, thus, can be used as a diagnostic for the correct specification of the return and variance dynamics.

In contrast, skewness has drawn far less scrutiny in empirical asset pricing, though skewness in financial markets appears to vary through time and also appears to possess systematic relation to expected returns and variance. The time-series variation in skewness can be viewed as analogous to heteroscedasticity. Moreover, estimating time-varying moments is important for testing asset-pricing models that impose restrictions across moments. Estimation of time-varying skewness may also be important in implementing models in option pricing. The

presence of skewness can also affect the time-series properties of the conditional mean and variance.

Modelling asymmetry of returns is becoming more important in economics and finance. Skewness that measures asymmetry of the distribution based on its standardized third central moment is observed in stock returns data [1, 14, 15]. A negative (positive) skewness reflects the prices change in such a way that there is a higher (lower) probability of a large increase in price than a large fall. By knowing the shape of the portfolio return distribution, investors will be able to make better judgments depending on their risk preferences.

Several studies in the empirical finance literature have reported evidence of two types of asymmetries in the joint distribution of stock returns. The first is skewness or asymmetry in the distribution of individual stock returns, which has been reported by numerous authors [1, 2]. The second asymmetry is in the dependence between stocks: stock returns appear to be more dependent during market downturns than during market upturns, a characteristic we refer to as asymmetric dependence. Evidence that stock returns exhibit some form of asymmetric dependence has been reported by several authors [16-17].

Moreover, there is strong evidence that standardised residuals from conditionally heteroskedastic models fitted to stock returns are asymmetrically distributed [15]. If there is predictability in the shape of the conditional distribution of stock returns, the implications for asset and derivative pricing and risk management are immediate. Even if the first two moments are sufficient for asset pricing, variation in the shape of the distribution may affect the estimation of the conditional mean and conditional variance of an asset return, just as (non-varying) asymmetry affects the estimation of the conditional mean and variance

Harvey and Siddique [1] were the first to study the conditional skewness of asset returns, and extend the traditional GARCH(1,1) model by explicitly modelling the conditional second and third moments jointly. Specifically, they present a framework for modelling and estimating time-varying volatility and skewness using a maximum likelihood approach assuming that the errors from the mean have a non-central conditional t distribution. They then use this method to model daily and monthly index returns for the U.S., Germany, and Japan, and weekly returns for Chile, Mexico, Taiwan, and Thailand; concurrently estimating conditional mean, variance and skewness. We also present a bivariate model of estimating co-skewness and covariance in a GARCH-like framework. We find significant presence of conditional skewness and a significant impact of skewness on the estimated dynamics of conditional volatility. Their results suggest that conditional volatility is much less persistent after including conditional skewness in the modelling framework and asymmetric variance appears to disappear when skewness is included.

In a later paper, Harvey and Siddique [2] offer the following intuition for including skewness in the asset-pricing framework. In the usual set-up, investors have preferences over the mean and the variance of portfolio returns. The systematic risk of a security is measured as the contribution to the variance of a well-diversified portfolio. However, there is considerable evidence that the returns distributions cannot be adequately characterised by mean and variance alone, which leads them to the next moment – skewness. Given the statistical evidence of skewness in returns, it is reasonable to assume that investors have preferences for skewness. With a large positive skewness (high probability of a large positive return), the investors may be willing to hold a portfolio even if its expected return is negative. As they show, this is still fully consistent with the Arrow-Pratt notion of risk aversion. Similarly,

variation in skewness risk should also be important for the cross-section of assets. Skewness may be important in investment decisions because of induced asymmetries in ex-post (realised) returns. At least two factors may induce asymmetries. First, the presence of limited liability in all equity investments may induce option-like asymmetries in returns. Second, the agency problem may induce asymmetries in index returns. That is, a manager has a call option with respect to the outcome of his investment strategies. Managers may prefer portfolios with high positive skewness. So, if investors know that the asset returns have conditional skewness at time t , excess asset returns should include a component attributable to conditional skewness. Their asset-pricing model formalises this intuition by incorporating conditional skewness. This model is found to explain much of the time-series variation in the expected market risk premium.

Skewness in the returns of financial assets can arise from many sources. Lempériere et al [15] points out that managers have an option-like features in their compensation. The impact of financial distress on firms and the choice of projects can also induce skewness in the returns. More fundamentally, skewness can be induced through asymmetric risk preferences in investors. Harvey and Siddique [2] and Chen, Hong and Stein [18] are detailed studies into the determinants and economic significance of skewness in stock returns; stocks that are experiencing relatively high turnover and/or usually high returns over previous periods tend to be more negatively skewed. Stock capitalisation also appears to be important in explaining the degree of skewness in stock returns.

Perez-Quiros and Timmermann [19] relate time-varying skewness to business cycle variation. The skewness in stock returns is economically significant. Chen, Hong and Stein [18] used cross-sectional regressions of skewness in the daily stock returns of individual firms, measured over six month periods, and found that periods of high return and unusually high turnover tend to be followed by periods of negative skewness, indicating that the asymmetry they find in stock returns changes options prices substantially.

The question of predictability in the shape of a variable's conditional distribution is usually framed in terms of predictability in conditional skewness, or 'conditional heteroskewness', with attention focusing primarily on predictability using the variable's past history. Recent investigations have uncovered some evidence of such predictability in stock returns. Specifying the conditional distribution of the standardised residuals of a GARCH-M model as a non-central t -distribution, with skewness depending on the conditional skewness in the previous period, Harvey and Siddique [1] present evidence of skewness in the conditional distributions of daily stock index returns in the US, German, Japanese, Chilean, Mexican, Taiwanese and Thai markets, and that this asymmetry in the shape of the distribution depends on the degree of skewness in previous periods. Harvey and Siddique [2] incorporate time-varying conditional skewness into an asset-pricing model and find that ignoring skewness results in significant pricing errors.

Perez-Quiros and Timmermann [19] found time-variation in the skewness of size-sorted portfolios US stocks. They investigated the determinants of skewness in the daily stock returns using an autoregressive conditional density model with an asymmetric Generalized- t distribution to estimate the time-varying skewness and the time-varying effects of prior return/turnover on skewness. Using NYSE and AMEX data from 1962 to 2000, the authors find that if today's return is positive and turnover ratio is relatively high, investors would expect tomorrow's return to be from a more positively skewed distribution.

In terms of European evidence, El Babsiri and Zakoian [20] develop an original set of GARCH

models which allow for time-varying skewness and kurtosis (hetero-skewness and hetero-kurtosis) and two kinds of asymmetry: (i) different volatility processes for up and down moves in equity markets (contemporaneous asymmetry); (ii) asymmetric reactions of these volatilities to past positive and negative changes (dynamic asymmetry or leverage effect). In an application to a daily French stock index returns, they found that this model with conditional hetero-skewness, hetero-kurtosis, and leverage effects in volatility improves upon models without these effects.

Overall, the evidence of predictability in the skewness of stock returns is, however, difficult to interpret, particularly its implications for risk management. The majority of studies on this issue proceed by fitting a model that allows for predictability in skewness, and testing if the parameters that embody conditional heteroskewness are statistically significant. However, little is known about the behaviour of models with time-varying conditional skewness. In particular, these models may not be robust to outliers. On the other hand, the models may not be able to pick up predictability in extreme realisations, even if predictability exists, as extreme realisations occur infrequently.

Earlier studies of mean and/or volatility spillovers assumed that the conditional distribution of stock returns to be symmetric about its conditional mean. As noted in the above studies, recent work suggests that dynamics in the conditional third moment is an empirically relevant feature of stock returns. Therefore, this paper extends the standard modelling of volatility spillovers to allow shocks have time-varying conditional skewness. Moreover, the presence of time-varying conditional skewness in equity returns raises a few questions concerning the measurement of the influence of global, regional and local factors on individual stock markets. We address whether incorporating time-varying skewness into an analysis of spillovers provide substantially different measurements of the relative importance of world and regional factors on the volatility of domestic equity returns.

DATA AND SUMMARY STATISTICS

We use daily equity market index returns from the first week of January 2002 to the last week of December 2017. The data are obtained from Thomson Reuters Datastream, and the daily percentage returns are calculated as the difference of log closing prices (multiplied by 100). The indexes used for the European markets in this study are the FTSE100, Xetra DAX, CAC40, and IBEX-35. For the world factor we use daily returns on the MSCI World Index. As a proxy for the regional factor, we use daily returns on MSCI's Europe Index. Table 1 below contains summary statistics on these weekly returns.

Table 1: Summary Statistics for Daily Stock Returns^a

	World	Region	UK	Spain	France	Germany
Mean	0.031	0.039	0.037	0.035	0.038	0.040
Std. Error	0.952	1.053	0.867	1.087	1.112	1.061
Skewness	-0.556***	-0.625***	-0.824***	-0.268**	-0.333***	-0.595***
Kurtosis	32.216***	8.571***	11.531***	5.837***	5.641***	9.435***
Jarque-Bera	146.8***	60.39***	1068***	81.01***	1064***	477.9***
$\rho_1(1)$	-0.069*	-0.052	-0.085**	-0.075*	-0.031	-0.063
Q1(10)	12.99	9.658	23.05**	11.80	17.47*	16.228*
$\rho_2(1)$	0.064	0.089**	0.314***	0.202***	0.340***	0.317***
Q2(10)	89.62***	193.9**	114.8***	179.1***	190.0***	135.5***
$\rho_3(1)$	-0.047	-0.001	-0.124***	-0.009	-0.244***	-0.236***
Q3(10)	40.73***	4.836	11.93	10.77	40.42	59.88**
$\rho_4(1)$	0.004	0.026	0.103**	0.068	0.259***	0.248***
Q4(10)	34.020***	2.315	6.558	68.08***	42.69	65.18***

^a *, **, and *** denote statistical significance at 10, 5, and 1% respectively. $\rho_j(1)$ is the 1st order autocorrelation of the returns to the j th power. $Q_j(10)$ is the Ljung-Box Q statistic at lag 10 for the returns to the j th power.

The Jarque-Bera test statistic clearly indicates that the returns are non-normal, and in all cases this is due to the presence of skewness and excess kurtosis. The data show strong evidence that negative shocks are more frequent than positive shocks (negative skewedness), that large shocks are more common than expected statistically (excess kurtosis) and that equity returns are autocorrelated. The statistically significant excess kurtosis is very likely due, at least in part, to the presence of autoregressive conditional heteroscedasticity, as evidenced by the prominent autocorrelations in the square of all the returns series. Significant autocorrelation in the returns taken to the third power is sometimes used as an indicator of the possible presence of autoregressive third moments. The statistics would then indicate the possible presence of autoregressive skewness in U.K., French and German returns.

MODELLING TIME-VARYING CONDITIONAL SKEWNESS

To confirm the presence of time-variation in conditional skewness, and to assess the need for and potential gains from using a framework that permits this, we fit univariate models of time-varying conditional skewness to these returns: the stock returns are modelled as following some AR-GARCH process, with the standardised residuals following a zero-mean unit-variance skewed t distribution developed in Hansen [21].

Letting $r_{i,t}$ represent the time t return on the equity index of market i , with $i = w, g, 1, 2, 3, 4$ representing the world, regional, and the four individual European markets respectively, we model returns as:

$$r_{i,t} = \alpha_{i,0} + \alpha_{i,1}r_{i,t-1} + \alpha_{i,2}r_{i,t-2} + \alpha_{i,3}r_{i,t-3} + \alpha_{i,4}r_{i,t-4} + \alpha_{i,5}r_{i,t-5} + \alpha_{i,6}r_{i,t-6} + \alpha_{i,7}r_{i,t-7} + \alpha_{i,8}r_{i,t-8} + \alpha_{i,9}r_{i,t-9} + \alpha_{i,10}r_{i,t-10} + \epsilon_{i,t} \quad (1)$$

$$\epsilon_{i,t}^2 = \omega_{i,t} + \beta_{i,1}\epsilon_{i,t-1}^2 + \beta_{i,2}\epsilon_{i,t-2}^2 + \beta_{i,3}\epsilon_{i,t-3}^2 + \beta_{i,4}\epsilon_{i,t-4}^2 + \beta_{i,5}\epsilon_{i,t-5}^2 + \beta_{i,6}\epsilon_{i,t-6}^2 + \beta_{i,7}\epsilon_{i,t-7}^2 + \beta_{i,8}\epsilon_{i,t-8}^2 + \beta_{i,9}\epsilon_{i,t-9}^2 + \beta_{i,10}\epsilon_{i,t-10}^2 + \gamma_{i,t}[\max(0, \epsilon_{i,t-1})]^2 \quad (2)$$

$$z_{i,t} \sim g(z_{i,t} | \omega_{i,t}, \gamma_{i,t}) \quad (3)$$

$$g(z_{i,t} | \omega_{i,t}, \gamma_{i,t}) = \begin{cases} \frac{1}{\Gamma(\frac{i+1}{2})} \left(\frac{b_{i,t}z_{i,t} + a_{i,t}}{1 + \frac{b_{i,t}z_{i,t} + a_{i,t}}{2}} \right)^{\frac{i+1}{2}} & \text{when } z_{i,t} < -a_{i,t}/b_{i,t} \\ \frac{1}{\Gamma(\frac{i+1}{2})} \left(\frac{b_{i,t}z_{i,t} + a_{i,t}}{1 + \frac{b_{i,t}z_{i,t} + a_{i,t}}{2}} \right)^{\frac{i+1}{2}} & \text{when } z_{i,t} \geq -a_{i,t}/b_{i,t} \end{cases} \quad (4)$$

The volatility equation (2) is the Glosten et al. [12] specification that allows for conditional

volatility $\sigma_{i,t}^2$ to react asymmetrically to the previous period's shock according to whether the shock is positive or negative. The conditional distribution of the standardised residuals $z_{i,t}$ is characterised by two parameters: η_i is a degree of freedom parameter and $\lambda_{i,t}$ determines the degree of asymmetry in the distribution; these are restricted to $2 < \eta_i < \infty$ and $-1 < \lambda_{i,t} < 1$. The values $a_{i,t}$, $b_{i,t}$ and c_i are defined as:

$$a_{i,t} = 4 \lambda_{i,t} c_i \frac{2}{i-1} \quad (5)$$

$$b_{i,t}^2 = 1 + 3 \lambda_{i,t}^2 a_{i,t}^2 \quad (6)$$

$$c_i = \frac{\frac{i+1}{2}}{\sqrt{\binom{i}{2} \frac{i}{2}}} \quad (7)$$

where $a_{i,t}$ and $b_{i,t}$ may vary over time as we specify $\lambda_{i,t}$ to be possibly time-varying with the following autoregressive specification:

$$\lambda_{i,t} = f(\lambda_{i,t-1}, \max(0, \lambda_{i,t-1})) \quad (8)$$

This distribution is fat-tailed, and is skewed to the left (right) when $\lambda_{i,t}$ is less (greater) than 0. It reduces to the student's t density when $\lambda_{i,t}$ is equal to zero. We refer to $\lambda_{i,t}$ as the "asymmetry parameter" or the "skewness parameter" as this parameter determines whether the distribution is symmetric or not. This parameter is, however, not the same as the coefficient of skewness; nonetheless, $\lambda_{i,t}$ and the conditional skewness coefficient measure the same thing, and we will refer to time-variation in $\lambda_{i,t}$ as time-variation in conditional skewness. Equation (8) that determines $\lambda_{i,t}$ will be referred to as the "asymmetry equation" or "skewness equation". The specification in (8) that we use differs from previous applications of the Autoregressive Conditional Density (ARCD) model as we allow for negative shocks and positive shocks to have different effects not just on volatility (the usual "leverage effect") but also on skewness.

The proof that a random variable with this distribution has zero mean and unit variance is in Hansen [21]. The models are estimated by maximum likelihood. In fitting the model, we impose the restrictions $-1 < \lambda_{i,t} < 1$ and $2 < \eta_i < \infty$ using the logistic transformations:

$$\lambda_{i,t} = 1 + \frac{2}{1 + \exp(\lambda_{i,t})} \quad (9)$$

$$\eta_i = 2 + \frac{30}{1 + \exp(\eta_i)}$$

and re-specify (8) as $\lambda_{i,t} = \alpha_0 + \alpha_1 \lambda_{i,t-1} + \alpha_2 \lambda_{i,t-1} + \alpha_3 \max(0, \lambda_{i,t-1})$

The results from this estimation exercise are shown in Table 2.

Table 2: Univariate Model with Time-Varying Conditional Skewness^a

	World	Region	UK	Spain	France	Germany
<i>Mean Eq.</i>						
$\alpha_{i,0}$	0.139 (0.069)**	-0.106 (0.109)	0.393 (0.151)***	-0.189 (0.155)	0.093 (0.126)	0.111 (0.117)
$\alpha_{i,1}$	-0.069 (0.039)*	-0.016 (0.047)	0.006 (0.047)	-0.072 (0.049)	0.040 (0.044)	0.014 (0.049)
<i>Variance Eq.</i>						
$\beta_{i,0}$	0.036 (0.025)	0.420 (0.333)	0.644 (0.325)**	0.070 (0.056)	0.471 (0.227)**	0.547 (0.327)*
$\beta_{i,1}$	0.954 (0.016)***	0.841 (0.095)***	0.818 (0.040)***	0.971 (0.022)***	0.836 (0.043)***	0.817 (0.070)***
$\beta_{i,2}$	0.068 (0.031)**	0.179 (0.093)*	0.186 (0.056)***	0.070 (0.021)***	0.233 (0.069)***	0.217 (0.087)**
$\beta_{i,3}$	-0.068 (0.044)	-0.165 (0.077)**	-0.085 (0.067)	-0.080 (0.017)***	-0.177 (0.063)***	-0.155 (0.075)**
<i>Skew Equation</i>						
$\gamma_{i,0}$	-0.208 (0.252)	-0.007 (0.158)	-0.195 (0.131)	0.025 (0.106)	0.035 (0.070)	-0.072 (0.130)
$\gamma_{i,1}$	0.117 (0.388)	0.525 (0.147)***	0.301 (0.214)	0.189 (0.266)	0.543 (0.382)	0.005 (0.397)
$\gamma_{i,2}$	0.301 (0.132)**	0.133 (0.069)*	0.062 (0.025)**	0.052 (0.021)**	0.010 (0.024)	0.043 (0.035)
$\gamma_{i,3}$	-0.470 (0.231)**	-0.117 (0.102)	-	-	-	-
<i>Degrees of Freedom</i>						
η	9.512 (3.769)**	6.879 (1.671)***	12.67 (5.567)**	14.77 (8.242)*	5.466 (1.234)***	7.441 (2.550)***
LR	5.808*	2.233	3.482	5.108*	0.414	1.514
Wald	5.712*	13.34**	9.367**	9.415**	4.381	1.484
K-S	0.019	0.024	0.033	0.024	0.023	0.023
$(q_t \bar{q})\rho_1$	0.026	0.001	0.021	0.064	0.030	0.029
Q1(10)	6.344	9.838	3.626	6.156	16.59*	18.888**

^aThe model being estimated consists of equations 1, 2, and 9. Standard errors are in parentheses, and *, **, and *** denote statistical significance at 10, 5, and 1% respectively. Wald and LR denote the Wald and Likelihood Ratio test statistics for the restriction $\gamma_{i,1} = \gamma_{i,2} = (\gamma_{i,3}) = 0$. $q_t = \int_{-\infty}^{y_t} g(u_t) du_t$. K-S is the Kolmogorov-Smirnov test for uniformity. ρ_j is the 1st order autocorrelation of $q_t \bar{q}$ to the j th power. Qj(10) is the Ljung-Box Q statistic at lag 10 for the returns to the j th power.

In Table 2, the goodness-of-fit measures for all returns series suggest that the models capture the dynamics of the returns well; the Kolmogorov-Smirnov test does not reject the null of uniformity in all cases, and the autocorrelations and Ljung-Box Q statistics show that, to a large extent, all the dynamics in the data have been accounted for. Both the return on the world and regional indexes show clear evidence of time variation in conditional skewness. The parameters $\gamma_{i,1}$, $\gamma_{i,2}$, and $\gamma_{i,3}$ in the asymmetry equation are mostly statistically significant at 5%. A Wald test on the joint significance of these parameters in each of the equation rejects the null that the parameters are zero. The results from the individual markets in our study are much less convincing. Individual and joint tests on the parameters $\gamma_{i,1}$, $\gamma_{i,2}$, and $\gamma_{i,3}$ in the asymmetry equation show mixed results, as do the Likelihood Ratio tests.

The world, regional and individual market returns in our study tend to be more negatively skewed during periods of high volatility. Table 6.3(a) displays the correlation between the

degree of skewness as measured by $\lambda_{i,t}$ and $\sigma_{i,t}^2$, the conditional volatility of returns from the univariate models.

Table 3(a): Correlation between $\lambda_{w,t}$ and $\sigma_{w,t}^2$, $\lambda_{g,t}$ and $\sigma_{g,t}^2$ and $\lambda_{i,t}$ and $\sigma_{i,t}^2$

World	Region	U.K.	Spain	France	Germany
-0.450	-0.697	-0.143	-0.049	-0.167	-0.142

The correlation of negative skewness with high volatility adds further weight to the economic significance of conditional skewness in the data, and the usefulness of refining our understanding of volatility spillovers to distinguish downside risks from overall volatility.

The correlations, shown in Table 3(b) below, between the estimated asymmetry parameters from the six univariate models suggest that a factor model would be an appropriate framework for such an analysis. The correlations are all fairly large and positive.

Table 3(b): Correlation between $\lambda_{w,t}$, $\lambda_{g,t}$ and $\lambda_{i,t}$ ^{a, b}

	World	Region	U.K.	Spain	France	Germany
World	1.000	0.558	0.248	0.111	0.191	0.236
Region		1.000	0.400	0.310	0.390	0.421
U.K.			1.000	0.376	0.447	0.686
Spain				1.000	0.270	0.401
France					1.000	0.517
Germany					-0.199	1.000

^{a, b} $\lambda_{w,t}$ and $\sigma_{w,t}^2$, $\lambda_{g,t}$ and $\sigma_{g,t}^2$ and $\lambda_{i,t}$ and $\sigma_{i,t}^2$ are the fitted asymmetry parameters and conditional variances obtained from the univariate models with time-varying conditional skewness (see note to Table 2).

The results from the univariate models strongly suggest that it will be productive to study the issue of volatility spillovers using a factor model with time-varying conditional skewness. We construct, in the spirit of Bekeart and Harvey [2] and Ng [7], the following sequence of models.

The world market returns series is assumed to follow the process described in equations (1) through (8) above, which are reproduced again below for convenience. The world factor is assumed not to depend on any of the individual markets in this study, or on the regional factor. The regional market returns series on the other hand is driven by a world shock, and a regional shock that is assumed to be independent of the world shock:

$$r_{g,t} = \alpha_{g,0} + \alpha_{g,1}r_{w,t} + \alpha_{g,2}r_{g,t} + \epsilon_{g,t} \tag{1}$$

$$\epsilon_{g,t} = \alpha_{g,1} \epsilon_{w,t} + e_{g,t}, e_{g,t} = z_{g,t} \tag{2}$$

$$z_{g,t} \sim \mathcal{G}(z_{g,t} | \alpha_{g,1}, \epsilon_{g,t}) \tag{3}$$

$$\sigma_{g,t}^2 = \alpha_{g,0} + \alpha_{g,1} \sigma_{g,t}^2 + \alpha_{g,2} e_{g,t}^2 + \alpha_{g,3} [\max(0, e_{g,t})]^2 \tag{4}$$

$$\epsilon'_{g,t} = \alpha_{g,0} + \alpha_{g,1} \epsilon'_{g,t} + \alpha_{g,2} e_{g,t} + \alpha_{g,3} \max(0, e_{g,t}) \tag{5}$$

The unexpected returns on individual markets are, in turn, assumed to depend on the world shock, the idiosyncratic portion of the regional shock, $e_{g,t}$, and a country-specific shock that is independent of both $e_{g,t}$ and $\epsilon_{w,t}$:

$$r_{i,t} = \alpha_{i,0} + \alpha_{i,1}r_{w,t-1} + \alpha_{i,2}r_{g,t-1} + \alpha_{i,3}r_{i,t-1} + \varepsilon_{i,t} \quad (6)$$

$$\varepsilon_{i,t} = \alpha_{i,1}w_{i,t} + \alpha_{i,2}e_{g,t} + e_{i,t}, e_{i,t} = z_{i,t} \quad (7)$$

$$z_{i,t} \sim \mathcal{G}(z_{i,t} | \alpha_{i,1}, \alpha_{i,2}) \quad (8)$$

$$\sigma_{i,t}^2 = \alpha_{i,0} + \alpha_{i,1}\sigma_{i,t-1}^2 + \alpha_{i,2}e_{i,t-1}^2 + \alpha_{i,3}[\max(0, e_{i,t-1})]^2 \quad (9)$$

$$\lambda_{i,t} = \alpha_{i,0} + \alpha_{i,1}\lambda_{i,t-1} + \alpha_{i,2}\lambda_{i,t-1} \quad (10)$$

Throughout, $\varepsilon_{i,t}$ is used to denote the time t unexpected return for each series (w, g, i) while $e_{i,t}$ denotes the idiosyncratic shock; $\sigma_{i,t}^2$ and $\lambda_{i,t}$ always denote the conditional variance and skewness of an idiosyncratic shock, while $h_{i,t}$ will refer to the conditional volatility of unexpected returns (which combines the idiosyncratic shock with the external factors).

We observe that $\lambda_{i,t}$ and $\sigma_{i,t}^2$ are connected through (9). The world shock affects the volatility and skewness of unexpected regional returns only through (2), while the world and idiosyncratic regional shocks influence the volatility and skewness of unexpected country returns through (7). These two equations are referred to as the *factor equations*.

The factor loadings $\phi_{i,1}$ and $\phi_{i,2}$ capture the impact of the global and regional factors on the volatility and skewness of country i 's return, and so in our analysis we consider the relative size and significance of these two parameters. To understand the economic significance of these factors, we calculate the proportion of variance in the market returns explained by the global and regional factors.

Since the conditional variance of country i 's stock return is

$$\left[\sigma_{i,t}^2 | \mathcal{I}_{t-1} \right] = h_{i,t} = \sigma_{i,1}^2 \sigma_{w,t}^2 + \sigma_{i,2}^2 \sigma_{g,t}^2 + \sigma_{i,t}^2 \quad (11)$$

we estimate the proportion of country i 's volatility accounted for by the factors by the average values of

$$VR_{i,t}^w = \frac{\hat{\sigma}_{i,1}^2 \hat{\sigma}_{w,t}^2}{\hat{h}_{i,t}} \quad \text{and} \quad VR_{i,t}^g = \frac{\hat{\sigma}_{i,2}^2 \hat{\sigma}_{g,t}^2}{\hat{h}_{i,t}} \quad (12)$$

EMPIRICAL RESULTS

As one of our aims is to evaluate how incorporating time-varying skewness into our analysis will affect the measurement of spillovers, we also present for comparison the corresponding parameter estimates from spillover models that restrict conditional skewness to be constant, i.e., a model where the world, regional and country returns are assumed to be generated by (1) - (4), (1) - (5), and (6) - (10) respectively, but where $\gamma_{ij} = 0 \forall j \neq 0, i = w, g, 1, \dots, 4$. Comparisons are made not just of the parameter estimates, but also of the variance ratios. We follow this with a discussion of the relative influence of global and regional factors in downside risk in the individual markets implied by the skewness coefficients and probabilities from the spillover models with time-varying conditional skewness.

Parameter Estimates for Constant Spillover Models

Tables 4 and 5 report the results for the spillover models. Table 4 presents the results from the model where conditional skewness of all idiosyncratic shocks is permitted to be time varying (including the idiosyncratic world and regional shocks). The parameter estimates of $\phi_{i,1}$ and $\phi_{i,2}$ in Table 4 show that the spillover effects of both the world and regional factors are

statistically significant.

Table 4: Constant Spillover Model with Time-Varying Conditional Skewness

	U.K.	Spain	France	Germany
Mean Equation				
$\alpha_{i,0}$	0.311 (0.117)***	-0.182 (0.149)	0.060 (0.114)	0.018 (0.105)
$\alpha_{i,1}$	0.018 (0.047)	-0.100 (0.047)**	-0.006 (0.045)	-0.054 (0.045)
$\alpha_{i,2}$	-0.244 (0.081)***	0.201 (0.110)*	0.010 (0.076)	-0.033 (0.066)
$\alpha_{i,3}$	0.245 (0.126)*	-0.095 (0.161)	0.214 (0.111)*	0.265 (0.099)***
Factor Equation				
$\phi_{i,1}$	0.854 (0.117)***	0.078 (0.138)	0.498 (0.113)***	0.654 (0.098)***
$\phi_{i,2}$	0.169 (0.092)*	0.513 (0.098)***	0.234 (0.083)***	0.280 (0.076)***
Variance Equation				
$\beta_{i,0}$	0.347 (0.188)*	0.099 (0.073)	0.202 (0.155)	0.198 (0.172)
$\beta_{i,1}$	0.812 (0.037)***	0.956 (0.019)***	0.869 (0.040)***	0.917 (0.059)***
$\beta_{i,2}$	0.191 (0.051)***	0.073 (0.019)***	0.188 (0.065)***	0.084 (0.046)***
$\beta_{i,3}$	-0.052 (0.068)	-0.065 (0.022)***	-0.122 (0.060)**	-0.056 (0.035)
Asymmetry Equation				
$\gamma_{i,0}$	-0.145 (0.098)	0.009 (0.077)	0.073 (0.069)	-0.075 (0.056)
$\gamma_{i,1}$	0.369 (0.224)*	0.398 (0.241)*	0.595 (0.226)***	0.652 (0.103)***
$\gamma_{i,2}$	0.100 (0.040)**	0.058 (0.019)***	-0.011 (0.022)	-0.043 (0.028)
Degrees of Freedom				
η	15.28 (8.094)*	10.76 (4.426)**	5.896 (1.469)***	6.220 (1.548)***

^a The model being estimated consists of equations 4.6, 4.7, 4.9 and 4.10 where $z_{i,t} \sim g(z_{i,t} | i, i_t)$ is the distribution as specified in equation 2.4. Standard errors are in parentheses, and *, **, and *** denote statistical significance at 10, 5 and 1% respectively.

The parameter estimates reported in Table 5 below are for the model restricting conditional skewness to be constant throughout the sample period. In both cases, we obtain the usual results concerning mean spillovers (defined in our models as persistent effects on individual markets of past information in global and regional returns). The global market in general displays larger spillover effects in the mean than the regional factor in all markets, except for Spain. For all markets, the coefficient estimates in the mean equation and the variance equation are to a close approximation the same in both the constant and time-varying conditional skewness models. The variance equation, which captures the evolution of the conditional variance of the idiosyncratic country shock, displays asymmetric effects of past

shocks on variance. The asymmetry equation also shows time-variation in the skewness of the idiosyncratic shock.

Table 5: Constant Spillover Model with Constant Conditional Skewness

	U.K.	Spain	France	Germany
Mean Equation				
$\alpha_{i,0}$	0.308 (0.116)***	-0.188 (0.150)	0.057 (0.114)	0.019 (0.104)
$\alpha_{i,1}$	0.014 (0.047)	-0.099 (0.048)**	0.000 (0.042)	-0.049 (0.047)
$\alpha_{i,2}$	-0.214 (0.083)***	0.190 (0.109)*	0.007 (0.076)	-0.053 (0.067)
$\alpha_{i,3}$	0.191 (0.127)	-0.078 (0.154)	0.198 (0.110)*	0.262 (0.104)**
Factor Equation				
$\phi_{i,1}$	1.027 (0.079)***	0.582 (0.104)***	0.731 (0.085)***	0.919 (0.073)***
$\phi_{i,2}$	0.166 (0.099)*	0.512 (0.099)***	0.231 (0.082)***	0.269 (0.074)***
Variance Equation				
$\beta_{i,0}$	0.289 (0.153)*	0.090 (0.070)	0.207 (0.156)	0.261 (0.267)
$\beta_{i,1}$	0.835 (0.038)***	0.959 (0.018)***	0.867 (0.040)***	0.892 (0.084)***
$\beta_{i,2}$	0.158 (0.055)***	0.073 (0.018)***	0.190 (0.066)***	0.103 (0.069)
$\beta_{i,3}$	-0.027 (0.061)	-0.069 (0.024)***	-0.124 (0.060)**	-0.065 (0.047)
Skewness Parameter				
λ	-0.217 (0.136)	0.014 (0.120)	0.189 (0.124)	-0.225 (0.135)*
Degrees of Freedom				
η	13.29 (6.175)**	9.695 (3.684)***	5.868 (1.450)***	6.571 (1.597)***

^aThe estimated model consists of equations 4.6, 4.7, and 4.9 where $z_{i,t} \sim g(z_{i,t} | i, i_t)$ is the distribution as specified in equation 2.4. Standard errors are in parentheses, and *, **, and *** denote statistical significance at 10, 5 and 1% respectively.

Compared with the estimates of the same parameters in the model that does not allow for time variation in skewness in Table 5, we find that in all cases the coefficient $\phi_{i,1}$ on the world factor is substantially smaller, especially in the case of Spain; the coefficient on the world factor for this market is close to zero and not statistically significant at conventional levels of significance. The coefficient of the regional factor has remained roughly the same from Table 4 to Table 5.

Spillover Effects in Variance

The results above suggest that when time-varying skewness is taken into account, risk in the six markets in our study seems to be driven more by regional factors than by world factors. To gain some insights into the economic significance of the results and to determine how well the model explains the time-variation of interdependencies, we calculate for each market the proportion of the movements in the conditional variance that can be attributed to the world and regional factors. Table 6 shows the average of the period t variance ratios for the world

and regional factors. The rows labelled 'World' and 'Region' respectively show the average value of $VR_{i,t}^w$ and $VR_{i,t}^g$ as described in (12).

Table 6: Average Variance Ratios for World and Regional Factors

	<u>UK</u>	<u>Spain</u>	<u>France</u>	<u>Germany</u>
<i>Spillover Model with Time Varying Conditional Skewness</i>				
World	0.209	0.001	0.081	0.160
Region	0.010	0.054	0.021	0.028
<i>Spillover Model with Constant Conditional Skewness</i>				
World	0.270	0.063	0.156	0.273
Region	0.007	0.049	0.015	0.023

The variance ratios for four models are displayed. The top panel lists the variance ratio for the spillover models, first with time-varying conditional skewness, and then with conditional skewness restricted to be constant. The spillover models show that the world factor plays an important role in explaining the variance of the unexpected returns for the London, Paris, and Frankfurt markets, whereas the regional factor accounts for only a very small fraction of the variance in all markets.

The world factor becomes negligible for all markets when the skewness is permitted to be time-varying, while in the constant skewness model the importance of the world factor increases substantially. The parameter estimates and estimates of the variance ratios from the spillover model with time-varying skewness seem more appealing than those from the non-time-varying skewness model. From Table 6, the variance ratios are generally between 0.1 and 0.2 for both sets of equations, indicating that the models manage to explain generally between 10% and 20% of the time variation of local returns for each period. Although these percentages might seem small, it should be kept in mind that the data has daily frequency, therefore including a lot of volatility. Moreover, these numbers compare very favourably to similar models conducted with either daily or weekly data, for example Ng [7].

CONCLUSIONS

We present new measurements of the relative importance of global, regional and local components of risk in equity markets, an issue with implications for important financial market activities, using a factor model that allows for time-varying conditional skewness. The inclusion of the latter factor follows the evidence first proposed by Harvey and Siddique [1-2] that such time-varying conditional skewness is priced in a variety of equity markets. The evidence we present is from four European markets, namely the U.K., Germany, France, and Spain, using daily data from 2002 to 2017, and using world and regional indexes as a proxy for world and regional factors. We consider the effects of omitting time-varying conditional skewness from an analysis of spillovers. We also explore spillovers both in terms of volatility as well as downside risks.

We find that incorporating time variation in skewness affects the measurement of the sources of risk substantially, and doing so results in a smaller measurement of the importance of the world factor in explaining risk in the individual equity markets considered. Incorporating time-varying skewness in the analysis also allows us to clarify the notion of volatility spillover effects. There is evidence that downside-risk spillover effects are due more to the world factor and not the regional factor. The variety in the source of risk in equity markets in our sample period re-emphasises the need to allow for time-varying spillovers, as in Bekeart and Harvey [5] and Ng [7]. We do not explore the specific reasons why spillover effects vary over time.

Another avenue of research is to explore the view that it is the nature of news that results in either the world factor or the regional factor playing the more important role. We could investigate the influence of key events on the role of world and regional factors on European equity markets. One way of exploring this might be to allow for Markov switching between the world and regional factor, and relating switches to specific news items.

Finally, more research into the economic reasons why asymmetry in the conditional distribution of stock returns is time varying is needed. Our lack of knowledge of the causes of time-varying conditional skewness notwithstanding, the results in this paper show that studies of spillovers and linkages between equity markets, and its various sources of risk, will benefit from explicitly incorporating predictability in conditional skewness.

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