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Theoretical Analysis of Optimal Vaccine Allocation under an Age-Structured Epidemic

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Abstract: This paper develops a theoretical model to analyze the optimal allocation of vaccines in an age-structured population during an infectious disease outbreak. The model explicitly incorporates both deaths caused by infection and mortality risks associated with vaccination, and examines how age-specific vaccination policies affect the total number of deaths over time. Infection dynamics are modeled as depending on the cumulative number of infected individuals and the remaining susceptible population, while vaccination permanently removes individuals from the susceptible pool. The optimal allocation is characterized by a threshold condition determined by age-specific infection fatality rates, population shares and the mortality risk from vaccination. Numerical examples show that, even when the elderly face higher infection fatality rates, prioritizing vaccination for the young can minimize total deaths by dynamically suppressing future infection spread. However, when the vaccination-related mortality risk for the elderly is sufficiently large, refraining from vaccinating the elderly becomes optimal.

Keywords: Vaccine allocation, Age-structured epidemic model, Infection fatality risk, Vaccination side effects, Optimal policy under epidemics

INTRODUCTION

Global spread of the novel coronavirus disease (COVID-19) has raised critical policy questions regarding the containment of infection and the optimal allocation of vaccination. In particular, under limited vaccine supply, issue of whether younger or older individuals should be prioritized for vaccination has attracted substantial theoretical and empirical interest from the perspectives of suppressing infection and minimizing mortality. This study aims to theoretically analyze the optimal allocation of vaccines by explicitly modeling the dynamic process of infection spread in a population with an age structure and by incorporating the trade-off between deaths caused by infection and the mortality risk associated with vaccination.

Economic analyses of infectious diseases have evolved from the classical SIR-type framework to models that incorporate interactions between infection dynamics and economic behavior. Seminal work of Kermack and McKendrick (1927) provided a mathematical foundation for modeling the spread of infectious diseases and laid the groundwork for subsequent epidemiological models. More recently, a rapidly growing literature has examined the macroeconomic impacts of epidemics. For example, Eichenbaum et al. (2020) and Alvarez et al. (2021) develop macroeconomic models that incorporate the trade-off between infection risk and economic activity. Furthermore, Acemoglu et al. (2021) analyze optimal policies that account for age-specific infection risks and differences in economic roles, highlighting the importance of policy design that explicitly considers population age structure.

Research on vaccination policy has also advanced considerably in recent years. Bubar et al. (2021) analyze optimal vaccine allocation by considering age-specific infection rates and fatality rates, suggesting that prioritizing younger individuals with higher contact frequencies may be effective in suppressing infection spread. Similarly, Buckner et al. (2021) and Matrajt et al. (2021) numerically investigate optimal allocation under constraints on vaccine effectiveness and supply using dynamic infection models. While these studies evaluate optimal vaccine allocation from the perspective of infection containment, relatively few theoretical models explicitly incorporate the risk of adverse side effects or mortality associated with vaccination itself.

Several theoretical studies have analyzed optimal policy responses to epidemics by endogenizing behavioral responses. Fenichel (2013) and Reluga (2010) present models that capture the interaction between individual behavioral changes and infection dynamics. Toxvaerd (2020) examines the relationship between individual preventive behavior and government intervention under epidemics, demonstrating the importance of policy timing. Although these studies emphasize the endogeneity of infection risk, they do not fully address the optimal allocation problem that simultaneously considers age-specific fatality differences and mortality risks from vaccination.

Building on this literature, this study constructs a dynamic model that jointly incorporates two key elements: differences in infection fatality rates across age groups and the mortality risk associated with vaccination. Specifically, under a population composed of younger and older individuals, we formulate a process in which infection spreads depending on both the cumulative number of infected individuals and the remaining susceptible population. We then analyze how the age-specific allocation of vaccinations in each period affects the total number of deaths arising from infection and from vaccination. This framework allows us to explicitly evaluate the trade-off between the infection-suppression effect of vaccination and the risk of vaccine side effects, and to derive theoretical threshold conditions for the optimal allocation.

This study makes three main contributions. First, by presenting a theoretical model that integrates deaths due to infection and deaths due to vaccination, it provides a new perspective for evaluating vaccination policies. Second, it explicitly analyzes how differences in vaccination targets affect future infection dynamics through the dynamic effect of reducing the susceptible population. Third, it derives threshold conditions under which the optimal vaccine allocation changes discretely depending on parameter values, thereby clarifying the theoretical implications of age structure and differential fatality rates for policy choice.

The remainder of the paper proceeds as follows. We first present basic model and derive dynamic equations governing the number of infected individuals and deaths. We then theoretically determine the optimal vaccine allocation that minimizes the total number of deaths from infection and vaccination, and characterize the corresponding threshold conditions. Finally, using numerical examples, we illustrate the conditions under which prioritizing younger individuals or older individuals is optimal, and discuss the policy implications of the model.

BASIC MODEL

We consider a region with a population of N individuals, where the proportion of young and elderly individuals is given by $n:(1-n)$. In this region, if a young individual becomes infected with COVID-19, a fraction $y \times 100\%$ develops symptoms and dies, while the remaining $(1-y) \times 100\%$ remain asymptomatic. In contrast, if an elderly individual becomes infected, a fraction $e \times 100\%$ develops symptoms and dies, while the remaining $(1-e) \times 100\%$ remain asymptomatic. Infected individuals are assumed to be capable of transmitting the infection to others regardless of whether they exhibit symptoms.

Let $I_Y(t)$ denote the number of newly infected young individuals from the beginning to the end of period t, and $I_E(t)$ denote the number of newly infected elderly individuals during the same period. We define the total number of newly infected individuals as $I(t) = I_Y(t) + I_E(t)$. Similarly, let $B_Y(t)$ denote the number of young individuals who die from infection between the beginning and the end of period t, and $B_E(t)$ the number of elderly individuals who die from infection during the same period.

To simplify the analysis, we assume that individuals who receive the vaccine become permanently immune and never get infected thereafter. In each period, $(V-x)$ young individuals and x elderly individuals are vaccinated. However, when an elderly individual is vaccinated, death occurs with probability $p \times 100\%$, while vaccination does not cause death among young individuals. Let $C(t)$ denote the number of elderly individuals who die from vaccination between the beginning and the end of period t.

Mechanism of infection spread is as follows. When an individual becomes infected, that individual contacts, on average, a other individuals, regardless of age group, and transmits the infection to those who are susceptible. Here, “susceptible individuals” refer to those who are not yet infected and have not received vaccination.

Let $S_Y(t)$ denote the number of susceptible young individuals who may become infected from the beginning to the end of period t, and $S_E(t)$ the corresponding number of susceptible elderly individuals. Under the above assumptions, the following relations hold:

$$S_Y(t) = S_Y(t-1) - I_Y(t-1) - (V-x), \tag{1}$$

$$S_E(t) = S_E(t-1) - I_E(t-1) - x. \tag{2}$$

Let $S(t) = S_Y(t) + S_E(t)$ denote the total number of susceptible individuals during period t.

Let $D(t)$ denote the cumulative number of deaths at the beginning of period t. The evolution of cumulative deaths is then given by

$$D(t+1) = D(t) + B_Y(t) + B_E(t) + C(t). \tag{3}$$

Finally, let $A(t)$ denote the cumulative number of infected individuals at the beginning of period t. As an initial condition, we assume $A(0) = 1$ and that this initial infected individual is young.

Each infected individual contacts a other individuals on average and transmits the infection to those who are susceptible. Hence, the number of newly infected individuals from the beginning to the end of period t, denoted by $I(t)$, is given by

$$I(t) = a\frac{S(t)}{N}A(t). \tag{4}$$

New infections are allocated between the young and the elderly in proportion to the number of susceptible individuals in each group, i.e., according to the ratio $S_Y(t){:}S_E(t)$. Therefore, we have

$$I_Y(t) = aA(t)\frac{S_Y(t)}{N}, \tag{5}$$

$$I_E(t) = aA(t)\frac{S_E(t)}{N}. \tag{6}$$

Since the infection fatality rate is $y \times 100\%$ for the young and $e \times 100\%$ for the elderly, the number of deaths due to infection is given by

$$B_Y(t) = yI_Y(t), \tag{7}$$

$$B_E(t) = eI_E(t). \tag{8}$$

DERIVATION OF THE NUMBER OF DEATHS IN EACH PERIOD

Next, the evolution of the cumulative number of infected individuals satisfies

$$A(t) = A(t-1) + I(t-1). \tag{9}$$

Substituting (10), obtained from (4),

$$I(t-1) = a\frac{S(t-1)}{N}A(t-1), \tag{10}$$

into (9), we obtain

$$A(t) = \left(1+a\frac{S(t-1)}{N}\right)A(t-1). \tag{11}$$

Thus, the general solution for $A(t)$is

$$A(t) = A(0)\prod_{\tau=0}^{t-1}\left(1 + a\frac{S(\tau)}{N}\right). \tag{12}$$

Using the initial condition $A(0) = 1$, we obtain

$$A(t) = \prod_{\tau=0}^{t-1}\left(1 + a\frac{S(\tau)}{N}\right). \tag{13}$$

Substituting (5), (6), and (13) into the difference equations (1) and (2) for $S_Y(t)$and $S_E(t)$, we obtain

$$S_Y(t) = \left(1-a\frac{A(t-1)}{N}\right)S_Y(t-1) - (V-x), \tag{14}$$

$$S_E(t) = \left(1-a\frac{A(t-1)}{N}\right)S_E(t-1) - x. \tag{15}$$

Define

$$\Phi(t) = \prod_{\tau=0}^{t-1}\left(1 - a\frac{A(\tau)}{N}\right), \tag{16}$$

$$\Phi(t-1,k) = \prod_{\tau=k+1}^{t-1}\left(1 - a\frac{A(\tau)}{N}\right). \tag{17}$$

Then we obtain

$$S_Y(t) = \Phi(t)S_Y(0) - (V - x)\sum_{k=0}^{t-1}\Phi(t-1,k), \tag{18}$$

$$S_E(t) = \Phi(t)S_E(0) - x\sum_{k=0}^{t-1}\Phi(t-1,k). \tag{19}$$

Furthermore, defining

$$\Psi(t) = \sum_{k=0}^{t-1}\Phi(t-1,k), \tag{20}$$

we obtain

$$S_Y(t) = \Phi(t)S_Y(0) - (V - x)\Psi(t), \tag{21}$$

$$S_E(t) = \Phi(t)S_E(0) - x\Psi(t). \tag{22}$$

For deaths due to infection, substituting (5) and (6) into (7) and (8) yields

$$B_Y(t) = yaA(t)\frac{S_Y(t)}{N}, \tag{23}$$

$$B_E(t) = eaA(t)\frac{S_E(t)}{N}. \tag{24}$$

Number of elderly individuals who die from vaccination between the beginning and the end of period t, denoted by $C(t)$, is given by

$$C(t) = px. \tag{25}$$

Hence, the total number of deaths from infection and vaccination during period t, $B_Y(t) + B_E(t) + C(t)$, is obtained by substituting (23)-(25):

$$B_Y(t) + B_E(t) + C(t) = a\frac{A(t)}{N}(yS_Y(t) + eS_E(t)) + px. \tag{26}$$

Substituting (21) and (22) into (26), we obtain

$$B_Y(t) + B_E(t) + C(t) = E(t) + \left(p - \frac{a(e-y)}{N}A(t)\Psi(t)\right)x, \tag{27}$$

where

$$E(t) = a\frac{A(t)}{N}(y(\Phi(t)S_Y(0) - V\Psi(t)) + e\Phi(t)S_E(0)). \tag{28}$$

Since $E(t)$ is independent of x, we have the following result.

Lemma 1

$\boldsymbol{B_Y(t) + B_E(t) + C(t)}$ is linear in $\boldsymbol{x}$.

From (27), it follows that:

1. If the coefficient of x is positive, the optimal choice is $x = 0$.
2. If the coefficient of x is negative, the optimal choice is $x = 100$.
3. If the coefficient of x is zero, any $x \in [0,100]$is optimal.

Since the coefficient of x is

$$p - \frac{a(e-y)}{N} A(t)\Psi(t),$$

we obtain the following result.

Lemma 2

1. If $\boldsymbol{A(t)\Psi(t) < \frac{pN}{a(e-y)}}$, then $\boldsymbol{x = 0}$ is optimal.
2. If $\boldsymbol{A(t)\Psi(t) > \frac{pN}{a(e-y)}}$, then $\boldsymbol{x = 100}$ is optimal.
3. If $\boldsymbol{A(t)\Psi(t) = \frac{pN}{a(e-y)}}$, then any $\boldsymbol{x \in [0, 100]}$ is optimal.

NUMERICAL EXAMPLE 1

To clarify the implications of the model, we specify the parameters as follows.

The population consists of 10,000 individuals, of whom 80% (8,000) are young and 20% (2,000) are elderly. If a young individual is infected with COVID-19, 90% remain asymptomatic while the remaining 10% develop symptoms and die. In contrast, if an elderly individual is infected, 50% remain asymptomatic and the remaining 50% develop symptoms and die.

Total number of vaccinations per period is 100, and when an elderly individual is vaccinated, death occurs with probability 0.01%. When an individual becomes infected, that person contacts, on average, 0.02 other individuals regardless of age.

In this case, the threshold condition becomes

$$A(t)\Psi(t) = \frac{0.0001}{0.0000008} = 125.$$

Therefore, it follows that

1. If $A(t)\Psi(t) < 125$, then $x = 0$ is optimal.
2. If $A(t)\Psi(t) > 125$, then $x = 100$ is optimal.
3. If $A(t)\Psi(t) = 125$, then any $x \in [0{,}100]$ is optimal.

Starting from the initial values $S(0) = 9999$and $A(0) = 1$, we obtain

$$A(55)\Psi(55) \approx 121.996,$$

and the coefficient of x,

$$p - \frac{a(e-y)}{N} A(55)\Psi(55) \approx +2.40 \times 10^{-6} > 0.$$

At $t = 56$,

$$A(56)\Psi(56) \approx 125.331,$$

and the coefficient of x,

$$p - \frac{a(e-y)}{N} A(56)\Psi(56) \approx -2.65 \times 10^{-7} < 0.$$

Hence, we have

$$x^*(t) = \begin{cases} 0 & (t \leq 55), \\ 100 & (t \geq 56). \end{cases}$$

From the above, we obtain the following proposition.

Proposition 1

When, in terms of population share, infection fatality, and vaccination-related mortality, the effective impact is greater for the elderly than for the young, it is optimal—so as to minimize the total number of COVID-related deaths—to vaccinate the young first and subsequently vaccinate the elderly.

Intuition behind this result is as follows. Young individuals constitute a larger share of the population and play a significant role in the chain of transmission that drives future infection spread. Therefore, prioritizing vaccination for the young reduces the pool of susceptible individuals and suppresses future infection dynamics, thereby decreasing the opportunities for infection among the elderly. Since the model assumes that vaccination provides permanent immunity, vaccinating the young in the early stage effectively blocks transmission pathways over the entire future horizon.

Accordingly, Proposition 1 theoretically demonstrates that a policy that simply prioritizes the group with the higher fatality rate is not necessarily optimal.

NUMERICAL EXAMPLE 2

Assume the same parameter values as in Numerical Example 1, except that the probability that an elderly individual dies upon vaccination is 0.1.

In this case, performing calculations similar to those in Numerical Example 1 yields the threshold condition

$$A(t)\Psi(t) = \frac{0.0001}{0.00000008} = 1250.$$

Hence, we have

1. If $A(t)\Psi(t) < 1250$, then $x = 0$ is optimal.
2. If $A(t)\Psi(t) > 1250$, then $x = 100$ is optimal.
3. If $A(t)\Psi(t) = 1250$, then any $x \in [0,100]$ is optimal.

Value of t satisfying $A(t)\Psi(t) = 1250$ is approximately $t \approx 441$. However, this solution assumes the absence of a population constraint. In reality, since the total population is 10,000, infections would saturate before reaching this point. Therefore, we obtain the following proposition.

Proposition 2

When the effective impact—taking into account population shares, infection-induced mortality and vaccination-induced mortality—is greater *for the elderly than for the young, and when the number of deaths caused by vaccinating the elderly is sufficiently large, it is optimal, in order to minimize the total number of COVID-related deaths, that the elderly not be vaccinated.*

Proposition 2 indicates that if the mortality risk associated with vaccination for the elderly is sufficiently large, refraining from vaccinating the elderly may lead to a lower total number of deaths. This result arises from the trade-off that, while vaccination reduces infection risk, it also entails a certain probability of death. Even when the infection fatality rate among the elderly is high, if the direct mortality from vaccination exceeds the future benefits from infection suppression, vaccinating the elderly may actually increase total deaths.

In this model, the infection-suppression effect accumulates dynamically over time, so the choice of vaccination target has long-run dynamic implications. Nevertheless, when the vaccination-related mortality risk is sufficiently high, even after accounting for these dynamic benefits, vaccinating the elderly does not become optimal. This highlights the critical importance of evaluating vaccine safety in the optimal design of vaccination policies.

Therefore, Proposition 2 theoretically clarifies the limitation of determining vaccination priority solely on the basis of higher infection fatality rates when the risk of vaccine side effects is non-negligible.

CONCLUSION

This study developed a theoretical model that integrates the dynamic process of infection spread with the problem of vaccine allocation in a population composed of young and elderly individuals. A key feature of the model is that it explicitly incorporates both deaths caused by infection and mortality risks associated with vaccination, and evaluates, under this trade-off, how the age-specific allocation of vaccines in each period affects the total number of deaths.

The analysis shows that the optimal vaccine allocation is determined in a threshold manner depending on age-specific infection fatality rates, the population composition and the magnitude of vaccination-related mortality risk. In other words, whether vaccination should prioritize the young or the elderly is theoretically determined by the balance between differences in infection-induced mortality risk and the dynamic effect of vaccination in suppressing future infection spread.

The numerical examples in this study demonstrate that even when the infection fatality rate is higher among the elderly, if there exists a non-negligible risk of adverse effects from vaccination, prioritizing vaccination for the young—thereby suppressing the spread of infection—may ultimately minimize total deaths. Furthermore, when the probability of death associated with vaccinating the elderly is sufficiently large, it becomes desirable to refrain from vaccinating the elderly. These results provide important theoretical implications for the design of vaccine allocation policies in the context of infectious disease control. In real-world policy debates, prioritizing the elderly is often intuitively supported due to their higher fatality rates. However, this study theoretically shows that, once the dynamic chain of infection spread is taken into account, prioritizing younger individuals can contribute to reducing total mortality.

Nevertheless, the analysis has several limitations. First, the model assumes identical contact rates across age groups, whereas in reality younger individuals may have more social interactions. Endogenizing this feature could alter the optimal allocation conditions. Second, the model adopts the simplifying assumption that vaccination completely eliminates the risk of infection, while in practice vaccine effectiveness may change over time. Third, the analysis uses the number of deaths as the sole measure of social loss; it could be extended to a broader welfare analysis that incorporates wider social costs such as healthcare system congestion and economic disruption.

Future research directions include incorporating age-specific differences in contact behavior, accounting for uncertainty in vaccine effectiveness, and formulating the problem as an optimal control problem for government policy. Such extensions would provide a more realistic and comprehensive theoretical foundation for the design of vaccine allocation policies in response to infectious diseases.

In sum, this study offers a new perspective to the economics of infectious diseases and public policy analysis by dynamically capturing the interaction between two risks—death from infection and death from vaccination—and by presenting clear theoretical conditions for the optimal age-specific allocation of vaccines. Given the practical challenge of designing vaccination strategies under limited medical resources, the theoretical framework developed here can serve as a useful benchmark for policy evaluation.

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